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Contributions of the Stochastic Shape Wake Model to Predictions of Aerodynamic Loads and Power under Single Wake Conditions

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Abstract. The contribution of wake meandering and shape asymmetry to load and power estimates is quantified by comparing aeroelastic simulations initialized with different inflow conditions: an axisymmetric base wake, an unsteady stochastic shape wake, and a large-eddy simulation with rotating actuator-line turbine representation. Time series of blade-root and tower base bending moments are analyzed. We find that meandering has a large contribution to the fluctuation of the loads. Moreover, considering the wake edge intermittence via the stochastic shape model improves the simulation of load and power fluctuations and of the fatigue damage equivalent loads. These results indicate that the stochastic shape wake simulator is a valuable addition to simplified wake models when seeking to obtain higher-fidelity computationally inexpensive predictions of loads and power.

1. Introduction

Aerodynamic loads refer to the forces exerted by the wind on a turbine. While ultimate loads are brought on by extreme events, fatigue loads are associated with a large number of smaller amplitude stress cycles which in time also lead to structural failure. The fluctuations of these loads around a mean value are a result of atmospheric turbulence and shear, and represent the largest contribution to fatigue damage [1]. Modeling these loads is important for design standards [2] and for designing control algorithms [3].

Since the highest turbulence within a wind power plant is often found in wakes, it is crucial to accurately quantify wake-induced aerodynamic fatigue loads. This can be done by coupling an aeroelastic and a fluid dynamics model. Such models can vary widely in fidelity and computational cost. Unsteady models based on an axisymmetric wake shape [4] are computationally inexpensive tools to simulate flow behind turbines beyond the near wake region. One such model, the dynamic wake meandering (DWM) model, has been shown by [5] to accurately predict mean load statistics and fatigue damage equivalent loads (DEL) in below-rated wind speeds. However, the DWM may over-estimate the root-mean-squared (rms) moments when compared to field measurements and large-eddy simulations (LES) [1]. Improving the fidelity of wake simulations while maintaining the low computational cost is valuable for more accurate load and power estimation, especially for wind speeds below rated [6].

We propose a stochastic wake shape (SWS) model that moves away from Gaussian axial symmetry and towards a physical description of the wake that accounts for intermittence along

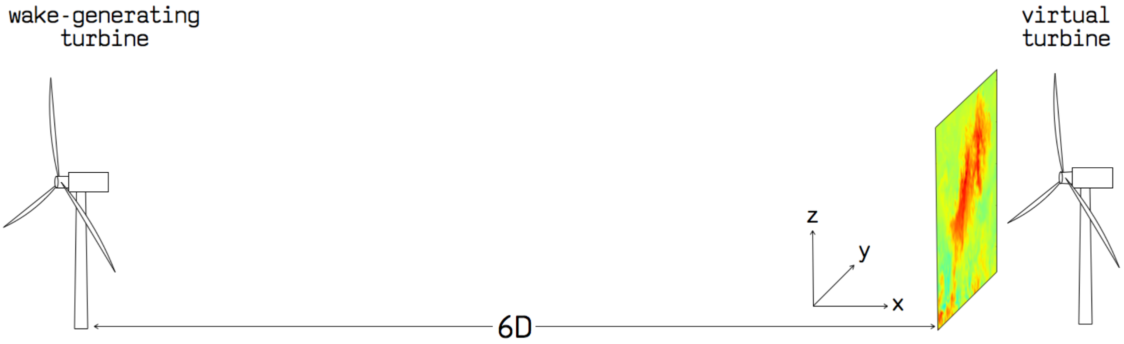


Figure 1: Schematic of coordinate system showing wake-generating turbine, the yz transect of wind speeds at 6D and the virtual turbine embedded in the wake.

the wake edges. Following the initial model development presented in [7], the objective of the present work is to quantify the contribution of the proposed model to simulations of loads and power. This is done by evaluating the relative importance of distinct wake components in wind turbine wake simulations. The models used are described in Section 2, the results in Section 3 and the final discussion in Section 4.

2. Data and Methods

In this work, wind turbine wakes are simulated with three different methods which are described in Section 2.1. The simulations are run for a total of 10 minutes and results are saved at a temporal resolution of 1 second. The velocity fields are sampled in transverse-vertical planes six rotor diameters (D) downstream of a turbine in the free stream as shown by the schematic in Fig. 1. These planes are then used as input to an aeroelastic simulation that calculates the loads experienced by a wind turbine subjected to each of the three velocity fields. The coordinate system is aligned with the mean wind and x, y and z are the streamwise, cross-stream and vertical directions respectively.

