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A Synthesis of Numerical Methods for Modeling Wave Energy Converter-Point Absorbers

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A synthesis of numerical methods for modeling wave energy converter-point absorbers

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Abstract:

During the past few decades, wave energy has received significant attention for harnessing ocean energy. Industry has proposed many topologies such as an oscillating water column, a point absorber, an overtopping system, and a bottom-hinged system. In particular, many researchers have focused on modeling the floating-point absorber, which is thought to be the most cost-efficient technology to extract wave energy. To model such devices, several modeling methods have been used such as the analytical method, the boundary-integral equation method, the Navier-Stokes equations method, and the empirical method. To assist the development of wave energy conversion (WEC) technologies, this report extensively reviews the methods for modeling the floating-point absorber.

Keywords: Wave Energy Converter; Wave Theory; Floating-Point Absorber; Numerical Modeling; Wave Body Interaction; Computational Fluid Dynamics
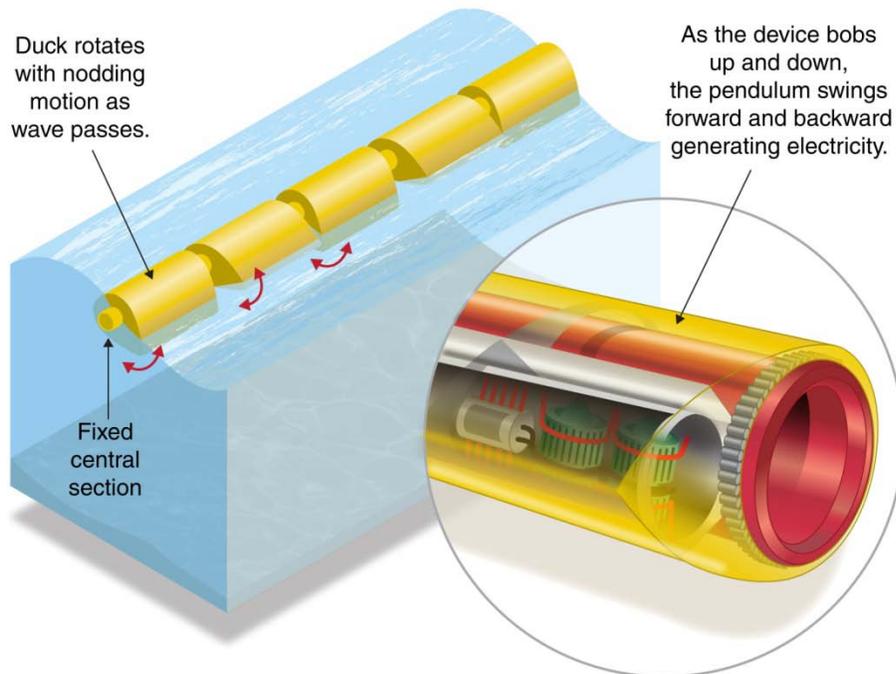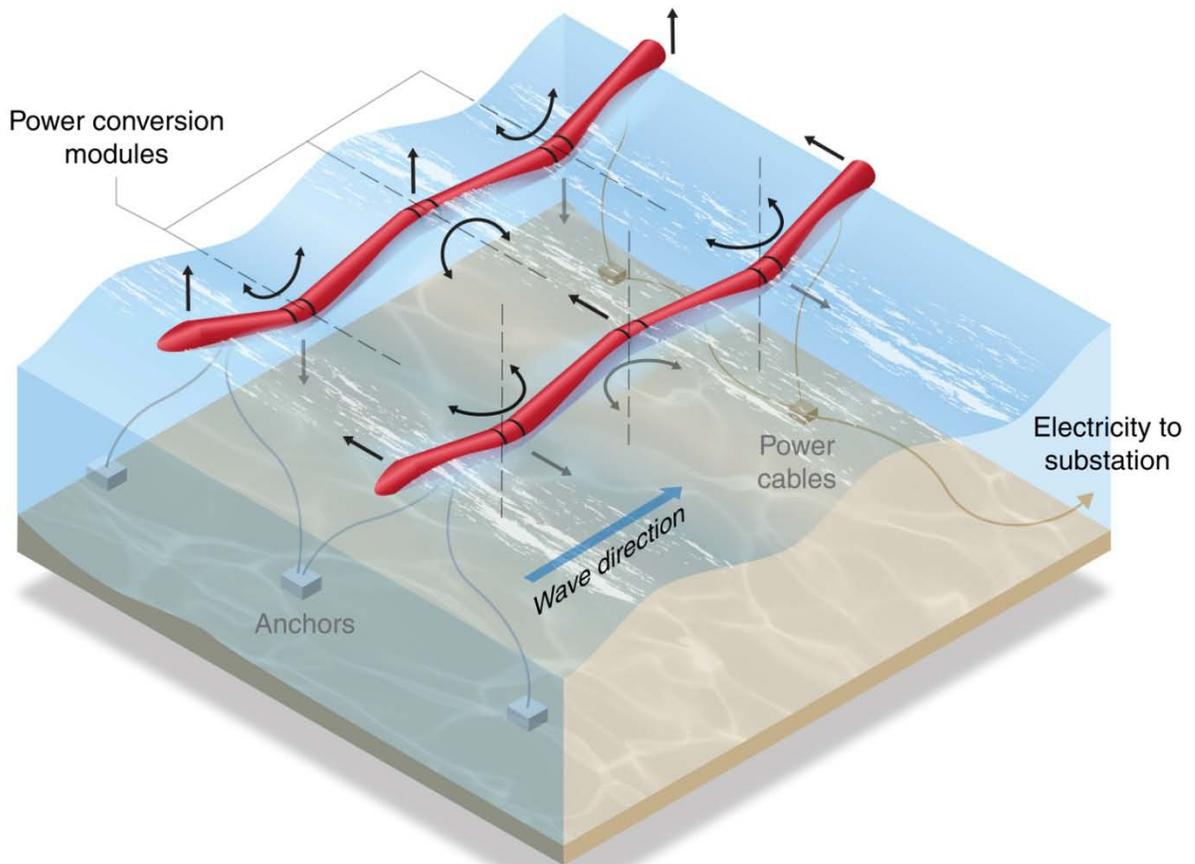
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1 Introduction

While development of modern wave energy converter dates back to 1799 [1], the technology did not receive worldwide attention until the 1970's when an oil crisis occurred and Stephen Salter published a notable paper about the technology in *Nature* in 1974 [2]. In the early 1980s, after a significant drop in oil prices, technical setbacks and a general lack of confidence, progress slowed in the development of wave energy devices as a commercial source of electrical power. In the late 1990s, awareness of the depletion of traditional energy resources and the environmental impacts of the large utilization of fossil fuels significantly increased, thereby facilitating the development of green energy resources. The development of the wave energy technology grew rapidly, particularly in oceanic countries such as Ireland, Denmark, Portugal, the United Kingdom, and the United States. Quite a few pre-commercial ocean devices were deployed. For example, a United States company, Ocean Power Technology, deployed one of their 150kW wave energy conversion (WEC) systems in Scotland in 2011 [3]. An Irish company, Wavebob, tested a one-quarter scale model in Galway Bay, Ireland, in 2006 [4]. In Denmark, the half scale 600kW Wave Star energy system was deployed at Hanstholm in 2009 [5], and a quarter and a half size model Wave Dragon was tested at Nissum Bredning in 2003 [6]. Furthermore, international organizations, such as the International Energy Agency and the International Electrotechnical Commission (IEC), are heavily involved in the development of wave energy devices. In 2001, the International Energy Agency established an Ocean Energy System Implementation Agreement to facilitate the coordination of ocean energy studies between countries [7]. In 2007, the IEC established an Ocean Energy Technical Committee to develop ocean energy standards [8].

1.1 Device Design

To date, there are more than one hundred prototypes of various WEC systems [7]. They can be sorted into a few types. According to their energy conversion principles, WEC systems include oscillating water column, overtopping, pitching, membrane, and point absorber. Most of these can be both bottom-mounted and floating. One may note that the focus of this paper is on floating-point absorbers.

The floating-pitching device, whether it is composed of a single body or a number of connecting bodies, has rotational freedom (Fig. 1). The device converts wave energy from its pitching motion. The principal axis for floating-pitching devices is either perpendicular (terminator) or parallel (attenuator) to the wave direction. Among various developments since the 1970s, Salter proposed one of the most significant inventions in 1974 [2], which became the well-known Salter's Duck (also referred to as the Edinburgh Duck). The Salter's Duck and PS Frog Mk 5 [9] are two examples of the terminator, and McCabe Wave Pump [10] and Pelamis [11] are examples of the attenuator. A wave farm consisting of 750 kW Pelamis devices was tested in northern Portugal in 2008 and another farm was deployed in Orkney, Scotland, in 2010.

Power conversion
modules

Electricity to
substation

Power
cables

Wave direction

Anchors

Duck rotates
with nodding
motion as
wave passes.

As the device bobs
up and down,
the pendulum swings
forward and backward
generating electricity.

Fixed
central
section

Figure 1. An illustration of the floating-pitching devices: (top) multiple degree of freedom and (bottom) single degree of freedom. *Illustration by Al Hicks, NREL*

The bottom-hinged pitching system includes a paddle, or a flap, that is connected to a hinge deflector on the seabed; the top of the device is generally above the free surface (Fig. 2). It is sometimes called a wave surge converter, because it converts wave energy from its pitching motion. Unlike the floating-pitching device, one end of the bottom-hinged device is fixed. That is, it shares a similar working principle as the floating terminators, e.g., Salter's duck in Fig.1b. Examples of the bottom-hinged pitching system include the swing mace, also proposed by Salter in the 1990s [12], and the Aquamarine Power Oyster [13], which was connected to the electrical grid and tested in Scotland in 2009 [14].
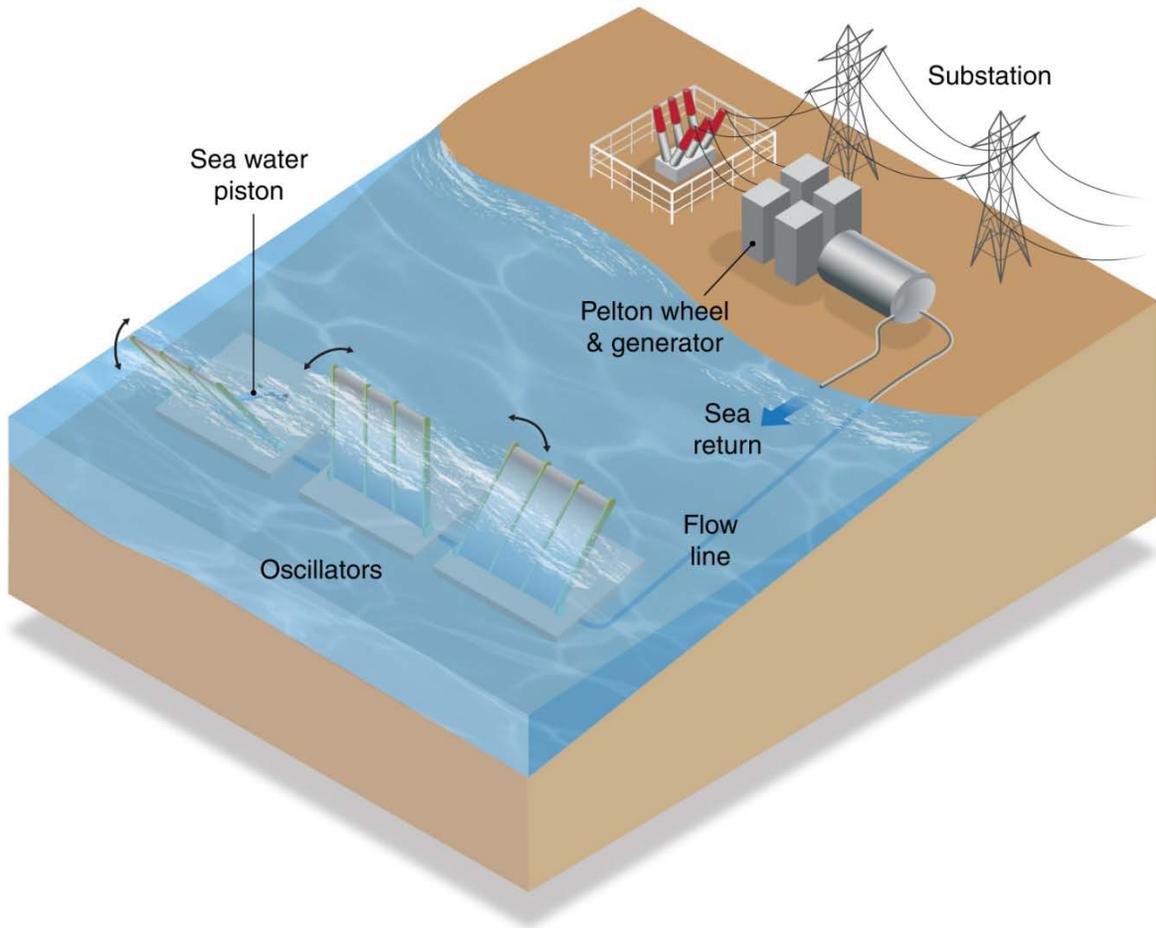
Figure 2. An illustration of the bottom-hinged system. *Illustration by Al Hicks, NREL*

