

Optimization Under Uncertainty for Wake Steering Strategies

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Abstract.

Wind turbines in a wind power plant experience significant power losses because of aerodynamic interactions between turbines. One control strategy to reduce these losses is known as “wake steering,” in which upstream turbines are yawed to direct wakes away from downstream turbines. Previous wake steering research has assumed perfect information, however, there can be significant uncertainty in many aspects of the problem, including wind inflow and various turbine measurements. Uncertainty has significant implications for performance of wake steering strategies. Consequently, the authors formulate and solve an optimization under uncertainty (OUU) problem for finding optimal wake steering strategies in the presence of yaw angle uncertainty. The OUU wake steering strategy is demonstrated on a two-turbine test case and on the utility-scale, offshore Princess Amalia Wind Farm. When we accounted for yaw angle uncertainty in the Princess Amalia Wind Farm case, inflow-direction-specific OUU solutions produced between 0% and 1.4% more power than the deterministically optimized steering strategies, resulting in an overall annual average improvement of 0.2%. More importantly, the deterministic optimization is expected to perform worse and with more downside risk than the OUU result when realistic uncertainty is taken into account. Additionally, the OUU solution produces fewer extreme yaw situations than the deterministic solution.

1. Introduction

Wind power plant control can be used to efficiently increase wind energy production by maximizing power in wind plants that are already installed [1,2]. It can also be used to mitigate structural loads to maximize the lifetime of the turbines and better integrate wind energy into the energy market. One wind power plant control strategy, known as wake steering, has been explored extensively in the literature [3–5]. Wake steering involves intelligently offsetting wind turbine yaw alignments from the incoming wind so that wakes are deflected away from downstream turbines. This control strategy may allow wind plant operators to produce more power than by controlling each turbine to maximize its own energy capture by yawing directly towards the incoming flow.

Accurately measuring and controlling yaw misalignment can be an issue when designing wake steering schemes. Complex phenomena such as vorticity caused by the turbines’ blades, the speed-up effect around the nacelle, and sensor errors introduce significant uncertainty in measuring each turbine’s yaw angle relative to the incoming wind. Several studies have examined wind-power-plant-level control strategies and have reported significant gains in annual energy



production [3, 4, 6]. These studies use deterministic formulations of the wind energy system, assuming perfect information on parameter inputs. The uncertainty associated with wind plant model parameters such as inflow direction, yaw misalignment, or average speed may cause a wake steering strategy in the field to perform significantly worse than anticipated. One potential solution to this problem is optimization under uncertainty (OUU). This technique has been used in a variety of studies to provide a robust solution to a problem under varying levels of uncertainty [7–9]. Although other studies have modeled flow within a wind power plant using uncertainty in wind direction [7, 10], the authors are unaware of any previous yaw steering optimization that has explicitly taken uncertainty into account.

In this paper, we propose a methodology to explicitly take uncertainty in yaw position into account during the process of optimizing turbines' yaw offsets for wake steering schemes. As a first step, we examined how large yaw misalignment uncertainty affects the optimal solution. We used an engineering wake model to simulate the flow within a wind power plant, described in Section 2.1. In Section 2.2, a stochastic optimization approach is formulated to determine the optimal turbine control settings under uncertain yaw misalignment. Two case studies are used to further understand the effects of yaw misalignment uncertainty, detailed in Section 2.3. The first case study uses two turbines, in which the front turbine directly wakes the downstream turbine. The second case study considers the Princess Amalia Wind Farm, wherein we assume a single wind speed for each direction. The Princess Amalia Wind Farm was used in a previous wake steering study [3]. The results of these case studies are discussed in detail in Section 3. Finally, Section 4 addresses some conclusions and future work.