The wake is defined as a function of the velocity deficit

$$vd = 1 - \frac{U}{U_\infty} \tag{1}$$

where U is the horizontal wind speed behind the turbine and U_∞ the free stream wind speed. Vertical wind components are assumed to be negligible. This section describes the three wake models (Section 2.1), the methodology used for modeling meandering (Section 2.2), the design of the virtual turbine which will be subjected to the modeled wakes (Section 2.3), and details of the aeroelastic simulations (Section 2.4).

2.1. Wake Models

2.1.1. Axisymmetric Wake For these simulations, the wake shape and velocity deficit (vd) distribution in the yz plane are axisymmetric (AS) around the wake center (Fig. 2, left). A Gaussian radial vd profile is initialized for $x = 2$ D as

$$vd(r) = vd_0 \exp\left[-3.56\,(r/b)^2\right] \tag{2}$$

where r is the radial direction from the wake center, the subscript 0 refers to $r = 0$, and b is the wake width. This profile is used as boundary conditions for the steady Ainslie [8] solution which is found by integrating the thin shear layer approximation to the Reynolds-Averaged Navier Stokes and the continuity equations in the radial and streamwise directions up to $x = 6$ D. To allow for a comparison with the LES wake, the the initialization parameters vd_0 and b are set to values that once integrated, yield a wake profile at 6 D that best matches the one from the LES.

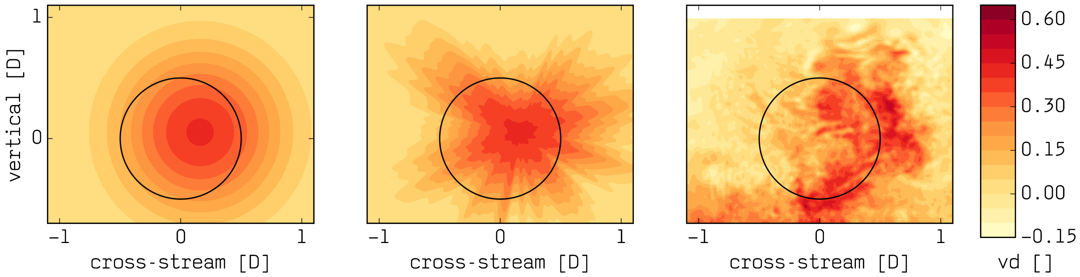


Figure 2: Instantaneous single wake with synchronized meandering 6 D downstream of a wind turbine, looking upstream in a fixed frame of reference. AS with Ainslie solution (left), SWS with Ainslie solution (middle), and LES (right). Scale bar gives velocity deficit (unitless). Black circles mark the turbine rotor circumference. Axes are centered at the turbine hub.

2.1.2. Stochastic Shape Wake A large number of metrics are needed to accurately define both power losses and loads arising from wind turbine wakes. Assuming these can be expressed in terms of vertical slices at discrete distances downstream of the turbine, they include: wake width and height, velocity deficit distribution, wake shape asymmetry and wake meander. In [?] stochastic and spectral methods for defining wake characteristics are developed based on a LES of single wakes in an offshore wind farm. These methods constitute the base of the SWS model, which produces a wake that is unsteady and asymmetric in shape (Fig. 2, center).

At each second, a wake shape is simulated on a polar coordinate system at an azimuthal resolution of 1°. Each point along the wake edge is given by an azimuthal mean radius $\langle r_w \rangle(t)$ and an azimuthal series of perturbations about this radius $r'_w(\theta, t)$ (where t is time, θ the azimuth angle, and $\langle\rangle$ represents azimuthal averaging). At each iteration, the mean radius is determined using a first-order auto-regressive model thus incorporating temporal coherence in the wake shape time series. The perturbations about this mean are obtained from the inverse Fast Fourier Transform (FFT) of a complex series. The phases of the complex coefficients are randomly sampled from probability distribution functions at each iteration. The magnitudes of these complex coefficients are constant for a fixed distance downstream. They are obtained from mean spectra which describe the relative contribution of different wave numbers to the fluctuations in the wake shape. Low wave numbers represent the contribution of large-scale features such as wake skewness due to vertical wind shear and veer. High wave numbers describe the effects of small-scale turbulence on the wake edges.