The oscillating water column includes a special chamber with a bidirectional turbine inside (Fig. 3). One end of the chamber has an inlet that allows the incident wave to enter and the other end contains the turbine. The device converts wave power by utilizing the wave elevation to compress or decompress the air in a chamber. The compressed air goes through a bi-directional turbine. The turbine is a Wells turbine conceived by Professor Alan Wells of Queen's University, in Belfast, in late 1970. A wide variety of oscillating water column devices include the LIMPET (shoreline system) [15], Sakata (integrated into breakwater) [16], and the floating Ocean Energy buoy.
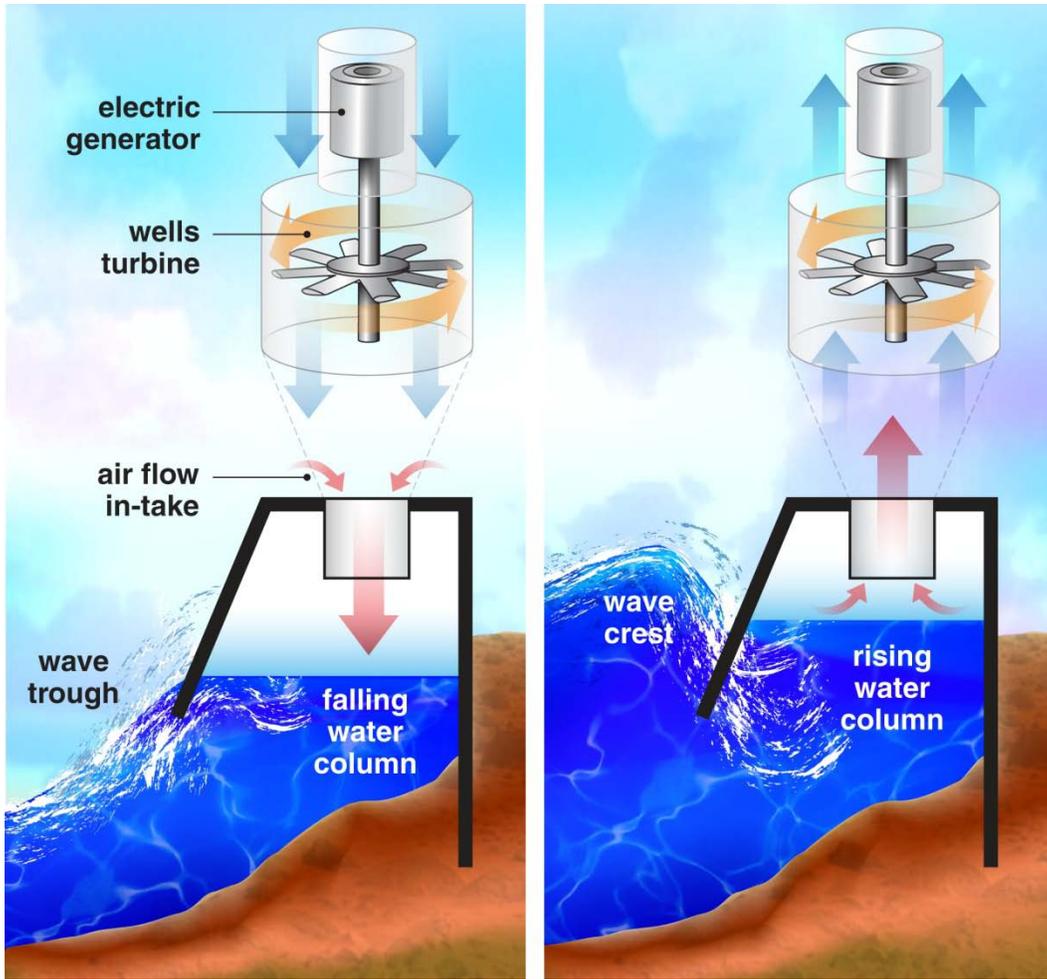
Figure 3. An illustration of the oscillating water column. *Illustration by Al Hicks, NREL*

The overtopping device includes a large structure that embraces the incident wave and an outlet with turbines inside the large structure (Fig. 4). The device converts wave power by utilizing the wave overtopping phenomenon to let the water fall through the outlet of the designed structure. When the water falls through the outlet, it passes one or more turbines similar to a traditional hydro dam; the potential energy is converted to electric power. The design involves both kinematic energy and potential energy in the conversion process. Examples include the Tapchan (shoreline system) [17], the Wave Dragon (offshore floating system) [18], and the Seawave Slot-Cone Generator (SSG), which was integrated into a breakwater [19].
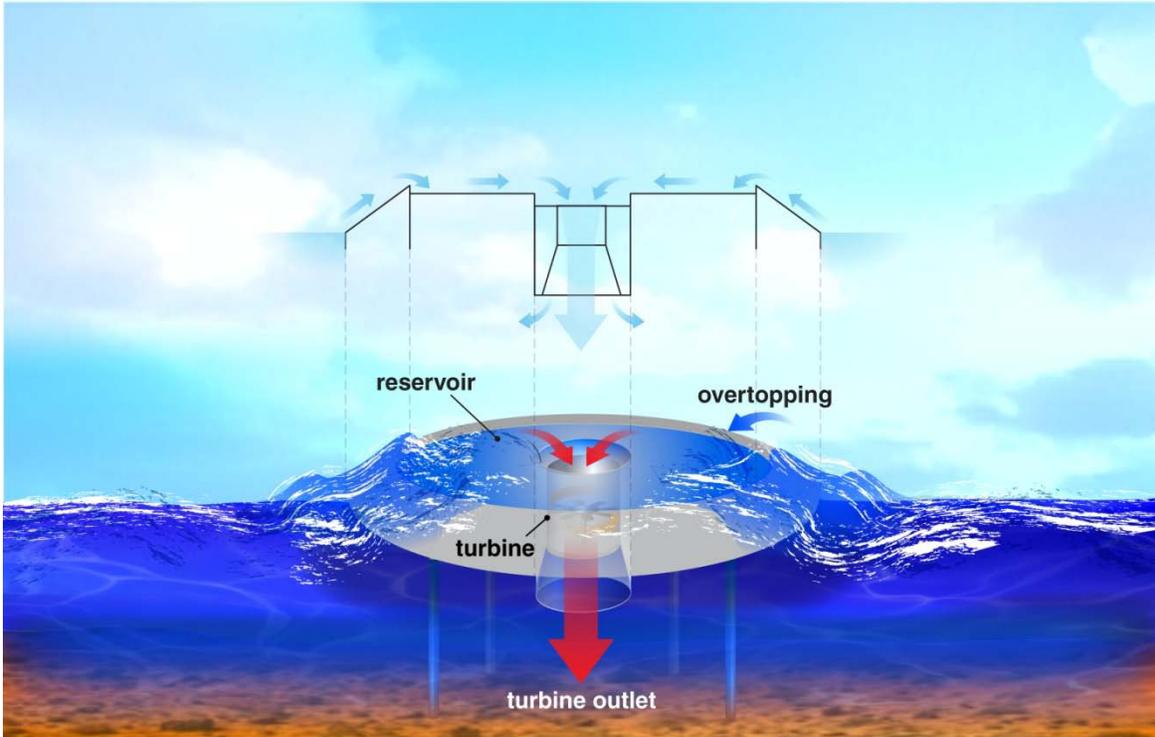
Figure 4. An illustration of the overtopping device. *Illustration by Al Hicks, NREL*

The membrane device includes a membrane structure and a power conversion system that can be a turbine, piezoelectric, or other system (Fig. 5). The device converts wave power by utilizing the dynamic pressure change during wave propagation (e.g., Lilypad [20]).
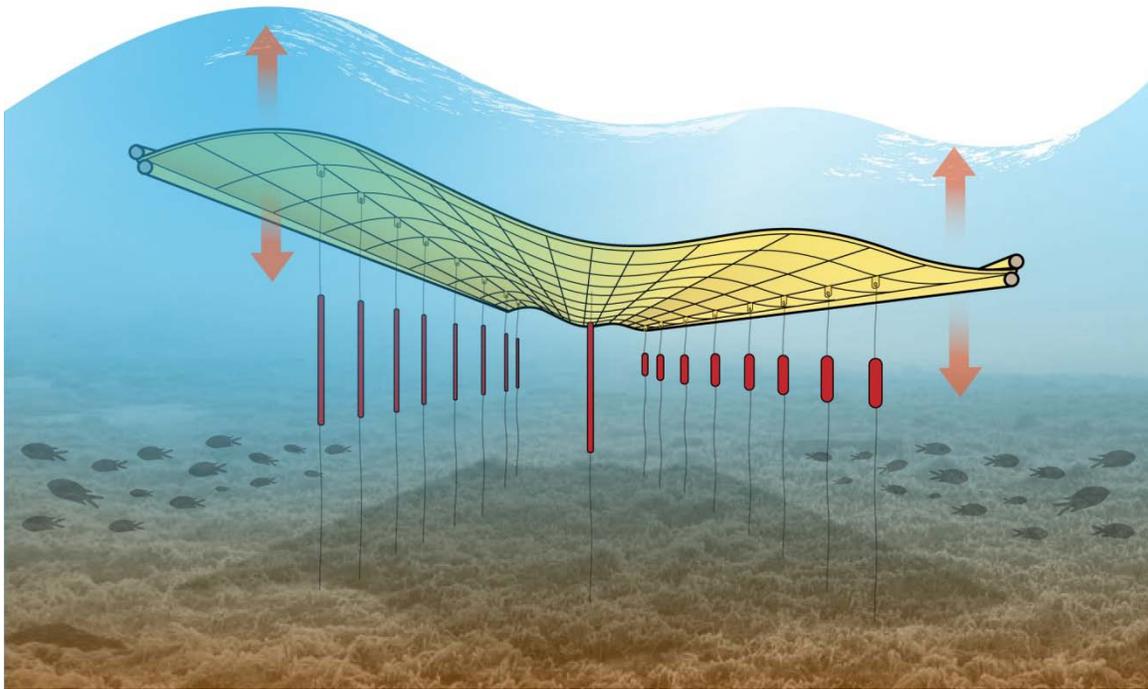
Figure 5. An illustration of the membrane device. *Illustration by Al Hicks, NREL*