2. Methodology, application, and approach

In this study, we applied the FLOW Redirection and Induction in Steady State (FLORIS) engineering wake model to a two-turbine test case and the Princess Amalia Wind Farm to broadly quantify potential benefits of explicitly taking yaw misalignment into account when designing wake steering schemes. For this study, we assumed a constant and extreme yaw position error $\omega \sim \mathcal{N}(0^\circ, \sigma^2)$, which represents a Gaussian distribution of yaw misalignment errors centered at 0° offset and with a standard deviation σ equal to 20° . This random yaw position error was added to the design yaw offsets during the OUU uncertainty quantification routine, simulating 30,000 possible scenarios resulting from different misalignment errors in each optimization iteration. A convergence study was used to identify the number of samples required to sufficiently minimize the variance of the expected power production. The turbine yaw controls were bounded from -30° to 30° and the yaw misalignment uncertainty was unbounded.

2.1. Engineering Wake Model: FLORIS

The FLORIS model, developed in [4], is an extension of Jensen's model [11], which approximates wake deflection caused by yaw offsets [12]. Further modifications have been made to better model the wake velocity profile and effects of partial wake overlap. The steady-state power of each turbine i , denoted as P_i , is given by the FLORIS model as:

$$P_i = \frac{1}{2} \rho A_i C_P(a_i) U_i^3 \quad (1)$$

where ρ is the air density, A_i is the area of the rotor, C_P is the power coefficient, and U_i is the effective wind speed at turbine i . The effective velocity at the downstream turbine i is found by combining the effect of the wakes of the upstream turbine j , weighting the wake zones (near

wake zone, far wake zone, and mixing wake zone) by their overlap with the rotor as:

$$U_i = U_\infty \left(1 - 2 \sqrt{\sum_j \left[a_j \sum_{q=1}^3 c_{j,q}(X_i) \min \left(\frac{A_{j,i,q}^{overlap}}{A_i}, 1 \right) \right]^2} \right) \quad (2)$$

where U_∞ is the free-stream velocity, X_i is the x (streamwise direction) location of turbine i , $A_{j,i,q}^{overlap}$ is the overlap area of a wake zone q of a turbine i with the rotor of turbine j , and $c_{i,q}(x)$ is a coefficient that defines the recovery of a zone q to the free-stream conditions:

$$c_{i,q}(x) = \left(\frac{D_i}{D_i + 2k_e m_{U,q}(\gamma_i)[x - X_i]} \right)^2 \quad (3)$$

where $m_{U,q}$ is defined as:

$$m_{U,q}(\gamma_i) = \frac{M_{U,q}}{\cos(a_U + b_U \gamma_i)} \quad (4)$$

for $q = 1, 2, 3$ (corresponding to the three wake overlap zones), where a_U and b_U are tuned model parameters, D_i is the rotor diameter of turbine i , γ_i is the yaw offset of turbine i , and $M_{U,q}$ are tuned scaling factors that ensure that the velocity in the outer zones of the wake will recover to the free-stream conditions faster than in the inner zones. Additional details can be found in [4].

When a turbine is yawed, it exerts a force on the flow that causes the wake to deflect in a particular direction. Empirical formulas were presented in [12] and used in the FLORIS formulation [4]. In addition to wake deflection, there is a rotation-induced lateral offset that is caused by the interaction of the wake rotation with the shear layer [4]. The total lateral offset is represented as:

$$y_{w,i} = Y_i + \delta y_{w,yaw,i}(x) + \delta y_{w,rotation,i}(x) \quad (5)$$

where Y_i is the y (cross-stream direction) location of the turbine. $\delta y_{w,yaw,i}(x)$ and $\delta y_{w,rotation,i}(x)$ are defined in detail in [4].

A modification was made to the FLORIS model to allow for extreme yaw events in the uncertainty quantification routine. As $a_U + b_U \gamma_i$ approaches 90° , the denominator in Equation (4) approaches zero and $m_{U,q}$ approaches infinity. We introduce a simple modification: when $a_U + b_U \gamma_i$ is greater than 85° , we replace the $a_U + b_U \gamma_i$ term with 85° . Though we believe that this modification should be sufficient to capture general trends, future work is needed to model wakes caused by extreme yaws more rigorously.