In this work, the SWS is used to simulate a wake that is subsequently used to drive the aeroelastic simulations. Each instantaneous yz plane is given by the methodology briefly described above. The vd distribution is still axisymmetric, and identical to the one in the AS solution. A comparison of SWS and AS therefore allows for an estimation of the contribution of an unsteady, asymmetric wake shape to simulations of loads and power.

2.1.3. Large-Eddy Simulation Wake This wake is produced by using a rotating actuator-line model [9] for the wind turbine and by solving the filtered Navier-Stokes and continuity equations using the National Renewable Energy Laboratory's (NREL) Simulator for Wind Farm Applications [10]. Both the wake shape and vd distribution are unsteady and asymmetric (Fig. 2, right). This dataset has already been validated against observations [1]. It is therefore taken as the reference data set throughout the analysis, and assumed to be representative of actual conditions. We focus on the wake behind a turbine that is subject to free stream conditions, which are generated by running a precursor LES. In the free stream, the hub height streamwise and cross-stream wind components are $(u, v) \sim (9, 0)$ m s^{-1} and and the turbulence intensity is $\sim$ 4 %, with positive (negative) cross-stream shear below (above) the rotor. At 6 D, the temporal mean of the minimum wind speed in the wake is $\sim$ 3.8 m s^{-1}, and the mean wake width is $\sim$ 1.5 D.

2.2. Wake Meandering

The relative contribution of meandering to load and power simulations is assessed by deliberately turning it on or off. When meandering is included in the simulations, it is synchronized between the three models based on the meandering time series obtained from the LES simulation as follows: (i) a Gaussian profile is fit to the 20-minute mean LES wake; (ii) the vd corresponding to the 95% confidence interval of this Gaussian is used to identify the wake edge at each instantaneous yz slice (Fig. 3); (iii) once the wake edge is identified, the center of gravity of the vd distribution within the wake area is found; finally (iv) the wake center movement is tracked relative to the turbine hub location. This meandering time series is then used to move the AS and SWS wakes as passive tracers as in the example shown in Fig. 2.

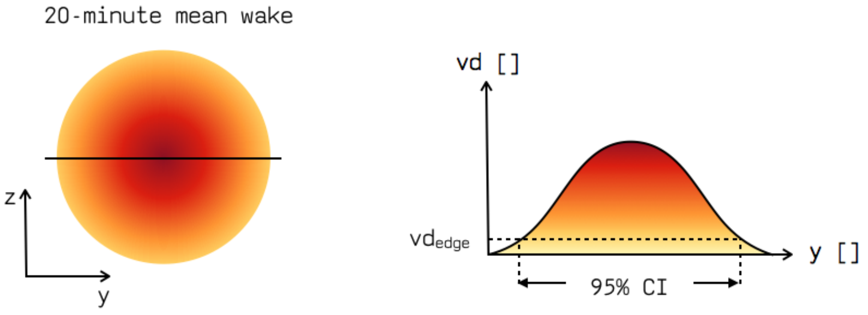


Figure 3: Schematic of method used to determine the wake edge at each instantaneous yz transect, based on the Gaussian fit to the velocity deficit distribution of the 20-minute mean LES wake.

Conversely, to assess the contribution of meandering in the LES slices it is necessary to remove the wake motion from the simulated fields. This is done by redefining the origin of the coordinate system at each second to follow the wake center and therefore keeping the wake on a meandering frame of reference instead of fixed at the hub. The subscripts MFoR and FFoR refer to a meandering and a fixed frame of reference respectively [11]. The MFoR follows the wake center and is therefore equivalent to a static wake, centered at the turbine hub. Conversely, the wake in a FFoR is dynamic and its center moves about the hub.

2.3. The Virtual Turbine

For the three simulations to be cross-compared and validated against measurements, a virtual turbine was designed to match the rotor size, hub height, and power curve of the turbine that is producing the wakes at the wind farm in the LES. The parameters for the virtual turbine were initially prescribed based on the WindPACT 3 MW turbine [12], and then modified to obtain a better fit to the desired power curve. Additionally to the cylinder at the blade root, three NREL airfoils were used. Some of the final parameters used are given in Table 1. A simple variable speed torque controller was used by setting the rated generator speed, torque, and slip percentage, and the torque constant in region 2. The pitch controller is irrelevant to the analyses because the wind speeds remain below rated where the blade pitch is kept constant.