A point absorber converts the wave energy by utilizing the heave motion (Fig.6). Typically, it is either a single-body that generates energy by reacting against a fixed seabed frame, or it is a multiple-body device that generates energy from the relative motion between the two bodies. There are many popular devices such as the OPT PowerBuoy [16], Wavebob [21], and Inter Project Service buoy [22,23,24]. In particular, two prototypes of the 40-kW OPT PowerBuoy were deployed, one in Santona, Spain, in 2008 and the other in Oahu, Hawaii, in 2009. Since the point absorber is the focus of the paper, its principles are described in greater detail in Section 2.
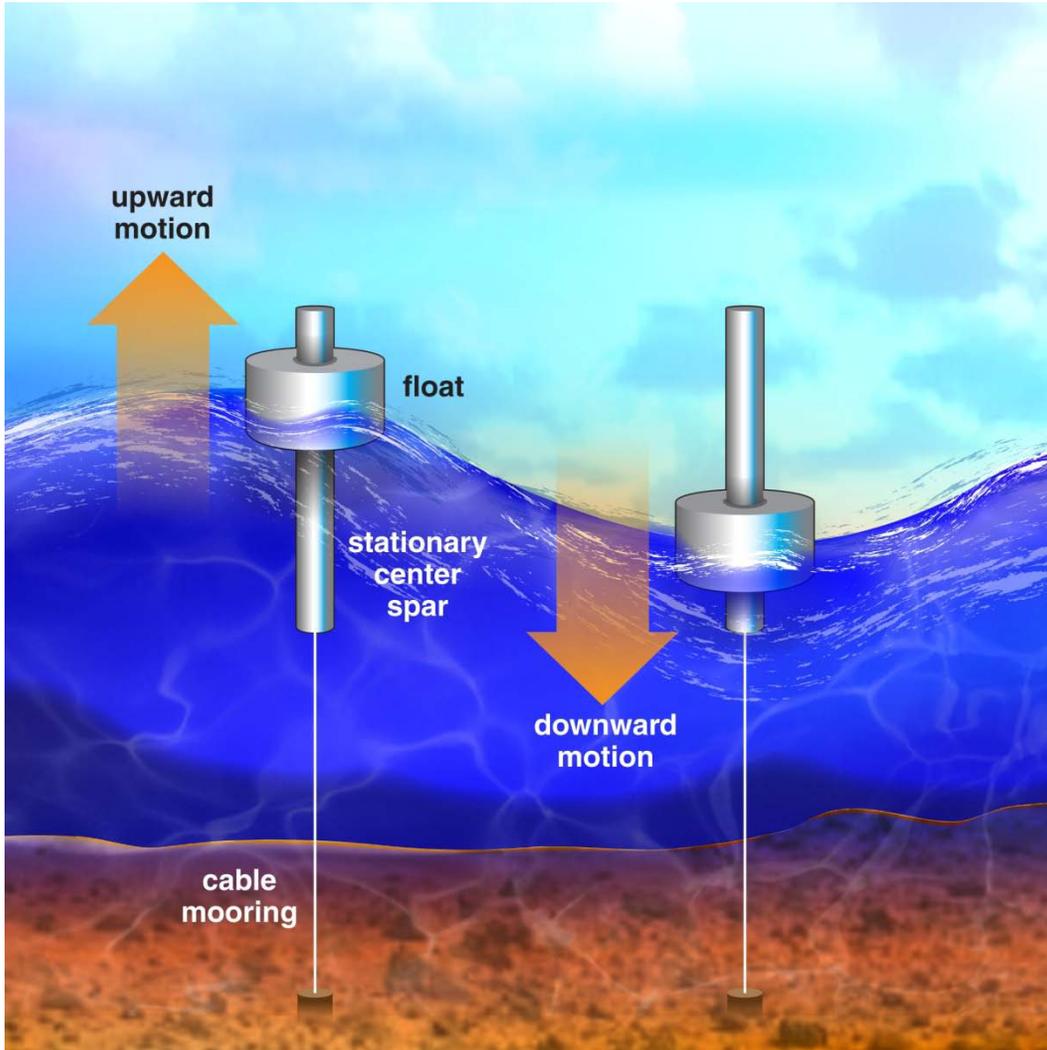
Figure 6. An illustration of the point absorber. *Illustration by Al Hicks, NREL*

1.2 Objective of this paper

Several literature reviews of WEC devices have been published providing information about various aspects of the technology. For example, Evans reviewed the analytical derivation of the absorber's motion [25]. Falcão provide a comprehensive overview of the history and status of the development of WEC systems [26]. Sarmento discussed the non-technical aspects of the technology during its commercialization path [27]. The U.S. Department of Energy (DOE) developed a database to document various technology characteristics [28]. Some concise reviews

reporting interim progress were written by Falnes, about theoretical limit calculation [29], and by Khan et al. [30], about interconnection issues. There also are introductory textbooks on this subject such as *Wave Energy Conversion* by Brooke [31], *Ocean Wave Energy* by Cruz [32], *Ocean Wave Energy Conversion* by McCormick [33], and *Ocean Waves and Oscillating Systems* by Falnes [34].

However, to date, there has been no systematic review of modeling methods. The rapid development has not led the industry to a commercial stage yet. From a pre-commercial stage to a full-scale commercial stage, a good understanding of device performance and reliability is required. This requirement has not been met yet. In addition to these non-technology barriers, it is believed that a lack of understanding and quantification of the device behavior is one of the main reasons industry development has been delayed.

During the development of wave energy technology, many methods for modeling WECs were developed during the technology's evolution. To understand point absorber behavior, precise modeling tools are needed to simulate the dynamics of the device, along with a well-defined combination of modeling methods. Technology developers, engineers, and researchers need a guideline of which methods to employ for a specific purpose. To facilitate the development of the technology and help to model the device behavior more cost-effectively, we conducted a systematic review of various methods for modeling WECs. The reviews are summarized in this paper. Since there are a great number of device types, we will focus on the floating-point absorber as an example. Additionally, we focus on the hydrodynamic modeling methods in this paper. Other modeling methods, such as electrical or control modeling, are not discussed, although they are important as well.

Specifically, this paper summarizes the numerical methods used for simulating point absorbers. After presenting the geometry and working principles of the point absorber, the discussion details the existing modeling work that uses analytical approaches, empirical methods, and numerical methods. In addition to these hydrodynamic modeling efforts, we also discuss other wave energy related modeling work, such as power take off, resource assessment, and environmental impacts. At the end, we compared the advantages and disadvantages of different hydrodynamic modeling methods.

2 Fundamentals of floating-point absorbers

The simulation of floating-point absorbers is a wave-body interaction topic. It requires knowledge of body dynamics and wave theory. Before we review the modeling methods for floating-point absorbers, we review the fundamentals of body dynamics and wave theory.

2.1 Point Absorber Dynamics
The motion of a point absorber in six degrees of freedom is plotted in Fig. 7. The dynamics of the device can be expressed by the following equations,

$$m_b \mathbf{a}_t = \mathbf{F}_d + \mathbf{F}_r + \mathbf{F}_{ext},$$
$$\mathbf{I}_g \mathbf{a}_\Omega + \mathbf{\Omega} \times \mathbf{I}_g \mathbf{\Omega} = \mathbf{M}_d + \mathbf{M}_r + \mathbf{M}_{ext},$$

(1)

where m_b is the mass of the body, $\mathbf{a}_t$ is the acceleration vector for the translation, $\mathbf{\Omega}$ and $\mathbf{a}_\Omega$ are the angular velocity and acceleration vectors, $\mathbf{I}_g$ is the moment of inertia tensor at the center of gravity, $\mathbf{F}$ and $\mathbf{M}$ are the resulting force and moment acting on the body. The subscripts 'd' and 'r' represent the hydrodynamic excitation component and the restoring component, respectively. The subscript 'ext' represents the external forcing terms induced by the mooring system and the Power Take Off (PTO) mechanism. Faltinsen [35] comprehensively reviewed the wave-induced motion and loads on offshore structures due to linear, nonlinear, and viscous effects.
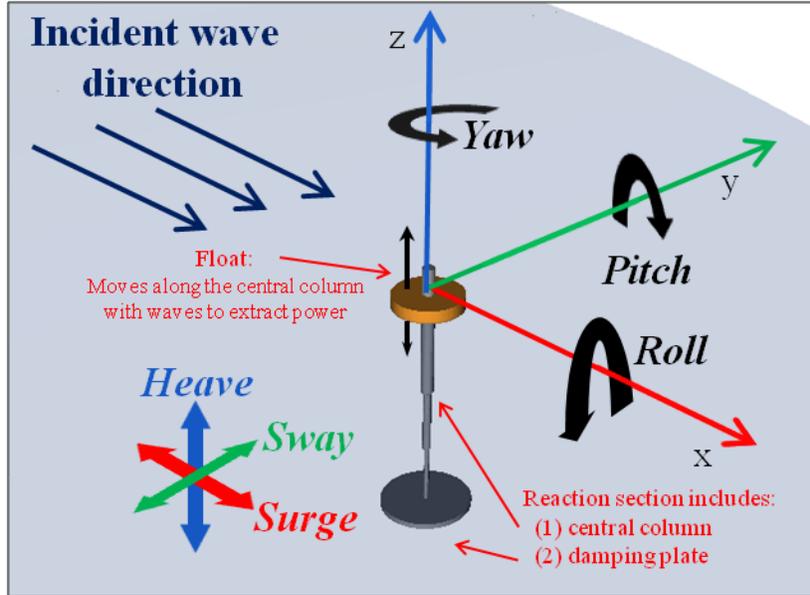
Figure 7. The translation and rotation of the body (6 degrees of freedom)

To generate power, this type of WEC system generally uses a two-body heaving system, which converts energy from the relative motion between the two oscillating bodies, a float section, and a reaction section. The float is only allowed to move freely in one degree of freedom, with respect to the reaction section (generally in heave). When the PTO component was included, the additional PTO force F_{PTO} was considered. A mass-spring-damper system can be used to represent the PTO mechanism, and PTO force becomes

$$F_{PTO} = -C_{PTO}u_{rel} - kz_{rel},$$

(2)

Where z_{rel} and u_{rel} are the relative motion and velocity of the two bodies, k is the spring stiffness, C_{PTO} is the power absorption (damping) coefficient. The equations of motion for the float and reaction sections in their relative motion direction become

$$\text{Float: } m_F a_F = Fz_F + F_{PTO},$$
$$\text{Reaction: } m_R a_R = Fz_R - F_{PTO}.$$

(3)

Where a is the acceleration in their relative motion direction, m is the mass of the body, Fz_F and Fz_R are the forces on the floater and reaction plate. The generated power from PTO is defined as

$$P_{PTO} = C_{PTO} u_{rel}^2,$$

(4)

which is proportional to the square of the relative translational velocity of the two sections. To optimize the power generation, different control strategies can be applied to the system. An introduction to latching phase-control was given in [36], and a review on the implementation of a spring in phase control, which is called reactive phase-control, was described in [26].