2.2. Problem Formulation

Using the FLORIS model described earlier, the power production of a wind power plant can be computed for the whole year (i.e., the annual energy production). The annual energy production can be used in the cost function to optimize the yaw angles of the turbines in the wind plant with and without uncertainty. The remainder of this section formulates the yaw steering OOU problem using the average annual power production as the object of interest.

The average annual power production is a linear sum of each speed-and-direction-specific energy production value:

$$\Pi(\mathbf{Y}, \boldsymbol{\omega}) = \sum_{i=1}^n \sum_{j=1}^m P(\theta_i) P(u_{ij}) \pi(\mathbf{Y}_{ij} + \boldsymbol{\omega}_{ij}, \theta_i, u_{ij}) \quad (6)$$

where $P(\theta_i)$ is the discrete probability mass function of θ_i , $P(u_{ij})$ is the discrete probability mass function of u_{ij} , n is the number of directional bins, m is the number of wind speed bins,

\mathbf{Y} is an m by n by T tensor, where T is the number of turbines and each \mathbf{Y}_{ij} is a speed-and-direction-specific vector of T yaw offsets for controlling the entire wind power plant, and $\boldsymbol{\omega}_{ij}$ is a speed-and-direction-specific array of T random variables representing yaw misalignment errors: $\mathbf{Y}_{ij} = [\mathbf{Y}_{ij,1}, \mathbf{Y}_{ij,2}, \dots, \mathbf{Y}_{ij,T}]$ and $\boldsymbol{\omega}_{ij} = [\boldsymbol{\omega}_{ij,1}, \boldsymbol{\omega}_{ij,2}, \dots, \boldsymbol{\omega}_{ij,T}]$. Each $\boldsymbol{\omega}_{ijk}$ entry is a Gaussian distribution centered at zero with standard deviations of 20° . $\pi(\mathbf{x}, \theta, u)$ is the power production assuming unidirectional θ degree inflow with average wind speed u , and \mathbf{x} is a speed-and-direction-specific steering scheme vector of length T , and $\Pi(\mathbf{Y}, \boldsymbol{\omega})$ is annual power production averaged across wind direction and speed frequencies.

In the OUU formulation [13], we optimize the expected value of Π :

$$\mathbb{E}[\Pi(\mathbf{Y}, \boldsymbol{\omega})] = \sum_{i=1}^n \sum_{j=1}^m P(\theta_i) P(u_{ij}) \int \dots \int \pi(\mathbf{Y}_{ij} + \boldsymbol{\omega}_{ij}, \theta_i, u_{ij}) P(\boldsymbol{\omega}_{ij}) d\boldsymbol{\omega}_{ij} \quad (7)$$

where \mathbb{E} is the expected value and $P(\boldsymbol{\omega}_{ij})$ is the probability density function of the multivariate normal vector of yaw misalignment, $\boldsymbol{\omega}_{ij}$.

Because of the intensive computational nature of the problem, we chose to only examine the average wind speed for 72 directional bins in the Princess Amalia Wind Farm wake steering strategy optimization. We plan to expand the problem to consider multiple speeds in each direction bin in future work. Thus, our problem is reduced to the following:

$$\mathbb{E}[\Pi(\mathbf{X}, \boldsymbol{\omega})] = \sum_{i=1}^n P(\theta_i) \int \dots \int \pi(\mathbf{X}_i + \boldsymbol{\omega}_i, \theta_i) P(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i \quad (8)$$

where $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n]$, where each \mathbf{X}_i is an inflow-direction-specific steering scheme for each of the wind power plant's T turbines, so, for each i directional bin, $\mathbf{X}_i = [\mathbf{X}_{i,1}, \mathbf{X}_{i,2} \dots \mathbf{X}_{i,T}]$.

In the OUU setup, we approximate the expected value using a sample average approximation with K samples:

$$\widehat{\mathbb{E}}[\Pi(\mathbf{X}, \boldsymbol{\omega})] = \sum_{i=1}^n P(\theta_i) \frac{1}{K} \sum_K \pi(\mathbf{X}_i + \boldsymbol{\omega}_i^k, \theta_i) \quad (9)$$

where $\widehat{\mathbb{E}}$ is an approximation of the true expected value and $\boldsymbol{\omega}_i^k$ is the k^{th} evaluation of the random vector $\boldsymbol{\omega}_i$.