Table 1: Basic parameters describing the virtual turbine.

Rotor diameter	90 m	Generator rating	3 MW
Rotor inertia	1.86E7 kg m^2	Generator speed	1680 rpm
Hub height	70 m	Nominal revolutions	16.10 rpm
Coning angle	-2.5°	Tower diameter	$\in [2.3, 4.2]$ m
Blade pitch	2.6°	Tower thickness	$\in [15.0, 26.0]$ m

2.4. Aeroelastic Simulations

The virtual turbine was subjected to wakes from the three simulations, with and without meandering according to the wake modeling component being analyzed. Loads and power for the virtual turbine were calculated using NREL's Fatigue, Aerodynamics, Structures, and Turbulence (FAST) model version 8.12 [13]. The inflow conditions were the transverse-vertical planes of data generated with the models described in Section 2.1. The simulations were run for $\sim$ 10 minutes with a time step of 0.005 s, allowing for a transient time of 1 minute. The modules for structural dynamics, inflow wind, aerodynamic loads, and control dynamics were used. The analysis focuses on time series of blade root (BR) out-of-plane (OoP) and in-plane (IP) bending moments, tower base side-to-side (SS) and fore-aft (FA) bending moments, and the respective DEL. For the blade moments, the values analyzed are an arithmetic average across the three blades.

3. Results

This section analyzes the loading and power of the virtual turbine under each simulation, and investigates the relative contribution of different wind turbine wake modeling components.

3.1. Virtual Turbine Loading

The aeroelastic simulations initialized with the AS and SWS wakes reproduced well the mean blade root (Fig. 4) and tower (Fig. 5) loading produced by the unsteady, asymmetric LES fields. The maxima and minima in the time series were not captured, and the variance was underestimated as will be further discussed in Section 3.4. It is evident from the time series that including wake shape asymmetry consistently increases the magnitude and variability of the bending moments considered, but a more realistic simulation requires an asymmetric representation of the velocity deficit distribution within the wake as well.

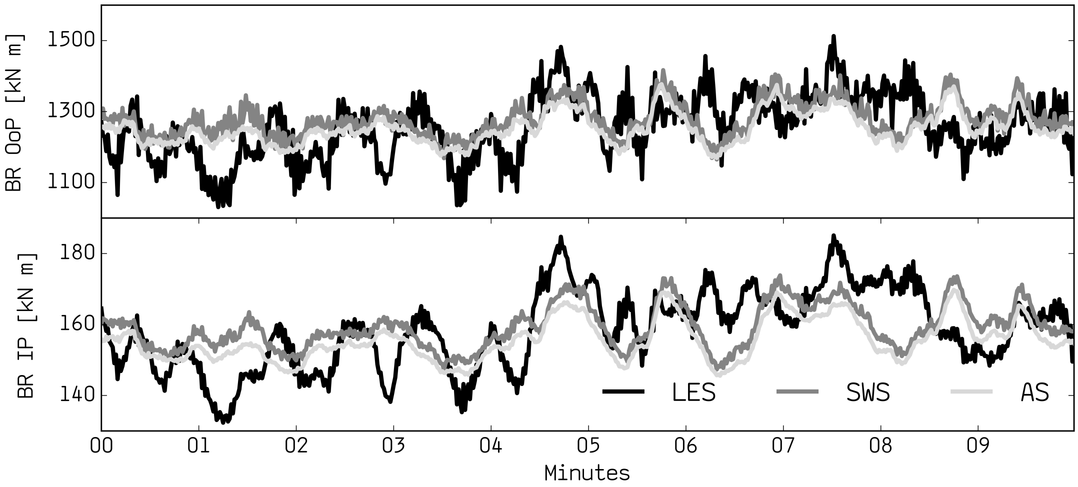


Figure 4: 10-minute time series of blade root out-of-plane (top) and in-plane (bottom) bending moments for the three simulations (with meandering).