2.2 Free surface wave theory

The free surface waves are a representation of various forces, e.g., the wind or a ship, acting on and deforming the fluid surface against the action of gravity and surface tension. Naturally, waves occur in all sizes and forms. Depending on their magnitude, they act on the fluid along with other environmental conditions, such as bottom topography, temperature and fluid density. In general, waves can be described as linear or nonlinear, regular or irregular, unidirectional or omni-directional. The analytical solutions of free surface waves were derived based on the potential flow method, where the flow is assumed to be incompressible and irrotational. The velocity potential $\phi(\mathbf{X},t)$ therefore, is obtained by solving the Laplace equation

$$\nabla^2 \phi = 0,$$

(5)

where the boundary condition at the seabed is given as

$$\frac{\partial \varphi}{\partial n} = 0.$$

(6)

By following [37,38], the kinematic free surface boundary condition (KFSBC) is

$$\frac{D\eta}{Dt} = w \text{ at } z = \eta,$$

(7)

where $D/Dt=(\partial/\partial t)+\nabla\phi\,\nabla$ is the material derivative, w is the vertical flow velocity, and η is the free surface elevation. The dynamic free surface boundary condition (DFSBC) assumes the pressure is continuous over the free surface. The DFSBC is then obtained by following the Bernoulli's equation,

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + g\eta = 0 \text{ at } z = \eta,$$

(8)

where g is gravity.

The exact solution of the above equation is very complex. We start the review at the linear wave theory, which is obtained using simplified assumptions. Linear wave theory, also referred to as

small amplitude wave theory and airy wave theory [39,40], is the simplest solution (first-order approximation) for the flow field. For progressive gravity waves of period T, amplitude H, and water depth d, linear wave theory assumes H is much smaller than d and wavelength λ, and the boundary conditions are linearized. The free surface boundary conditions are then reduced to

$$\text{KFSBC: } \frac{\partial \phi}{\partial z} - \frac{\partial \eta}{\partial t} = 0 \text{ at } z = 0, \tag{9}$$

$$\text{DFSBC: } -\frac{\partial \phi}{\partial t} + g\eta = C(t) \quad \text{at } z = 0,$$

where both the linearized KFSBC and DFSBC are defined at the mean free surface. The flow properties of linear waves can be expressed as

$$\text{Wave elevation: } \eta = \frac{H}{2}\cos[k(x - ct)], \tag{10}$$

$$\text{Velocity potential: } \phi = \frac{gH}{2\omega}\frac{\cosh[k(z + d)]}{\cosh(kd)}\sin[k(x - ct)], \tag{11}$$

$$\text{Dispersion relation: } c^2 = \frac{\omega^2}{k^2} = \frac{g}{k}\tanh(kd), \tag{12}$$

$$\text{Horizontal flow velocity: } u = \frac{\pi H}{T}\frac{\cosh[k(z + d)]}{\sinh(kd)}\cos[k(x - ct)], \tag{13}$$

$$\text{Vertical flow velocity: } w = \frac{\pi H}{T}\frac{\sinh[k(z + d)]}{\sinh(kd)}\sin[k(x - ct)], \tag{14}$$

where x and z are the horizontal and vertical positions, k ($=2\pi/\lambda$) is the wave number, c ($=\lambda/T$) is the wave speed, and ω is the wave frequency. Depending on the water depth, the waves can be categorized as shallow water waves when $d/\lambda < 1/20$, as intermediate-depth waves when $1/20 < d/\lambda < 1/2$, and as deep water waves when $d/\lambda > 1/2$.

When the wave amplitude is not small, the waves become nonlinear. The linearized KFSBC and DFSBC are not satisfied, and the nonlinear KFSBC and DFSBC can be written as

$$\text{KFSBC: } \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} = -\frac{\partial \phi}{\partial z} \quad \text{at } z = \eta(x,t), \tag{15}$$

$$\text{DFSBC: } \frac{P}{\rho} + \frac{1}{2}[(\frac{\partial \phi}{\partial x})^2 + (\frac{\partial \phi}{\partial z})^2] - \frac{\partial \phi}{\partial t} + gz = C(t) \quad \text{at } z = \eta(x,t),$$

The nonlinear solution can only be obtained by solving Eqs. (5~8). The nonlinear higher-order solution is generally the superposition of additional components, which are at higher wave frequencies, on the fundamental linear wave-theory terms. There have been numerous nonlinear

higher-order wave theories developed since the 1800s. The Stokes theory [41,42] is the most well known and studied. An extension of the fifth-order Stokes waves was presented in [43], and it has been widely used in engineering applications. The wave profile and the velocity potential of the fifth-order Stokes waves are given as

$$\text{Wave elevation: } k\eta = \sum_{n=1}^{5} A_n \cos[nk(x-ct)], \tag{16}$$

$$\text{Velocity potential: } \frac{k\varphi}{c} = \sum_{n=1}^{5} C_n \cosh[nk(z+d)]\sin[nk(x-ct)]. \tag{17}$$

More details and the coefficients A_n and C_n are described in [43]. The applicability of various wave theories was studied in [44,45], and the figure plotted by Le Méhauté is shown in Fig. 8. It illustrates the validity limitation of these approximation approaches for different wave conditions. A review of the introduction of wave theories and the applications to wave load prediction on offshore structures is provided in [46], and a more comprehensive review of the theories and the applications of linear and nonlinear waves is given in [47].

In the real world, waves are not monochromatic. There are a large variety of waves with different frequencies, phases, and amplitudes. For an adequate description of the free surface, a large number of waves must be superimposed together. Usually, we use spectra to describe them. Furthermore, the behavior of waves is rather random. In this situation, the waves are called irregular waves. The random wave field can be approximated by using an infinite sum of sinusoidal components propagating with various wavelengths. Pierson-Moskowitz spectrum [48] and Jonswap spectrum [49] are the two most widely used spectra profiles. Refer to textbooks such as [47] and [50] for wave theory and [35,46,51] for wave-body interactions.
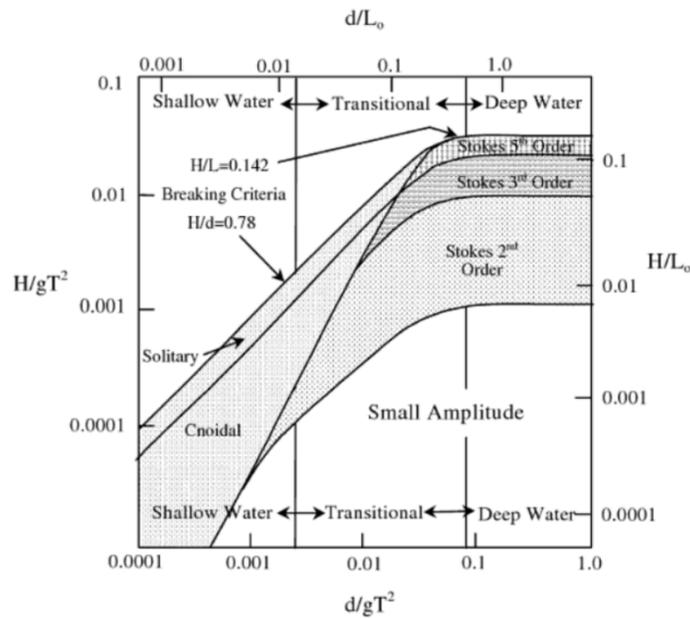

Figure 8. The applicability of various wave theories [44,46]

3 Modeling Methods

The main purpose of hydrodynamics modeling of floating-point absorbers is to understand the interaction between waves and floating bodies, as well as the dynamic response of the mooring system. The dynamics of the floating-point absorbers can be calculated by solving Eq. (1). Theoretically, the hydrodynamic forces that affect the absorber motion can be obtained analytically[2], empirically or numerically, depending on the modeling purpose, the operational condition, and the detailed design. The boundary integral equation method (BIEM) and the Navier-Stokes equation method (NSEM) are the two numerical methods that are often used for numerical modeling. These modeling methods evolved for modeling offshore floating structures, such as offshore platforms and ships. Therefore, we apply the methods of offshore floating structures when we review modeling methods for floating-point absorbers.

3.1 Analytical Methods

Analytical methods can be sorted, with or without detailed descriptions, by WEC systems. The first analytical study of a floating-point absorber's power output focused on maximum efficiency, without the detailed description of the device, in 1975 [52,53]. The linearized equation of motion for the floating-point absorber device in heave is

$$(m + m_r)\ddot{z} + R_r\dot{z} + Sz = F_{PTO} + F_{ext}, \tag{18}$$

where m is the mass of the body, m_r is the hydrodynamic coefficient of added mass, R_r is the radiation damping coefficient, S is stiffness of hydrostatic restoring force, F_{PTO} is the vertical force due to power-take-off mechanism, and F_{ext} is excitation force. The maximum energy absorption P_{max} for different incident wave frequencies can be reached when the oscillating body system is at resonance. Based on the linear deep-water wave assumptions, the time averaged optimal power absorption at resonance is

$$P_{max} = \frac{1}{8R_r}|F_{ext}|^2. \tag{19}$$

An absorption width L_{max} is defined as

$$L_{max} = \frac{P_{max}}{J} = \frac{\lambda}{2\pi}, \tag{20}$$

$$J = \frac{\rho g^2 T H^2}{32\pi}, \tag{21}$$

where J is the wave energy flux for linearized deep water waves. Note that Evans [54], Mei [55] and Newman [56] also independently derived the same result in Eq. (20) using similar approaches in 1976.

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[2] The analytical method can be derived by an equivalent linearization of the empirical calculation. Therefore, the former can be viewed as a special case of the latter.

The maximum power that can be absorbed from waves and the absorption width is derived based on linear wave theory and consideration of wave radiation. Budal and Falnes [57] proposed another upper limit (also referred to as Budal's upper bound) for wave power that is absorbed by a given submerged body volume V

$$\frac{P_u}{V} < \frac{\pi \rho g H}{4T},$$ (22)

where the diffraction effect is neglected, and the size of the body is assumed to be small compared with the wave length. The two curves represent the theoretical prediction of the maximum wave energy that can be captured by a semi-submerged heaving body with optimum heaving amplitude.

The power output for a practical device is below the two upper limits, and the maximum power that can be absorbed is about half of P_{max}, when a phase control method is applied [29]. In reality, the wave condition is often beyond linear wave theory, and the viscous damping effect can be significant. More details on the prediction of maximum converted useful power, energy loss due to viscous damping, and PTO, as well as the optimal control of the WEC systems can be found in [34]. Moreover, some engineering constraints, such as mooring lines, are unavoidable. Evans [58] and Pizer [59] showed that additional constraints may further reduce the motion response of the device.

In addition to a single body, this method has been used to study arrays of WEC systems. For example, through the use of linear theory and multiple-scale approximation, Garnaud and Mei [60] presented an analytical solution to the wave energy extraction using infinite strips of buoys and a circular array. The study showed that an array of heaving wave-absorbing buoys can have a broader range of wave frequencies than a single large buoy, with good energy-absorbing efficiency.

