Online Data-Enabled Predictive Control

Chin-Yao Chang   Stefanos Baros   Andrey Bernstein

Background

Data-Enabled Predictive Control (DeePC)

- Follow the streamflow of data-driven rather than model-based approach: an alternative for model predictive control (MPC)
- Learn system "behaviour" instead of classical approaches that constructing parametric models
- DeePC is equivalent to MPC for linear time invariant (LTI) systems [J Coulson, J Lygeros, F Dörfler, 2019]
- The advantage over MPC may be limited for LTI cases as system identification together with predictive control for LTI systems is well established
- The potential of DeePC lies in *nonlinear or time-varying black-box systems that post challenges on system identification for MPC*

Behavioural Model for DeePC

1. Given a input/output sequence (u_t, y_t), $t = 1, \cdots, T$ of a system, construct the Hankel matrices in the following ($T_0 < L < T$)

$$\begin{bmatrix} U_p \\ U_f \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & \cdots & u_{T-L+1} \\ \vdots & \vdots & \cdots & \vdots \\ u_{T_0} & u_{T_0+1} & \cdots & u_{T-L+T_0} \\ \hline u_{T_0+1} & u_{T_0+2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ u_L & u_{L+1} & \cdots & u_T \end{bmatrix}, \quad \begin{bmatrix} Y_p \\ Y_f \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{T-L+1} \\ \vdots & \vdots & \cdots & \vdots \\ y_{T_0} & y_{T_0+1} & \cdots & y_{T-L+T_0} \\ \hline y_{T_0+1} & y_{T_0+2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & \cdots & y_T \end{bmatrix}$$

2. For LTI systems,
   - If $[U_p^\top, U_f^\top]^\top$ is of full row rank, then for every possible input/output trajectory $[u_{\text{ini}}^\top, y_{\text{ini}}^\top, u^\top, y^\top]^\top$, there exists g such that

$$Hg = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \\ y \end{bmatrix}, \quad H := \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} \tag{1}$$

   - If T_0 is large enough, then for a given $[u_{\text{ini}}^\top y_{\text{ini}}^\top]^\top$, then there exists a unique output y associated with control u

Contributions

Update the Behavioural Model Online

Update H in (1) based on the real-time input/output data $\Longrightarrow$ **applicable to nonlinear or time-varying black-box systems** in principle

Address the Complexity Issues

- Model (1) can involve a high dimensional H as (1) can be viewed as a "lifted linear model" for a nonlinear system
- Solving the predictive control problem with a high dimensional model (1) can be computational prohibited in real time
- Proposed a computational efficient method based on **primal-dual gradient method** and **Fast Fourier Transform (FFT)** to solve the predictive control problem in real time

Predictive Control based on the Behavioural Model

Solve the predictive control problem with time horizon N and the behavioural model constructed by real-time data

$$\underset{g\in\mathbb{R}^\kappa, u\in\mathcal{U}, y\in\mathcal{Y}}{\text{minimize}} \sum_{k=0}^{N-1} f(u_k, y_k), \quad \text{subject to } H^t g = \begin{bmatrix} u_{\text{ini}}^t \\ y_{\text{ini}}^t \\ u \\ y \end{bmatrix} \tag{2}$$

- The equality constraint replaces the state-space dynamical system in MPC
- Only the first element of the control of u, u_0, is implemented on the system, $u = [u_0^\top, \cdots, u_{N-1}^\top]^\top$
- Whenever the control u_0 is implemented, update the optimization for $t+1$ by updating H^t, u_{ini}^t and y_{ini}^t
- **Online control with (2) requires solving and updating optimization (2) frequently $\longrightarrow$ computational prohibit in real-time**

Online DeePC

Apply primal-dual gradient method and implement the control u_0 with N_I number of iterations (even if the algorithm has not reached the optimum)

$$z^{\tau+1} = \text{Proj}_{\mathcal{U}\times\mathcal{Y}\times\mathbb{R}^{\kappa+N_v}}\{z^\tau - \alpha^\tau \Psi^\tau(z^\tau)\} \tag{3}$$

- τ captures the algorithmic iteration number
- $z^\tau = [u^{\tau\top}\ y^{\tau\top}\ g^{\tau\top}\ v^{\tau\top}]^\top$ and v^τ is the dual variable