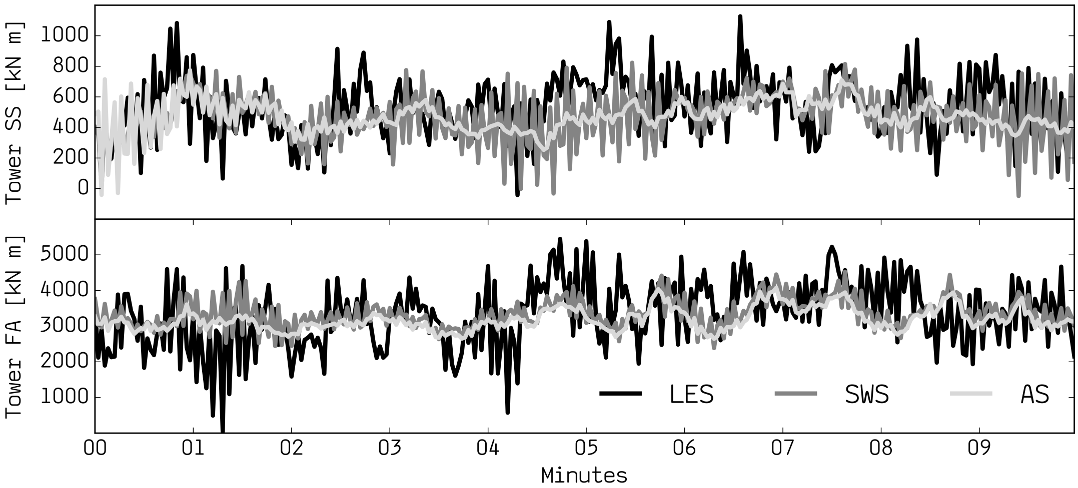


Figure 5: 10-minute time series of side-to-side (top) and fore-aft (bottom) tower base bending moments for the three simulations (with meandering).

3.2. Virtual Turbine Power

The results for generator power are similar to those seen for the turbine loading. The AS and SWS simulations reproduced the mean generator power but failed to capture the maxima and minima, and the magnitude of the fluctuations. While the asymmetric stochastic shape improved the estimate of the load fluctuations, it did not significantly improve the simulation of power fluctuations because the deficit distribution within the yz transect remains the same for AS and SWS.

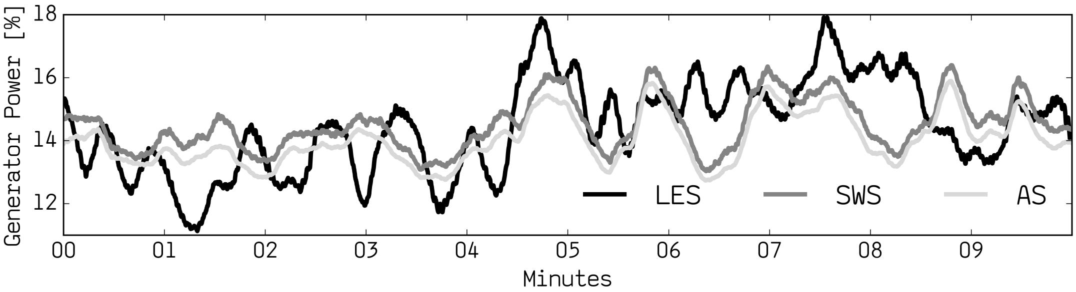
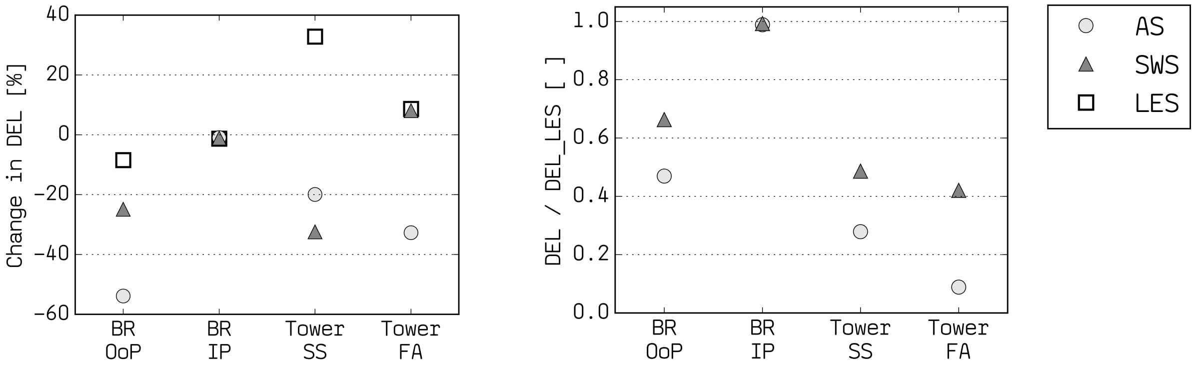


Figure 6: 10-minute time series of generator power (normalized relative to rated power) for the three simulations with meandering.