Thus, we formulate the wake steering OUU problem as follows:

$$\mathbf{X}_{OUU}^* = \arg \max_{\mathbf{X}} \widehat{\mathbb{E}}[\Pi(\mathbf{X}, \boldsymbol{\omega})] = \arg \max_{\mathbf{X}} \left(\sum_{i=1}^n P(\theta_i) \frac{1}{K} \sum_K \pi(\mathbf{X}_i + \boldsymbol{\omega}_i^k, \theta_i) \right) \quad (10)$$

Similarly, the deterministic wake steering optimization is formulated as:

$$\mathbf{X}_{det}^* = \arg \max_{\mathbf{X}} \Pi(\mathbf{X}, \mathbb{E}[\boldsymbol{\omega}]) = \arg \max_{\mathbf{X}} \left(\sum_{i=1}^n P(\theta_i) \pi(\mathbf{X}_i, \theta_i) \right) \quad (11)$$

Because a metric is needed to assess the quality of each solution, we introduce the value of the stochastic solution (VSS). Our definition is similar to the VSS metric introduced by Birge and Louveaux [13], but is expressed as a percent increase in production rather than an absolute value increase. We define the VSS as the ratio of the expected values of deterministic and OUU solutions, given yaw uncertainty:

$$VSS = \frac{\mathbb{E}[\Pi(\mathbf{X}_{OUU}^*, \boldsymbol{\omega})]}{\mathbb{E}[\Pi(\mathbf{X}_{det}^*, \boldsymbol{\omega})]} \quad (12)$$

where VSS is the value of the stochastic solution evaluated across the wind rose; ω is an uncertain n by T matrix of yaw misalignment, normally distributed around zero with standard deviation equal to 20° ; \mathbf{X}_{OUU}^* is the optimal yaw strategy obtained via OUU, assuming unbiased, Gaussian yaw misalignment ω ; and \mathbf{X}_{det}^* is the optimal yaw strategy obtained via deterministic optimization, assuming perfect control of yaw alignment.

In the Princess Amalia Wind Farm case, it was useful to report direction-specific VSS values, defined as:

$$vss_i = \frac{\mathbb{E}[\pi_i(\mathbf{X}_{OUU,i}^* + \omega)]}{\mathbb{E}[\pi_i(\mathbf{X}_{det,i}^* + \omega)]} \quad (13)$$

where: vss_i is the value of the stochastic solution considered in the i^{th} direction.

2.3. Case studies

We explored the benefits of OUU in a two-turbine test case as well as for the Princess Amalia Wind Farm, explicitly taking yaw misalignment uncertainty into account during the optimization process.

The two-turbine case has unidirectional flow with a spacing of 10 rotor diameters. The turbines are both 5 MW and the inflow is 10.6 m/s and orientated such that the front turbine directly wakes the back turbine. We investigated the two-turbine OUU setup for various levels of yaw misalignment uncertainty. We performed a parameter sweep across possible values of the front turbine with a nested Latin hypercube sampling routine—simulating yaw misalignment uncertainty in each turbine—to find the optimum steering strategy at various levels of uncertainty, recording the VSS and optimum front turbine angle associated with each level of yaw misalignment uncertainty.