All of these studies either use existing calculations of the coefficients or assume that the optimal coefficients can be obtained. In the real world, the geometry is more complicated and the scale is different. The addition of the mass and damping coefficient should be re-evaluated. A method for calculating the added mass and damping coefficient for offshore engineering applications was proposed by Yeung [61]. The paper suggests a method to calculate the coefficient using the Eigen functions. The proposed method has been widely used in ocean engineering problems, especially for an array of cylinders [62] [63]. In the past decade, many researchers have used this method to calculate the power output of point absorbers [64] [65] [66].

In general, analytical methods are very efficient for providing a quick performance estimation of floating-point absorbers using simple geometry. If one needs to conduct a more detailed analysis, or to study devices with a more complicated geometry, analytical methods are not appropriate.

3.2 Empirical Prediction

In addition to the methods presented above, for sinusoidal wave conditions, the empirical solution is an acceptable approach as well. The most popular empirical approach is Morison's equation. The method was developed by four authors, J.R. Morison, M.P. O'Brien, J.W. Johnson and S.A. Schaaf, at the University of California, in 1950 [67]. The approach presumes that the

force exerted by unbroken waves on a cylinder is the sum of the drag and inertial force components. These loads are determined based on the flow velocity and acceleration, and Morison's equation assumes that the flow acceleration is almost uniform at the location of the body. When $D/\lambda > 0.2$, the inertia force is no longer in phase with the flow acceleration, the effects of wave radiation and diffraction should be taken into account. Therefore, the Morison's equation is not applicable for larger bodies. The empirical force on the body, obtained from Morison's equation, is calculated by using appropriate drag C_d and inertia C_m coefficients, where $C_m = 1 + C_a$, and C_a is the added-mass coefficient. The simplest form for the instantaneous in-line force on a vertical cylinder can be given as

$$F = \int_L (1 + C_a) \rho \frac{\pi D^2}{4} \dot{u} - C_a \rho \frac{\pi D^2}{4} \dot{u}_b \, ds + \int_L \frac{1}{2} C_d \rho D |u - u_b| (u - u_b) \, ds \tag{23}$$

in which ρ indicates the density of sea water, u is the flow velocity, L is the length of the cylinder, and u_b is the velocity of the body. More details of the use of Morison's equation on predicting the force on a fixed or an oscillating cylinder, as well as the appropriate values for the empirical hydrodynamic coefficients, can be found in [46,68].

Many modifications and applications of Morison's equation have been proposed over the years. In particular, by introducing a buoyancy term and by changing the floating body volume with time, the Morison type approach has successfully predicted the wave load of a submerged horizontal cylinder [69]. Moreover, a hybrid model has been developed by Zhang et al. [70]; it combined the Morison's equation with a second-order wave theory in which the effects of the interaction between the wave components of irregular waves were considered up to the second-order. An application of this hybrid model, with six degrees of freedom, to a mini-tension leg platform was presented in [71].

For WEC system modeling, Brekken et al. [72] applied a Morison-type approach to calculate the excitation force on an oscillating body for optimal control analysis. Elwood et al. [73]. also used the dynamic response modeling of a Taut-moored dual-body WEC device (SeaBea I). They coupled the Morison type calculation, with a finite element method based commercial code OrcaFlex, and applied the method to analyze the dynamics of the mooring system.

3.3 Boundary Integral Equation Methods

BIEM is an advanced potential flow method that can be used for handling more complicated geometries. Given that the velocity potential of the boundary value problem is solved by discretizing along the boundary surfaces, BIEM reduces the dimension of the problem so that it can be solved numerically. The governing equation, Eq. (5), is formulated in a boundary integral equation form, which is derived by introducing a Green's function G ($\mathbf{p}$,$\mathbf{q}$), where $\mathbf{p}$ and $\mathbf{q}$ represent the field point and the source point, respectively. The boundary integral equation obtained through the use of Green's third identity gives

$$\alpha(\mathbf{p})\phi(\mathbf{p}) + \int_S \phi(\mathbf{p}) G_n(\mathbf{p},\mathbf{q}) - \phi_n(\mathbf{p}) G(\mathbf{p},\mathbf{q}) ds = 0, \tag{24}$$

where S indicates the boundaries of the fluid domain, $\alpha(\mathbf{p})$ is the internal angle formed at the boundaries, $G_n = \nabla G \cdot \mathbf{n_q}$ and $\phi_n = \nabla \phi \cdot \mathbf{n_q}$, with $\mathbf{n_q}$ being the normal vector at $\mathbf{q}$. By using Dirichlet- and Neumann-type boundary conditions, the potential flow field can be obtained by solving the resulting system of linear equations.

After solving the potential flow field, the pressure on the body surface is evaluated using Bernoulli's equation,

$$p = -\rho \left(\phi_t + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gz \right).$$
(25)

The forces $\mathbf{F}$ and moment $\mathbf{M}$ acting on the body are calculated by integrating the pressure on the immersed body surface S_B,

$$\mathbf{F} = \int_{S_B} p\mathbf{n}ds; \mathbf{M} = \int_{S_B} p\mathbf{n} \times \mathbf{r}ds,$$
(26)

where $\mathbf{n}$ indicates the unit normal vector on the body surface, and $\mathbf{r}$ is the distance vector from the center of rotation.

Regarding simulation of the interaction between waves and floating bodies, two types of approaches have been used for modeling. One follows a weak nonlinear theory that uses a perturbation expansion for the solution and specifies the boundary conditions at the mean free-surface and body-surface through the use of Taylor series expansion. The other method uses a fully nonlinear time-domain approach.

The weakly nonlinear approach has the advantage of having the same coefficient matrix for the system of equations solved at every time step. One can solve the problem in the time domain [74] or in the frequency domain, which is the method used in the panel code WAMIT [75]. In the frequency domain calculation, radiation, diffraction, and excitation forces are calculated through the use of the linear superposition principle, which is limited to weak nonlinear waves [76]. The complexity of the method increases significantly for higher order schemes. Studies, for example, that used this weak nonlinear approach for wave radiation and diffraction problems can be found in [71,77,78,79,80,81,82]. A discussion on the use of linear and second-order approaches for predicting the wave excitation of floating bodies was given in [83].

The weak nonlinear type approach has been widely used for modeling the dynamics of heaving or pitching WEC systems. For example, the BIEM-based code, WAMIT, was applied to obtain the frequency-dependent hydrodynamic coefficients in the study of the pitching plate [84] of the PS Frog Mk. 5 WEC device [9], and for heaving axisymmetric bodies in irregular waves [85]. It also was adopted by Cruz and Salte [86], to model a modified version of the Salter's Duck, in which the numerical results were compared with those measured from experimental tests. Moreover, several researchers have used WAMIT in the study of control strategies for a heaving WEC device in irregular seas [87] and for the analysis of array systems for floating-point absorbers [88,89]. In these, the interaction effect was considered through the use of a multiple-scattering method. A similar approach was adopted by Vicente et al. [90] to analyze the

dynamics of the array system of WEC buoys, with inter-body connections and bottom slack-mooring lines.

The fully nonlinear time domain approach for wave analysis was first proposed by Longuet-Higgins and Cokelet, using a Mixed Eulerian-Lagrangian (MEL)-type method for modeling surface waves [38,91]. Fully nonlinear boundary conditions were applied on the instantaneous water free surface and body surface, and the free surface was computed using a tracking method. With BIEM, the computational domain is discretized along the domain boundaries, including the free surface and the body surface. Therefore, the coefficient matrix for the system of equations should be updated at every time step, and the method is only solved in the time domain. A high order Runge-Kutta scheme is often used to update the time varying free surface and body surface. The nonlinear time-domain approach is more accurate for modeling the highly nonlinear interaction between waves and floating bodies. Using this approach, researchers successfully predicted the nonlinear irregular wave radiation [92] and diffraction [93] of vertical cylinders, as well as the ventilating entry of surface-piercing hydrofoils [94].

Additionally, modeling a floating-point absorber in a numerical wave tank requires a boundary that generates waves and absorbs reflected waves at the same time, and a boundary that absorbs outgoing waves. Over the years, many efforts have been made to develop a numerical method to simulate these types of boundaries, without creating additional numerical disturbances. A review on unbounded domains, with various types of artificial boundary conditions and absorbing methods, was given in [95]. Most of the studies between the 1970s and the 1990s were based on potential flow theory. In particular, a sponge-layer method was proposed by Israeli and Orszag [96], in which an artificial damping layer (zone) was implemented to attenuate the outgoing wave before it reached the outer boundary. Similar approaches also were proposed in [97,98].

3.4   Navier-Stokes Equation Methods

The viscous effects of boundary layer separation, turbulence, wave breaking, and overtopping are important to the prediction of hydrodynamic forces of the devices. The potential flow method cannot capture them. Therefore, modelers often turn to a fully viscous solution by implementing the Navier-Stoke Equation Methods (NSEM). The velocity vector $\mathbf{U}$ and pressure p of the incompressible flow field are obtained by solving the continuity equation and the momentum equations, which are given as

$$
\nabla \cdot \mathbf{U} = 0,
$$
$$
\rho(\mathbf{U}_t + \mathbf{U} \cdot \nabla \mathbf{U}) = -\nabla p + \mathbf{F}_b + \nabla \cdot \mathbf{T},
$$

(27)

where $\mathbf{U}_t$ is the time derivative of $\mathbf{U}$, $\mathbf{F}_b$ is the body force vector (such as gravity), and $\mathbf{T}$ indicates the stress tensor. The computational domain can be discretized using a finite difference/volume/element method, and the resulting system of linear equations can be solved directly, or preferably, through iterations using an appropriate linear equation system solver. A well known Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm [99] and its variants have been widely used for solving Navier-Stokes equations. More details on the numerical discretization methods, as well as the physical phenomena, are given in [100,101]. The force and moment calculation is similar to that applied in the potential flow theory, except that the frictional force also is included in the calculation.

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Basically, using the Navier-Stokes equation-based method for modeling the hydrodynamics of floating bodies in waves involves the calculation of the free surface in a numerical wave tank and the simulation of the turbulent flow. In general, two types of approaches have been applied for predicting the free surface. One is the surface tracking method, which treats the free surface as a sharp boundary and updates it with time [102]. However, the method does not have the ability to model wave breaking and overtopping. The other is the interface-capturing method, where the simulation is performed on a grid that includes both air and water phases. Marker-and-Cell (MAC) method [103], Volume Of Fluid (VOF) method [104], and the level set approach [105,106] are the most often used interface-capturing methods. In particular, the VOF method is the most widely used for commercial and open source Computational Fluid Dynamics (CFD) codes. Review of these methods and their application to wave hydrodynamics can be found in [101,107]. The artificial damping layer method also is widely used in the Navier-Stokes equation-based approaches for absorbing the outgoing and reflecting waves in the wave-body interaction simulation. For example, Lara et al. [108] combined the sponge-layer method, with an internal-wave maker method [109], to model the wave generating-absorbing boundary and simulated the interaction between waves and permeable structures.