3.3. Meandering

Meandering was deliberately turned on and off in the aeroelastic simulations to assess its contribution to the loads experienced by the virtual turbine. The experiment was conducted using the three models: AS, SWS, and LES. The results are quantified by the difference in DEL, which was calculated for the 10-minute time series of bending moments output by the aeroelastic model, considering Wöhler exponents of 4 and 10 for steel and composite materials respectively. The differences are calculated as $\left(x_{\mathrm{MFoR}} - x_{\mathrm{FFoR}}\right)/x_{\mathrm{FFoR}}$ where x is the load being considered.

The results are shown in Fig. 7a and indicate that overall, removing meandering from the models reduced the loads experienced by the turbine. For the LES, the magnitudes of the changes were lowest because the base wake is already asymmetrical in shape and deficit distribution and meandering is only one of the modeling components contributing to loading on the turbine. On the other hand, meandering is the largest driver of loading for the AS simulation and therefore for this case, the reduction in the loads when meandering was removed is very large, especially for the BR OoP DEL. For the loads in the streamwise direction (BR OoP and Tower FA), the effect of meandering in SWS is much closer to the LES values than the more simplified AS simulation.



(a) Meandering. (b) Shape asymmetry.

Figure 7: Sensitivity of DEL to meandering (a) and wake shape asymmetry (b).

3.4. Wake Shape Asymmetry

The contribution of wake shape asymmetry was assessed by comparing the loads and power produced by three simulations: AS, SWS, and LES. For more realistic estimates, synchronized meandering was including in all of them. DEL for each simulation normalized relative to the LES values are shown in Fig. 7b. The added asymmetry in the SWS relative to the AS model improves the load simulations, and the BR OoP DEL is ∼70% of the LES value for the SWS wake, despite its axisymmetric deficit distribution.

The intermittence in the loads and power is quantified in terms of rms fluctuations. The values for each simulation are given in Table 2. Including the intermittent edges in the simplified wake model improves the simulations of the fluctuating loads relative to LES. However, these fluctuations are still underestimated and other unsteady wake modeling components (i.e., asymmetry in the velocity deficit distribution) should be added to the proposed SWS model to improve its predictive accuracy.

Table 2: RMS values of fluctuating loads and power for AS, SWS and LES simulations with meandering.

	unit	AS	SWS	LES
BR OoP	[kN m]	45.4	58.7	110.7
BR IP	[kN m]	5.8	6.2	11.2
Tower Base SS	[kN m]	167.3	310.0	362.3
Tower Base FA	[kN m]	330.5	639.1	1568.3
Generator Power	[kW]	23.4	24.5	43.9

4. Conclusion

In this work, the role of different wind turbine wake modeling components was assessed. The main objective was to quantify the contribution of the proposed SWS model to simulations of loads and power, particularly their unsteady fluctuations. Tower base and blade root bending moments were considered and compared across three different simulations with varying levels of fidelity.

It was found that meandering is a large contributor to the loads experienced by the turbine, and that removing it from the simulations can reduce the loading by more than 50% in the case of simplified models. The asymmetry in wake shape was also found to affect the load magnitudes, with the proposed SWS model producing DEL closer to the reference LES values. Even with an axisymmetric vd distribution, the SWS model produced BR OoP DEL values that are ∼70% of the LES.

While the SWS model improved the simulation of the fluctuating loads and power relative to the AS model, it still largely underestimates them especially for the FA tower base values. It also fails to capture minima and maxima in the load and power time series. This is likely due to the axisymmetric velocity deficit distribution in the SWS wake. The LES wakes include atmospheric and wake-generated turbulence, and generally show steep gradients at the wake edges where the velocity deficit distribution deviates from Gaussian at the tails.

The results obtained so far testify to the potential of the SWS model proposed in [7] to simulate wind turbine wakes for high frequency load and power estimates. Ongoing work focuses on increasing the fidelity of the proposed model while maintaining its low computational cost. Asymmetry in velocity deficit fields and lateral merging of wakes will be incorporated. These phenomena directly affect the turbulence characteristics and therefore the time series of loads and power on a downstream wind turbine. Future work will additionally validate the aeroelastic simulations relative to high-frequency turbine measurements.

Acknowledgments

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