# Impedance of Three-Phase Systems in DQ, Sequence, and Phasor Domains

IEEE PES Webinar, August 18, 2020

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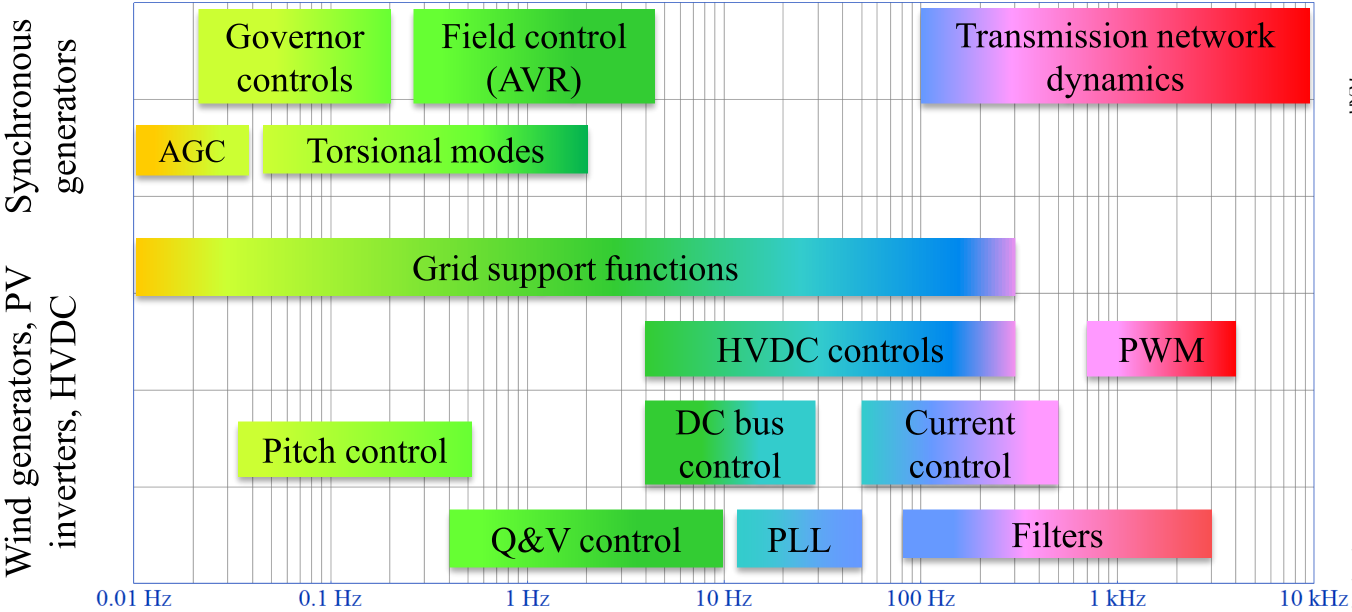
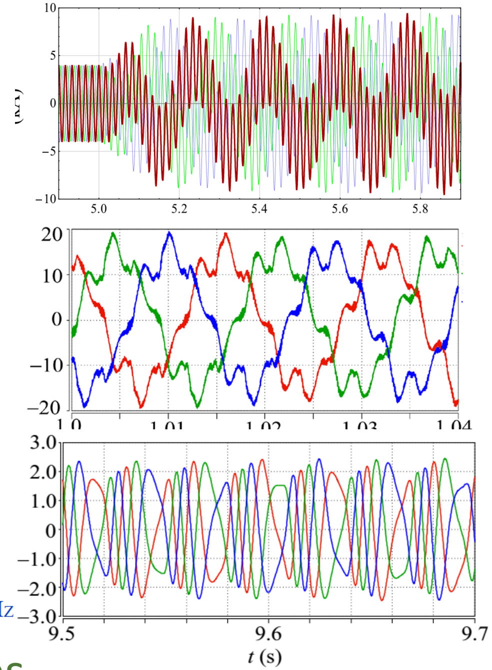
# Outline

- Impedance of three-phase systems
  - DQ and sequence domain
  - Correct definition of sequence impedance
    - Frequency coupling effects
  - Impedance measurement of a 1.9-MW wind turbine
- Impedance in phasor domain
- Coupling between AC and DC power systems