In general, four types of numerical methods are applied for turbulent flow modeling, including direct numerical simulations (DNS), large eddy simulations (LES), Reynolds-averaged Navier-Stokes (RANS) methods, and detached eddy simulations (DES). DNS gives a complete description of turbulent flow by directly solving the Navier-Stokes equations. However, the mesh resolution requirement, and the corresponding computational cost for DNS, is very high. Therefore, DNS is often viewed as a research tool in the study of turbulence [110]. LES, on the other hand, is a turbulent model that directly solves those time-dependent flow equations for mean flow and large scales of turbulence, and models the effect of the small scales of turbulence. LES is less computationally costly when compared to DNS. A review of LES was given in [111]. An example of applying LES to a vortex-induced vibration simulation for a truss spar geometry can be found in [112]. The computation of LES remains too costly to merit consideration for most floating body dynamics problems. Therefore, the RANS method is the most widely used scheme for modeling turbulent flow. The unsteadiness of the turbulence is time averaged and the computation of turbulent flow relies on simplified engineering approximations. DES is a hybrid method that combines the RANS and LES methods. It handles the near wall region, with a RANS-type model, and treats the rest of the flow field using a LES-type method. Different types of RANS models are designed for different turbulent flows, and the physical phenomena and the basic assumptions for each turbulence model were comprehensively reviewed in [113].

An example of applying a RANS model for analyzing the wave load of a jack-up platform under freak waves was presented by Moctar et al. [114], where the VOF method was adopted for the free-surface calculation and was coupled with a finite element model for the structure's stress analysis. The study demonstrated the significance of using a Navier-Stokes type approach, where the effects of wave run-up on the platform leg and the impact-related wave loads on the hull were all considered. Their study also showed that the difference between the shear and overturning moment for the platform under freak waves calculated from the Morison's approach, and those obtained from the RANS calculation, can increase up to 25%, especially for larger and breaking waves.

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A few studies on the fluid-structure interaction of WEC systems were performed using the Navier-Stokes-type approaches. A RANS code (COMET) was applied by Agamloh et al. [115] to model the dynamics of a single buoy and a double buoy system in waves. The PTO mechanism was considered as the damping system based on the energy that was captured. Studies on the Wave-driven, Resonant, Arcuate-action, Surging Point-Absorber (WRASPA) were presented by Bhinder et al. [116,117]. To investigate the hydrodynamics of a floating-point absorber wave energy system in real seas, Yu and Li performed a series of studies using a RANS model. In particular, for operational wave conditions, a two-body, heave-only floating-point absorber system was modeled. The power take off was represented using a mass-spring-damper system, and the maximum power that this particular device generated, with a constant value spring-damper system, was examined [118].

Another method being considered for solving the Navier-Stokes equation is the Smooth Particle Hydrodynamics (SPH) [119]. SPH is a mesh free Lagrangian method developed during the past couple of decades [120]. It uses a smoothing kernel to approximate a delta function, with many sampling points in the fluid domain.

4 Discussion

In Section 3, we have summarized the existing numerical methods for simulating a floating-point absorber resulting in the review of existing analytical methods including, Morison Equation methods, boundary BIEM, and NSEM. Here, we provide further insights into these methods.

In terms of cost-effectiveness, selecting a method for simulating floating-point absorber WEC devices involves determining the capability of the method to describe the physics of flow. Table 1 illustrates the features of each hydrodynamic modeling method. Depending on the purpose of the study, each method has its advantages. The analytical method and MEM only can be used for simple geometries while BIEM and NSEM can be used for studying a more complicated geometry. If one intends to conduct an optimization or real time analysis, the analytical method, the MEM, and the frequency domain BIEM have often been used. Note that the frequency domain BIEM is generally used for calculating the hydrodynamic coefficients of the excitation force, and these coefficients need to be carefully determined and adjusted near resonance. Time domain BIEM can be used for optimization, but the computational cost remains too high for the purpose of real time analysis, particularly for 3D simulations. Since NSEM requires longer computational time, it is used only for detailed analyses, particularly when wave breaking and overtopping and viscous damping effects are significant.

Table 1. Features of hydrodynamic modeling methods

Methods / Features	Analytical	Morison	Frequency-domain BIEM	Time-domain BIEM	NSEM
Viscous Effect	No (but can be input)	Yes (Empirical)	No (but can be input)	No (but can be input)	Yes
Wave/floating body interaction	Linear (when wave diffraction and reflection are considered)	Body motion has no effect on the waves	Weakly nonlinear	Fully nonlinear (time domain)	Fully nonlinear
Wave Breaking	N/A	N/A	N/A	N/A	Use VOF

General accuracy	Acceptable	Acceptable for mooring line	Good	Good	Very good
Accuracy around resonance	Barely acceptable	Acceptable (with modified hydrodynamic coefficients)	Barely Acceptable (higher order scheme)	Acceptable	Very good

Given that the BIEM is only discretized along the boundary surface, it reduces the dimensions of the problem to be solved numerically. The conventional two-dimensional BIEM is typically applied using a direct matrix solver to the system of linear equations. Even though the BIEM reduces the spatial dimension of the problem by one, and the resulting BIEM matrix is usually fully populated and asymmetric, the computational cost increases exponentially with the number of elements. Therefore, the three-dimensional BIEM generally uses accelerated iterative algorithms, such as fast multiple methods [121], and generalized minimal residual methods [122], to reduce the CPU time. Of course, one can choose the frequency domain BIEM instead of the time domain BIEM. The frequency domain BIEM solves the problem at the mean water and body surfaces. Conversely, the time domain method solves the problem at each time step, with instantaneous water surface profiles and body surface locations. The time domain BIEM is feasible for capturing the fully nonlinear interaction between waves and the floating body, except for wave breaking and overtopping. However, the total CPU time of a time domain BIEM simulation is much longer than a frequency domain BIEM simulation.

The NSEM numerical model requires much more CPU time than BIEM to model the details of the fluid. As typical NSEM solvers discretize across the fully three-dimensional computational domain, the CPU time depends on many factors, including the CPU speed, convergence criterion, mesh quality, parallel efficiency, and the complexity of the interaction between waves and the floating body. The latter generally involves the calculation of the body location and mesh morphing or the application of overset meshes. Due to the sparse nature of the linear equation system generated by the NSEM, iterative methods are widely used. The convergence rate of the iterative method and its parallel efficiency influence the total CPU time of an NSEM method significantly. An algebraic multi-grid method is commonly used in existing CFD packages for accelerating the convergence of an iterative method, and the details of the AMG method are described in [123].

An example for modeling the power generation performance of a two-body floating-point absorber system is presented to demonstrate the utilization of these methods. The two most commonly used methods, a frequency-domain BIEM and a NSEM (CFD), were selected. In the frequency-domain BIEM, the hydrodynamic forces were calculated using WAMIT, and the viscous damping coefficient was estimated based on the empirical solution of [46]. The details of the device geometry and the numerical settings in the CFD simulation were described in [118]. To compare the results obtained from the two methods, the computational time for each model is shown in Table 2, and the power generation performance is plotted against incident wave periods in Fig. 9. The study shows that the frequency-domain BIEM is much more cost-effective. However, the selection of viscous damping coefficients has a significant influence on the frequency-domain BIEM results, particularly near the resonance. Note that the maximum power generally occurs when floating-point absorber systems are close to the resonance. The

calculation of viscous damping is essential to the prediction of power generation, where the selection of viscous damping coefficients can result in a large uncertainty in the modeling. A study on the variation of the drag coefficients was presented in [121]. They showed that if the drag coefficients were varied by a factor of 4, the effect on power generation could be up to 30%.

Table 2. Run-time benchmark for modeling FPA WEC systems

Hydrodynamics Modeling	Discretization	Number of Panels/Meshes	Wall Clock Time
Frequency domain BIEM (WAMIT) & Quadratic Drag	Boundary Discretization	5096 (Low-order panels)	2804 seconds[3] (30 wave frequencies)
NSEM: Floating Body-Dynamics CFD	Computational Domain Discretization	≈1 million (Finite volume mesh)	12 hours on 64 cores[4] (one wave frequency)
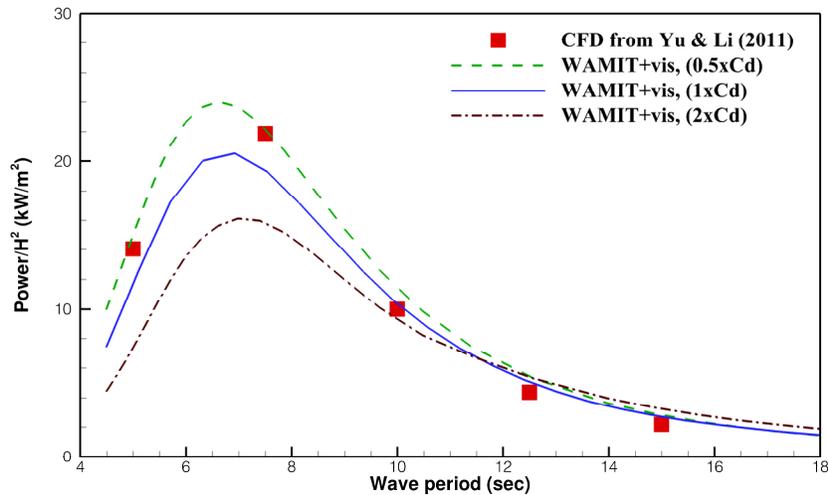
Figure 9. Power generation prediction from NSEM and BIEM (C_{PTO}=507 kNsm^{-1})

In general, a more cost-effective method should be developed for optimization purposes. Such a method should be able to provide sufficient accuracy, even in the resonance stage, with a minimal time requirement. In particular, this method should be able to work with the modeling tools of other disciplines. For example, control is very important for maximizing the power output of the wave energy converters in real seas. Many investigations have been published focusing on real time control strategies, e.g., [125,126,127]. In terms of hydrodynamic input, they take the solutions from analytical methods or BIEM in the frequency domain. With an advanced computational facility, it is possible to integrate the control strategies together with the time domain solutions, or at least add a high accuracy frequency domain solution near the resonance stage. Additionally, other disciplines, such as power electronics, hydrology, and environmental science, seek a better integration with the hydrodynamic modeling of wave energy devices.

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[3] On a 3.33 GHz Intel i5 processor with 8GB of memory

[4] Each compute node consists of dual socket/quad-core 2.93 GHz Intel Nehalem processor, with 12 GB of memory shared by all 8 cores.

In short, all these four methods have their own strength and weakness as summarized in Table 1. Simply speaking, analytical methods and MEM are suitable for optimization purposes, which are in the conceptual design stage. On the contrary, BIEM and NSEM are more appropriate for further analysis. Particularly, frequency domain BIEM may be used in between of two stages. Furthermore, lower order methods tend to be easy to be combined with the modeling techniques of other aspects of the floating-point absorbers such as the power system.

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References

[1] Ross, D., *Power from sea waves*, 1st ed. United Kindom: Oxford University Press, 1995.

[2] Salter, S., "Wave power," *Nature*, pp. 720-724, 1974.

[3] (2011, May) Ocean Power Technologies. [Online]. http://www.oceanpowertechnologies.com

[4] (2011, May) Wavebob. [Online]. http://www.wavebob.com/home/

[5] Wavestar. [Online]. http://wavestarenergy.com/

[6] Wave Dragon. [Online]. http://www.wavedragon.net/

[7] IEA-OES. (2011, May) International Energy Agency Ocean Energy System. [Online]. http://www.iea-oceans.org/index.asp

[8] IEC. (2011, May) International Electrotchnical Commission Marine Energy Technical Committee. [Online]. http://www.bsigroup.com/en/Standards-and-Publications/Committee-Members/Committee-member-news/Summer-2007/New-Committee-IECTC-114-Marine-Energy/

[9] McCabe, A., Bradshaw, A., Meadowcroft, J., and Aggidis, G., "Developments in the design of the PS Frog Mk 5 wave energy converter," *Renewable Energy*, vol. 31, no. 2, pp. 141-151, 2006.