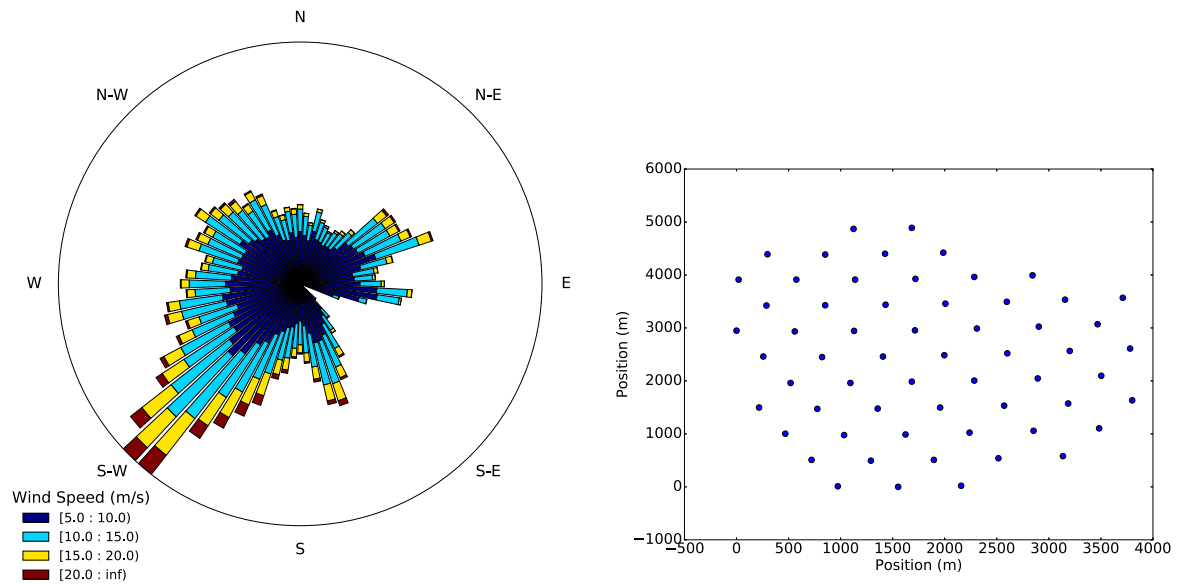


Figure 1. Princess Amalia Wind Farm characteristics. The wind rose is shown on the left. The turbine layout is shown on the right.

Next, we used the Princess Amalia Wind Farm specifications to compare wake steering schemes. We assumed a single wind rose with 72 directional bins (Figure 1), where each bin's wind characteristics are summarized by the average annual wind speed. The wind power plant has 60 five-MW turbines with rotor diameters of 126 m (Figure 1). We approached the Princess

Amalia Wind Farm wake steering OUU problem by nesting a semirandom sampling routine within an outer optimization. We chose to use the Latin hypercube sampling approach and the constrained optimization by using a linear approximation (COBYLA) optimization algorithm. The optimal yaw arrangement was found using each of the 72 recorded directions and the associated mean wind speeds in parallel optimizations with 10 multiple random optimization starts associated with each direction. The Latin hypercube samples the uncertain inputs semirandomly by dividing their bounded values into a grid to obtain sparsely selected random samples and avoid needlessly sampling the model [14]. The Latin hypercube sampling was highly parallelized using Sandia National Laboratories' DAKOTA analysis engine [14]. The COBYLA routine uses a sequential trust-region algorithm to create variable-sized linear approximations of the objective function and constraints to solve the optimization [14, 15]. Thus, COBYLA creates a constructed optimum, as opposed to other metaheuristic algorithms that search for optimums by semirandomly perturbing the candidate solution. We ran COBYLA for 1,000 iterations with a nested Latin hypercube sampling routine with 30,000 samples. For each direction-specific yawing scheme, we maximized the expected power production using this nested sampling approach.

3. Results

First, we performed the wake steering OUU on the two-turbine case, varying the level of yaw uncertainty. Next, using the Princess Amalia Wind Farm, we assumed a single yaw misalignment uncertainty. We report the average power produced by the Princess Amalia Wind Farm using deterministic and OUU optimal strategies, assuming a single wind speed per direction. We also examine the flow geometry, distribution of yaw misalignment, and distributions of potential power generation resulting from possible yaw misalignment errors for a direction that yielded large VSS.