Ψ^τ refers to two different mappings depending on whether (2) is updated based on the latest input/output pair (or the control $u_0^{\tau-1}$ is implemented on the system) :

- If (2) is not updated: Ψ^τ is the gradient of the augmented Lagrangian of (2)
- If (2) is updated: Ψ^τ is a composite mapping of element shifting of z^τ and the gradient of the augmented Lagrangian
- The element shifting of z^τ "initializes" the solution for optimization at $t+1$ from the one of t

Under assumptions of bounded variation of the optima over time, strong monotonicity and Lipschitz continuity of Ψ^τ, we show that (3) converges Q-linearly to a bounded neighborhood of the optimal point

Computational Efficient Fast Fourier Transform (FFT)

- The most computational heavy parts of (3) lies in the matrix-vector multiplications $H^t g^\tau$ and $H^{t\top} v^\tau$
- H^t is a concatenation of block Hankel matrices which have the structures such that FFT is useful
- ***Developed a FFT-based method to reduce the complexity of*** $H^t g^\tau$, $H^t \in \mathfrak{R}^{m\cdot T\times\kappa}$***, from*** $O(m\cdot T\cdot\kappa)$ ***to*** $O(m\cdot\max(T,\kappa)\log(\max(T,\kappa)))$
- Considerably faster than direct multiplication for large T and κ

Numerical Validation

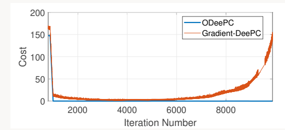
- Consider a time-varying dynamical system

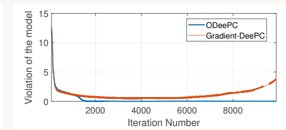
$$\begin{aligned} x_{t+1} &= A_t x_t + B_t u_t \\ y_t &= C x_t \end{aligned} \tag{4}$$

- Aim to track the output y_t to a reference signal. The objective function is given by

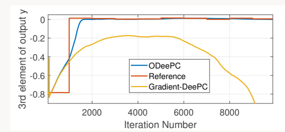
$$u^\top Q u + (y_t - r_t)^\top R (y_t - r_t), \quad Q \geq 0, \quad R > 0$$

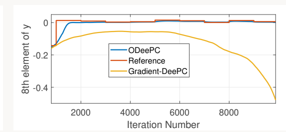
- A biased variation on B_t: the magnitude of each entry of B_t increases p% of its value whenever t changes to $t+1$, where p is randomly generated with a uniformly distribution in $[0, 0.01]$
- Initialize H by a sequence of input/output pair generated from (4)
- DeePC becomes infeasible with only a few updates of u_{ini} and y_{ini} (H kept unchanged)
- We simulate Gradient-DeePC which keeps H unchanged and updates z^τ by the gradient method





- As shown in the figures above, ODeePC keeps the cost close to zero while Gradient-DeePC gradually deviates from the optimum.
- Figures below show that the tracking performance of y for ODeePC and Gradient-DeePC. Gradient-DeePC again deviates from the reference due to the outdated behavioural model





- With the moderate size system of $x_t, u_t, y_t \in \mathfrak{R}^{10}$, FFT-based ODeePC takes 0.51 seconds on average with $N_I = 50$ inner gradient iterations, while the average for the direct multiplication is around 1.1 seconds.

Future Efforts

Address the Challenges for Real World Implementation

- High requirement on the measurement accuracy: the accuracy of the behavioural model drops noticeably with the increase of the measurement noise
- Can only be entirely model-free: the system may not be a black box in many applications. It could be grey box with limited knowledge of physics. DeePC or ODeePC can not incorporate those knowledge

This work was authored by the National Renewable Energy Laboratory, operated by Alliance for Sustainable Energy, LLC, for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. Funding provided by Funding provided by DOE Office of Electricity, Advanced Grid Modeling Program, through agreement NO. 33652. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

Forging Connections between Machine Learning, Data Science, & Power Systems Research
Alexandria, VA
March 5-6, 2020
NREL/PO-5D00-76263

Contact: {chinyao.chang,stefanos.baros,andrey.bernstein}@nrel.gov