[10] Kraemmer, D., Ohl, C., and McCormick, M., "Comparison of experimental and theoretical results of the motions of a McCabe wave pump," in *Proceedings of Fourth European Wave Power Conference*, vol. 1, Aalborg, Denmark, 2000.

[11] Pelamis Wave Power. [Online]. http://www.pelamiswave.com/

[12] Salter, S., "The swinging mace," in *Proceedings of Workshop Wave Energy R&D*, 1992, pp. 197-206.

[13] Whittaker, T., Collier, D., Folley, M., Osterried, M., Henry, A., and Crowley, M., "The development of Oyster--A shallow water surging wave energy converter," in *Proceedings of 7th European Wave and Tidal Energy Conference*, 2007, pp. 11-14.

[14] Aquamarine Power Oyster. [Online]. http://www.aquamarinepower.com/technology

[15] Heath, T., Whittaker, T., and Boake, C., "The design, construction and operation of the LIMPET wave energy converter," in *Proceedings of Proc. 4th European Wave Energy Conference*, Islay, Scotland, 2000, pp. 49-55.

[16] Ohneda, H., Igarashi, S., Shinbo, O., Sekihara, S., Suzuki, K., Kubota, H., Ogino, H., and Morita, H., "Construction procedure of a wave power extracting caisson breakwater," in *Proceedings of 3rd Symposium on Ocean Energy Utilization*, 1991, p. 171â€"179.

[17] Clément, A., McCullen, P., Falcão, A., Fiorentino, A., Gardner, F., Hammarlund, K., Lemonis, G., Lewis, T., Nielsen, K., and Petroncini, S., "Wave energy in Europe: current status and perspectives," *Renewable and Sustainable Energy Reviews*, vol. 6, no. 5, pp. 405-431, 2002.

[18] Kofoed, J., Frigaard, P., Friis-Madsen, E., and SÃ¸rensen, H., "Prototype testing of the wave energy converter wave dragon," *Renewable Energy*, vol. 31, no. 2, pp. 181-189, 2006.

[19] Vicinanza, D. and Frigaard, P., "Wave pressure acting on a seawave slot-cone generator," *Coastal Engineering*, vol. 55, no. 6, pp. 553-568, 2008.

[20] Lilypad. [Online]. http://www.energyisland.com/projects/lilypad/lilypad.html

[21] Weber, J., Mouwen, F., Parish, A., and Robertson, D., "Wavebob--research \& development network and tools in the context of systems engineering," in *Proceedings of Eighth European Wave and Tidal Energy Conference*, Uppsala, Sweden, 2009.

[22] Salter, S. and Lin, C.-P., "The sloped IPS wave energy converter," in *Proceedings of 2nd European Wave Energy Conference*, Lisbon, Portugal, 1995, pp. 337–344.

[23] Payne, G. S., Taylor, J. R. M., Bruce, T., and Parkin, P., "Assessment of boundary-element method for modelling a free-floating sloped wave energy device. Part 1: Numerical modelling," *Ocean Engineering*, vol. 35, no. 3-4, pp. 333-341, 2008.

[24] Payne, G. S., Taylor, J. R. M., Bruce, T., and Parkin, P., "Assessment of boundary-element method for modelling a free-floating sloped wave energy device. Part 2: Experimental validation," *Ocean Engineering*, vol. 35, no. 3-4, pp. 342-357, 2008.

[25] Evans, D., "Power from water waves," *Annual Review of Fluid Mechanics*, vol. 13, no. 1, pp. 157-187, 1981.

[26] Falcão, A. F. d. O., "Wave energy utilization: A review of the technologies," *Renewable and Sustainable Energy Reviews*, 2010.

[27] Sarmento, A. J. N. A., Neumann, F., and Brito-Meto, A., "NON-TECHNICAL BARRIERS TO LARGE-SCALE WAVE ENERGY UTILISATION," in *Proceedings of 2004 NEW AND RENEWABLE ENERGY TECHNOLOGIES FOR SUSTAINABLE DEVELOPMENT*, Evora, Portugal, 2004, pp. 225-233.

[28] Energy, U. S. D. o. (2010, December) DOE. [Online]. http://www1.eere.energy.gov/windandhydro/hydrokinetic/listings.aspx?type=Tech

[29] Falnes, J., "A review of wave-energy extraction," *Marine Structures*, vol. 20, no. 4, pp. 185-201, 2007.

[30] Khan, J., Bhuyan,G, and Moshref, A., "An Assessment of Variable Characteristics of the Pacific Northwest Region'sWave and Tidal Current Power Resources, and their Interaction with Electricity Demand & Implications for Large Scale Development Scenarios," PowerTech Lab, Surrey, Internal Report 2008.

[31] Brooke, J., *Wave Energy Conversion*.: Elsevier Science, 2003.

[32] Cruz, J., *Ocean Wave Energy*, 1st ed. Berlin, Germany: Springer, 2008.

[33] McCormick, M., *Ocean Wave Energy Conversion*, 2nd ed.: Dover, 2007.

[34] Falnes, J., *Ocean waves and oscillating systems*., 2002.

[35] Faltinsen, O., "Wave loads on offshore structures," *Annual Review of Fluid Mechanics*, vol. 22, no. 1, pp. 35-56, 1990.

[36] Drew, B., Plummer, A., and Sahinkaya, M., "A review of wave energy converter technology," *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, vol. 223, no. 8, pp. 887-902, 2009.

[37] Dussan, V. and others, "On the difference between a bounding surface and a material surface," *Journal of Fluid Mechanics*, vol. 75, no. 04, pp. 609-623, 1976.

[38] Longuet-Higgins, M. and Cokelet, E., "The deformation of steep surface waves on water-I. A numerical method of computation," *Proc. R. SOC. Lond. A350, 1*, vol. 26, 1976.

[39] Airy, G., "Tides and waves," *Encyclopedia Metropolitana*, vol. 5, pp. 24l-396, 1841.

[40] Clarke, J. A., Connor, G., Grant, A. D., and Johnstone, C. M., "Design and testing of a contra-rotating tidal current turbine," *Proceedings of the Institution of Mechanical Engineers Part a-Journal of Power and Energy*, vol. 221, no. A2, pp. 171-179, 2007.

[41] Stokes, G., "On the theory of oscillating waves," *Trans. Camb. Phil. Soc*, vol. 8, p. 441, 1847.

[42] Stokes, G., "On th effect internal friction of fluids on the motion of pendulums," *Mathematics and Physics Papers*, 1880.

[43] Skjelbreia, L. and Hendrickson, J., "Fifth order gravity wave theory," in *Proceedings of 7th conf. Coastal Eng*, 1961, pp. 184-196.

[44] Le Méhauté, B., *An introduction to hydrodynamics and water waves*.: Springer-Verlag New York, 1976.

[45] Dean, R., "Relative validities of water wave theories," *Journal of the Waterways, Harbors and Coastal Engineering Division*, vol. 96, no. 1, pp. 105-119, 1970.

[46] Sarpkaya, T., *Wave forces on offshore structures*.: Cambridge, 2010.

[47] Mei, C. C., Stiassnie, M., and Yue, D. K.-P., *Theory and Applications of Ocean Surface Waves: Nonlinear aspects*.: World Scientific, 2005.

[48] Pierson Jr, W. and Moskowitz, L., "A proposed spectral form for fully developed wind seas based on the similarity theory of SA Kitaigorodskii," *Journal of Geophysical Research*, vol. 69, no. 24, pp. 5181-5190, 1964.

[49] Hasselmann, K., Barnett, T., Bouws, E., Carlson, H., Cartwright, D., Enke, K., Ewing, J., Gienapp, H., Hasselmann, D., and Kruseman, P., "Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP)," *Ergnzungsheft zur Deutschen Hydrographischen Zeitschrift Reihe*, vol. 8, no. 12, 1973.

[50] Tucker, M. J. a. P. G., *Waves in Ocean Engineering*, 1st ed.: Elsevier, 2001.

[51] Newman, J. N., *Marine Hydrodynamics*, 1st ed. Cambridge,MA, USA: The MIT Press, 1977.

[52] Budal, K. and Falnes, J., "A resonant point absorber of ocean-wave power," *Nature*, vol. 256, pp. 478-479, corrigendum 257:626, 1975.

[53] Budal, K. and Falnes, J., "Power generation from ocean waves using a resonant oscillating system," *Marine Science Communication*, vol. 1, pp. 269-288, 1975.

[54] Evans, D., "A theory for wave-power absorption by oscillating bodies," *Journal of Fluid Mechanics*, vol. 77, no. 01, pp. 1-25, 1976.

[55] Mei, C. C., "Power extraction from water waves," *Journal of Ship Research*, vol. 20, pp. 63-66, 1976.

[56] Newman, J., "The interaction of stationary vessels with regular waves," in *Proceedings of 11th Symposium on Naval Hydrodynamics*, 1976, pp. 491-501.

[57] Budal, K. and Falnes, J., "Interacting point absorbers with controlled motion," *Power from sea waves*, pp. 381-399, 1980.

[58] Evans, D., "Maximum wave-power absorption under motion constraints," *Applied Ocean Research*, vol. 3, no. 4, pp. 200-203, 1981.

[59] Pizer, D. J., "Maximum wave-power absorption of point absorbers under motion constraints," *Applied Ocean Research*, vol. 15, pp. 227-234, 1993.

[60] Garnaud, X. and Mei, C., "Wave-power extraction by a compact array of buoys," *Journal of Fluid Mechanics*, vol. 635, pp. 389-413, 2009.

[61] Yeung, R. W., "Added mass and damping of a vertical cylinder in finite-depth waters," *applied ocean research*, vol. 3, no. 3, pp. 119-133, April 1980.

[62] Mclver, P., "Wave forces on arrays of floating bodies," *Journal of Engineering Mathematics*, vol. 18, no. 4, pp. 273-285, 1984.

[63] Siddorn, P. and Eatock Taylor, R., "Diffraction and independent radiation by an array of floating cylinders," *Ocean Engineering*, no. 13, pp. 1289-1303, 2008.

[64] Eriksson, M., Isberg, J., and and Leijon, M., "Hydrodynamic modelling of a direct drive wave energy converter," *International Journal of Engineering Science*, vol. 43, no. 17-18, pp. 1377-1387, 2005.

[65] Berggren, L. and Johansson, M., "Hydrodynamic coefficients of a wave energy device consisting of a buoy and a submerged plate," *Applied Ocean Research*, vol. 14, no. 1, pp. 51-58, 1992.

[66] Mavrakos, S. A. and Kalofonos, A., "Power Absorption by Arrays of Interacting Vertical Axisymmetric Wave-Energy Devices," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 119, no. 4, pp. 224-231, 1997.

[67] Morison, J., O'brien, M., Johnson, J., and Schaaf, S., "The force exerted by surface waves on piles," *Petroleum Transactions, AIME*, vol. 189, pp. 149-154, 1950.

[68] Chakrabarti, S., *Handbook of offshore engineering*.: Elsevier Science Ltd, 2005.

[69] Dixon, A., Salter, S., and Greated, C., "Wave forces on partially submerged cylinders," *Journal of the Waterway Port Coastal and Ocean Division*, vol. 105, no. 4, pp. 421-438, 1979.

[70] Zhang, J., Chen, L., Ye, M., and Randall, R., "Hybrid wave model for unidirectional irregular waves--part I. Theory and numerical scheme," *Applied Ocean Research*, vol. 18, no. 2-3, pp. 77-92, 1996.