3.1. Two-turbine test case

We examined a two-turbine case to better understand how yaw misalignment uncertainty affects the optimal solution. We found that the OUU solution generally prefers less steering and that the optimal front turbine yaw offset decreased with increases in yaw misalignment uncertainty (Figure 2). As expected, the VSS increased with yaw misalignment uncertainty. When the standard deviation of the Gaussian distribution of errors in both turbines' yaw misalignment reached 20° (the uncertainty assumed for the Princess Amalia Wind Farm case), the OUU strategy performed 1.1% better than the deterministically formulated strategy when both strategies were implemented with noise in the yaw sensors. The optimal expected power decreased from 6.9 to 5.7 GW when the standard deviation of the turbines' yaw misalignment increased from 0° to 30° . As yaw misalignment uncertainty increased, the optimum strategy shifted to extract more power from the front turbine. Thus, the downside risk of offsetting the front turbine increased and there was less utility in wake steering.

3.2. Princess Amalia Wind Farm case

In this section, we present the results of the OUU problem using the Princess Amalia Wind Farm. We found the optimal yaw configuration for this case using the deterministic and OUU optimization formulations developed in Section 2.2. If there is perfect knowledge of the alignment of each wind turbine's yaw relative to the average direction of the incoming wind, there is no need for the OUU approach. Given uncertainty in yaw alignments, there appear to be various benefits from OUU depending on the incoming wind direction (Figure 3). For example, considering the northern inflow (0°) direction, the OUU solution produces 1.4% more power than the deterministic solution, under the yaw misalignment uncertainty introduced in Section 2.2.

We expect the northern direction OUU solution to yield 1.4% more power than the deterministic strategy [Equation (9)]. The probability distributions and deterministic values

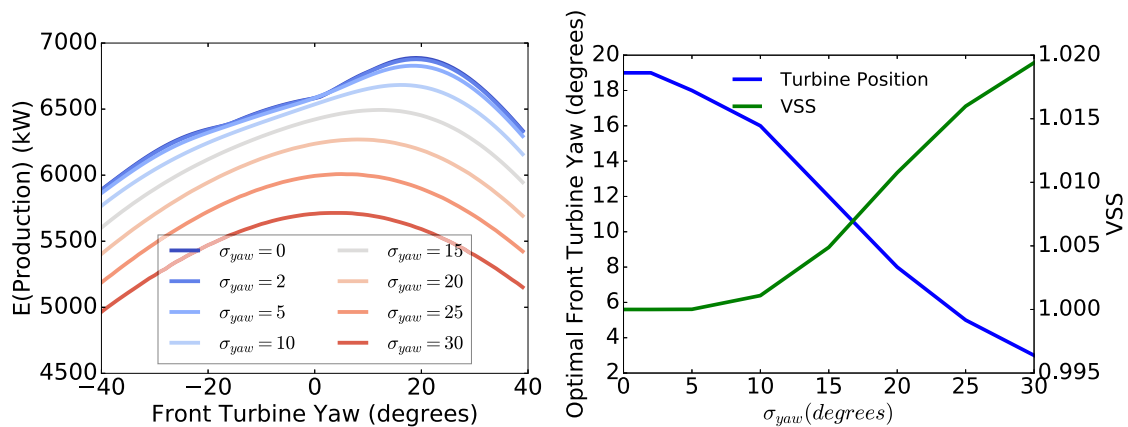


Figure 2. (Left) Power production resulting from different front turbine yaw angles. Contours of different yaw misalignment uncertainty values are shown. (Right) Optimal front turbine arrangement and VSS plotted against yaw misalignment uncertainty.