[71] Chen, X., Ding, Y., Zhang, J., Liagre, P., Niedzwecki, J., and Teigen, P., "Coupled dynamic analysis of a mini TLP: Comparison with measurements," *Ocean Engineering*, vol. 33, no. 1, pp. 93-117, 2006.

[72] Brekken, T., Jouanne, A. v., and Han, H., "Ocean Wave Energy Overview and Research at Oregon State University," 2009.

[73] Elwood, D., Yim, S., Amon, E., Jouanne, A. v., and Brekken, T., "Experimental Force Characterization and Numerical Modeling of a Taut-Moored Dual-Body Wave Energy Conversion System," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 132, p. 011102, 2010.

[74] Kim, Y., Kring, D., and Sclavounos, P., "Linear and nonlinear interactions of surface waves with bodies by a three-dimensional Rankine panel method," *Applied Ocean Research*, vol. 19, no. 5-6, pp. 235-249, 1997.

[75] Lee, C., "WAMIT theory manual," *Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, MA*, 1995.

[76] Denis, M., "Some Cautions on the Employment of the Spectral Technique to Describe the Waves of the Sea and the Response Thereto of Oceanic Systems," in *Proceedings of Offshore Technology Conference*, 1973.

[77] Kim, M. and Yue, D., "The complete second-order diffraction solution for an axisymmetric body Part 2. Bichromatic incident waves and body motions," *Journal of Fluid Mechanics*, vol. 211, pp. 557-593, 1990.

[78] Newman, J., "Second-harmonic wave diffraction at large depths," *Journal of Fluid Mechanics*, vol. 213, pp. 59-70, 1990.

[79] Newman, J., "The second-order wave force on a vertical cylinder," *Journal of Fluid Mechanics*, vol. 320, pp. 417-443, 1996.

[80] Wu, G. and Taylor, R., "The second order diffraction force on a horizontal cylinder in finite water depth*," *Applied Ocean Research*, vol. 12, no. 3, pp. 106-111, 1990.

[81] Ran, Z. and Kim, M., "Nonlinear coupled responses of a tethered spar platform in waves," *International Journal of Offshore and Polar Engineering*, vol. 7, pp. 111-118, 1997.

[82] Kim, M., "Second-order sum-frequency wave loads on large-volume structures*," *Applied Ocean Research*, vol. 13, no. 6, pp. 287-296, 1991.

[83] Lee, C. and Newman, J., "Computation of wave effects using the panel method. Numerical Models in Fluid-Structure Interaction," *Preprint, WIT Press, Southampton, UK*, 2004.

[84] McCabe, A., Aggidis, G., and Stallard, T., "A time-varying parameter model of a body oscillating in pitch," *Applied Ocean Research*, vol. 28, no. 6, pp. 359-370, 2006.

[85] McCabe, A., Stallard, T., Baker, N., and Yavuz, H., "Estimation of the responses of axisymmetric bodies in spread irregular waves," *Ocean Engineering*, vol. 34, no. 10, pp. 1371-1382, 2007.

[86] Cruz, J. and Salter, S., "Numerical and experimental modelling of a modified version of the Edinburgh Duck wave energy device," *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*, vol. 220, no. 3, pp. 129-147, 2006.

[87] Yavuz, H., Stallard, T., McCabe, A., and Aggidis, G., "Time series analysis-based adaptive tuning techniques for a heaving wave energy converter in irregular seas," *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, vol. 221, no. 1, pp. 77-90, 2007.

[88] Mavrakos, S. and Kalofonos, A., "Power absorption by arrays of interacting vertical axisymmetric wave-energy devices," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 119, p. 244, 1997.

[89] Ricci, P., Saulnier, J., and Antǿnio, F., "Point-absorber arrays: a configuration study off the Portuguese West-Coast," in *Proceedings of The 7th European Wave and Tidal Energy Conference* , 2007.

[90] Vicente, P., Falcão, A., Gato, L., and Justino, P., "Dynamics of arrays of floating point-absorber wave energy converters with inter-body and bottom slack-mooring connections," *Appl Ocean Res*, 2009.

[91] Longuet-Higgins, M. and Cokelet, E., "The deformation of steep surface waves on water. II. Growth of normal-mode instabilities," *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 364, no. 1716, p. 1, 1978.

[92] Bai, W. and Taylor, R. E., "Higher-order boundary element simulation of fully nonlinear wave radiation by oscillating vertical cylinders," *Applied Ocean Research*, vol. 28, no. 4, pp. 247-265, 2006.

[93] Ferrant, P., Touzé, D. L., and Pelletier, K., "Non-linear time-domain models for irregular wave diffraction about offshore structures," *International Journal for Numerical Methods in Fluids*, vol. 43, no. 10-11, pp. 1257-1277, 2003.

[94] Vinayan, V. and Kinnas, S., "A numerical nonlinear analysis of two-dimensional ventilating entry of surface-piercing hydrofoils with effects of gravity," *Journal of Fluid Mechanics*, in press.

[95] Tsynkov, S., "Numerical solution of problems on unbounded domains. A review* 1," *Applied Numerical Mathematics*, vol. 27, no. 4, pp. 465-532, 1998.

[96] Israeli, M. and Orszag, S., "Approximation of radiation boundary conditions," *Journal of Computational Physics*, vol. 41, no. 1, pp. 115-135, 1981.

[97] Grilli, S. and Horrillo, J., "Numerical generation and absorption of fully nonlinear periodic waves," *Journal of engineering mechanics*, vol. 123, no. 10, pp. 1060-1069, 1997.

[98] Romate, J., "Absorbing boundary conditions for free surface waves," *Journal of Computational Physics*, vol. 99, no. 1, pp. 135-145, 1992.

[99] Patankar, S. V. and Spalding, D. B., "A calculation procedure for heat, mass, and momentum transfer in three-dimensional flows," *Int. J. Heat Mass Transfer*, vol. 15, p. 1787, 1972.

[100] Patankar, S. V., "Numerical heat transfer and fluid flow," *Washington, DC, Hemisphere Publishing Corp., 1980. 210 p.*, 1980.

[101] Ferziger, J. H. and Peric, M., *Computational methods for fluid dynamics*.: Springer, 2002.

[102] Gentaz, L., Alessandrini, B., and Delhommeau, G., "2D nonlinear diffraction around free surface piercing body in a viscous numerical wave tank," in *Proceedings of 9th Int. Offshore and Polar Eng. Conf.*, vol. 3, 1999, pp. 420-426.

[103] Harlow, F. and Welch, J., "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface," *Physics of fluids*, vol. 8, no. 12, p. 2182, 1965.

[104] Hirt, C. and Nichols, B., "Volume of fluid (VOF) method for the dynamics of free boundaries* 1," *Journal of Computational Physics*, vol. 39, no. 1, pp. 201-225, 1981.

[105] Sussman, M., Smereka, P., and Osher, S., "A level set approach for computing solutions to incompressible two-phase flow," *Journal of Computational Physics*, vol. 114, no. 1, pp. 146-159, 1994.

[106] Sethian, J. and Smereka, P., "Level set methods for fluid interfaces," 2003.

[107] Lin, P. and Liu, P. L.-F., "Free surface tracking methods and their applications to wave hydrodynamics," *Advances in Coastal and Ocean Engineering*, vol. 5, pp. 213-240, 1999.

[108] Lara, J., Garcia, N., and Losada, I., "RANS modelling applied to random wave interaction with submerged permeable structures," *Coastal engineering*, vol. 53, no. 5-6, pp. 395-417, 2006.

[109] Lin, P. and Liu, P. L.-F., "Internal wave-maker for Navier-Stokes equations models," *Journal of waterway, port, coastal, and ocean engineering*, vol. 125, p. 207, 1999.

[110] Moin, P. and Mahesh, K., "Direct numerical simulation: a tool in turbulence research," *Annual Review of Fluid Mechanics*, vol. 30, no. 1, pp. 539-578, 1998.

[111] Lesieur, M. and Métais, O., "New Trends in Large-Eddy Simulations of Turbulence," *Annu. Rev. Fluid Mech*, vol. 28, pp. 45-82, 1996.

[112] Sirnivas, S., Wornom, S., Dervieux, A., Koobus, B., and Allain, O., "A study of LES models for the simulation of a turbulent flow around a truss spar geometry," , vol. 6, p. 25.

[113] Wilcox, D., *Turbulence modeling for CFD.*: DCW industries, 2006.

[114] El Moctar, O., Schellin, T., Jahnke, T., and Peric, M., "Wave Load and Structural Analysis for a Jack-Up Platform in Freak Waves," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 131, p. 021602, 2009.

[115] Agamloh, E., Wallace, A., and Jouanne, A. V., "Application of fluid-structure interaction simulation of an ocean wave energy extraction device," *Renewable Energy*, vol. 33, no. 4, pp. 748-757, 2008.

[116] Bhinder, M., Mingham, C., Causon, D., Rahmati, M., Aggidis, G., and Chaplin, R., "A joint numerical and experimental study of a surging point absorbing wave energy converter (WRASPA)," in *Proceedings of 28th Int. Conf. on Ocean, Offshore and Arctic Eng.*, 2009.

[117] Bhinder, M., Mingham, C., Causon, D., Rahmati, M., Aggidis, G., and Chaplin, R., "Numerical and Experimental Study of a Surging Point Absorber Wave Energy Converter," in *Proceedings of The 8th European Wave and Tidal Energy Conference* , 2009.

[118] Yu, Y.-H. and Li, Y., "A RANS Simulation for the Heave Response of a Two-Body Floating Point Wave Absorber," in *Proceedings of 21st International Offshore (Ocean) and Polar Engineering Conference*, Maui, Hi, United States, 2011.

[119] Cho, Y., Kim, M., Kim, M., and Sim, S., "On the flushing effect and the likelihood of the environment friendly vertical breakwater consist of immersed water channel and water chamber as a wave energy extraction measure," in *Proceedings of proceedings of the seventheeth intenational offshore and polar engineering conference*, Lisbon,Portugal, 2007, pp. 2621-2627.

[120] Monaghan, J. J., "An introduction to SPH," *Computer Physics Communication*, vol. 48, pp. 88-96, 1988.

[121] Babarit, a. H. J. M. M. J. K. A. M. T. K. J., *Numerical benchmarking study of a selection of wave energy converters*., 212, vol. 41.

[122] Nishimura, N., "Fast multipole accelerated boundary integral equation methods," *Applied Mechanics Reviews*, vol. 55, p. 299, 2002.

[123] Zhao, Y. and Graham, J., "An iterative method for boundary element solution of large offshore structures using the GMRES solver," *Ocean engineering*, vol. 23, no. 6, pp. 483-495, 1996.

[124] Shapira, Y., *Matrix-based multigrid: theory and applications*.: Springer Verlag, 2008, vol. 2.

[125] Falnes, J., "Optimum Control of Oscillation of Wave Energy Converters," in *Proceedings of Proceedings of the Eleventh International Offshore and Polar Engineering Conference,June 17-22*, Stavanger,Norway, 2001, pp. 567-574.

[126] Babarit, A. . C. H., "Optimal latching control of a wave energy device in regular and irregular waves," *Applied Ocean Research*, vol. 28, no. 2, pp. 77-91, April 2006.

[127] Falcão, A. F. d. O., "Modelling and control of oscillating-body wave energy converters with hydraulic power take-off and gas accumulator," *Ocean Engineering*, vol. 34, no. 14-15, pp. 2021-2032, Oct 2007.