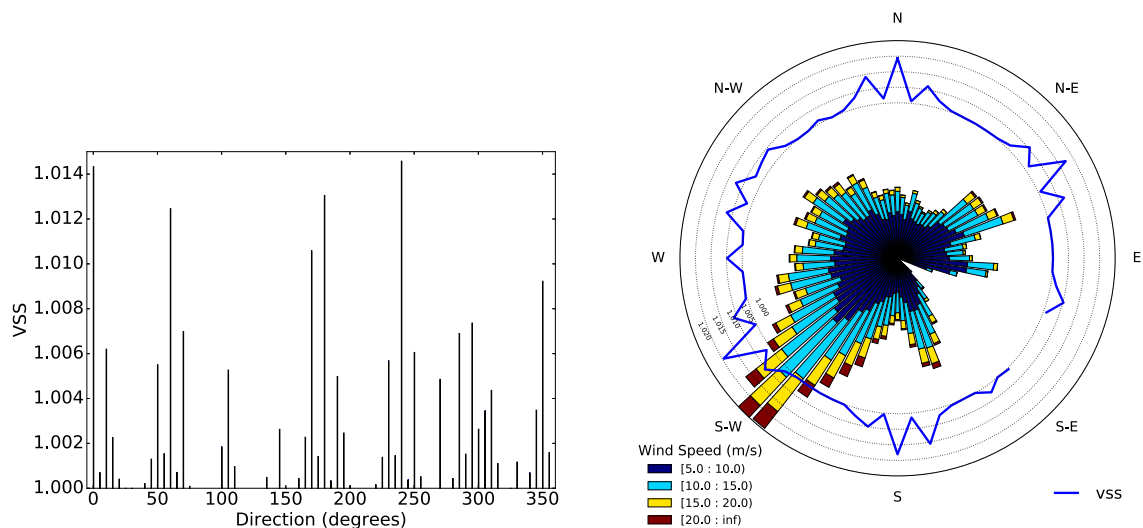


Figure 3. (Left) The vss for each direction considered. (Right) The vss plotted along the site wind rose. In some directions, the OUU solution outperformed the deterministic strategy's power production (under yaw misalignment uncertainty) by nearly 1.4%, whereas the OUUs associated with other directions yielded little or no improvements in expected power production.

of the OUU optimal strategy, deterministic optimal strategy, and baseline steering strategy are shown in Figure 4 (left) for the northern inflow case. The distribution of yaw offsets in the OUU and deterministic optimum strategies are shown in Figure 4 (right). Similarly to the two-turbine case, introducing uncertainty resulted in less utility to steering wakes. Consequently, the OUU strategy uses lesser offsets, which is attractive because lesser offsets reduce turbines' non-torque drivetrain loads and yaw moments. Figure 5 shows the flow fields for the northern inflow's baseline, deterministic, and OUU solutions, respectively. Figure 6 shows a close-up of the flow fields for the deterministic and OUU strategies to highlight the differences between the two solutions. The deterministic strategy steers wakes further away from downstream turbines, thus, the OUU solution prefers less yaw offsets.

We expect the robust design to produce 0.2% more average annual power than the deterministic design (Table 1). In each direction, the deterministic solution that performed best under certain yaw alignments was compared to the OUU solution that performed best under yaw alignment uncertainty. These results were generated using 10 random initial starting points per direction.

Table 1. Optimization results (GW).

	Deterministic Average Power Production	Stochastic Average Power Production
O UU Solution	110.4	100.6
Deterministic Solution	113.1	100.1
No Yaw Offsets	105.1	97.4

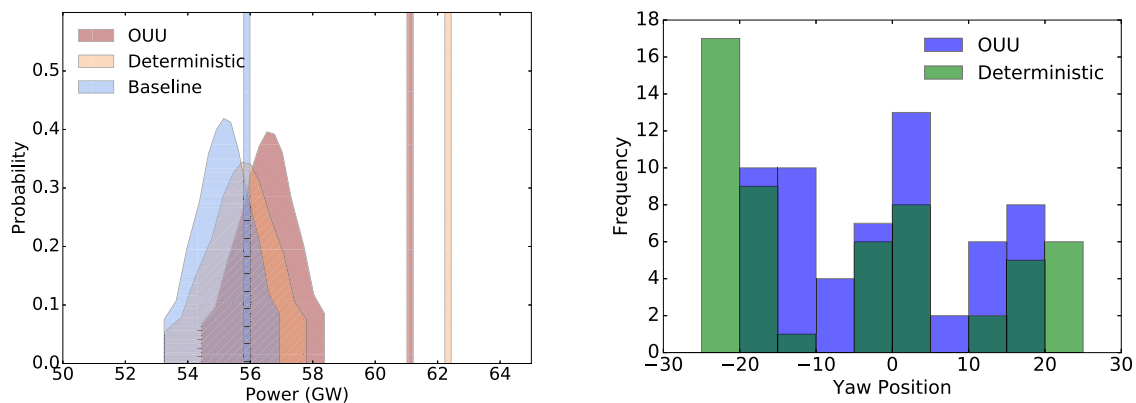


Figure 4. (Left) Empirical distributions of energy production under yaw misalignment uncertainty for O UU, deterministic, and baseline (no yaw offsets) steering strategies for the northern inflow case. The probability distributions show possible yaw misalignment. The straight bars show the forecasted production for each arrangement, assuming perfect knowledge of each turbine’s yaw alignment. (Right) Frequency occurrence of deterministic and O UU solution yaw positions for the northern inflow. O UU solutions generally produced strategies with less extreme offsets.

4. Conclusions

Robust optimization offers an attractive solution to the yaw misalignment problem when designing wake steering strategies. Deterministically, there is a natural optimum steering strategy balancing the trade-offs of losing power from the steering turbine’s offset and gaining power by steering the velocity deficit away from downstream turbines. When yaw misalignment uncertainty is considered, the optimal wake steering shifts towards lesser yaw offsets, extracting more power from the front turbines. Future work in this area could examine other sources of uncertainty. In addition, it may be useful to integrate wake steering strategies into the turbine placement design (this was done deterministically in [3]), explicitly taking measurement uncertainty into account during the design process.

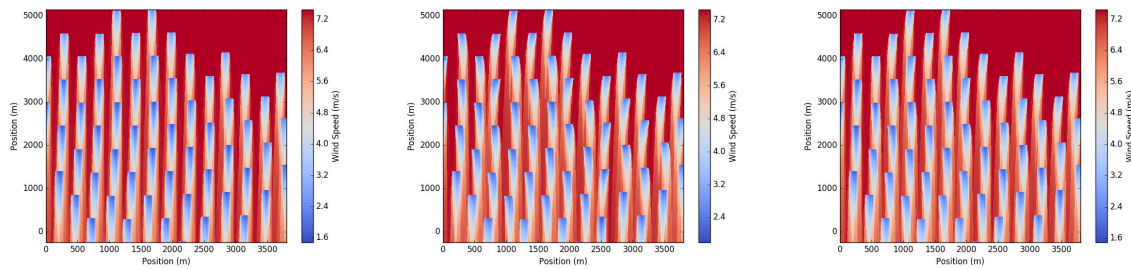


Figure 5. (Left) No steering with the northern inflow. (Middle) Deterministic optimal steering with the northern inflow. (Right) OUU optimal steering with the northern inflow.

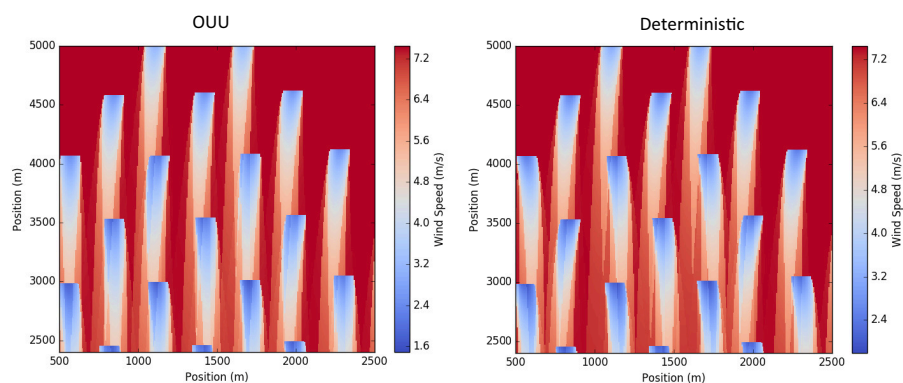


Figure 6. Close-up of the flow fields associated with the OUU and deterministic optimal yaw offset strategies. The deterministic strategy steers the wakes directly next to the downstream turbines, whereas the OUU solution requires less steering.

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