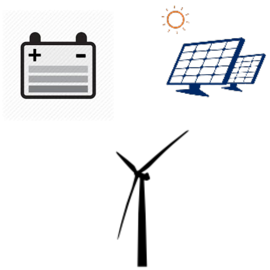
# New Stability Problems



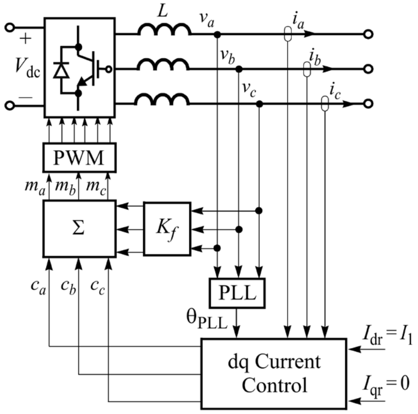



Challenges: (1) diversity of controls in inverter-based resources, (2) unavailability of high-fidelity dynamic models, and (3) complex dynamics

# Impedance-Based Stability Analysis



Inverter-based resource

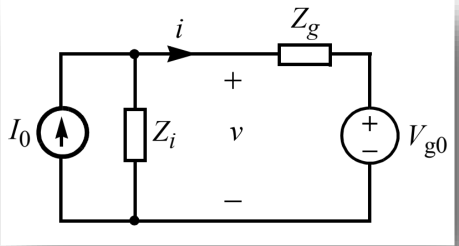


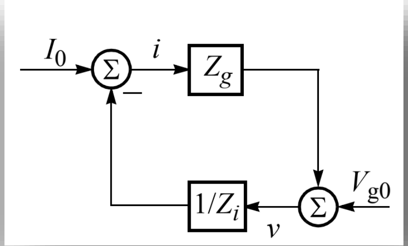

$Z_i(s)$

$Y_i(s)$

$Z_g(s)$

Grid




- Nyquist criteria is applied to $Z_g(s)/Z_i(s)$ or $Z_g(s) \cdot Y_i(s)$.
- **How are impedance $Z_i(s)$ or admittance $Y_i(s)$ defined?**

# Admittance of Three-Phase Systems

- DQ domain:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

where $\theta = 2\pi f_1 t + \phi_{\mathrm{v1}}$ = phase of $v_a$

$$\begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} = \begin{bmatrix} Y_{\mathrm{dd}}(s) & Y_{\mathrm{dq}}(s) \\ Y_{\mathrm{qd}}(s) & Y_{\mathrm{qq}}(s) \end{bmatrix}\begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix}$$

  - Reference frame defined by θ

- Sequence domain:

$$\begin{bmatrix} V_p(s) \\ V_n(s) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} V_a(s) \\ V_b(s) \\ V_c(s) \end{bmatrix}$$

where $a = e^{j2\pi/3}$

  - Uncoupled:

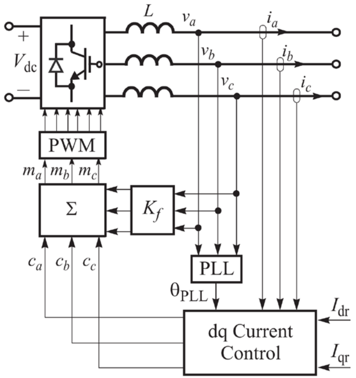
$$Y_p(s) = \frac{I_p(s)}{V_p(s)} \qquad Y_n(s) = \frac{I_n(s)}{V_n(s)}$$

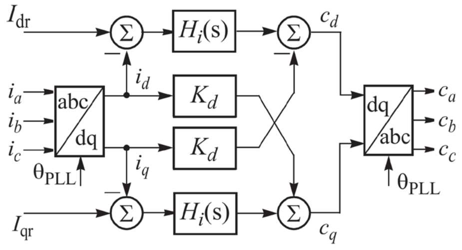
  - Coupled:

$$\begin{bmatrix} I_p(s) \\ I_n(s) \end{bmatrix} = \begin{bmatrix} Y_{\mathrm{pp}}(s) & Y_{\mathrm{pn}}(s) \\ Y_{\mathrm{np}}(s) & Y_{\mathrm{nn}}(s) \end{bmatrix}\begin{bmatrix} V_p(s) \\ V_n(s) \end{bmatrix}$$ ?????

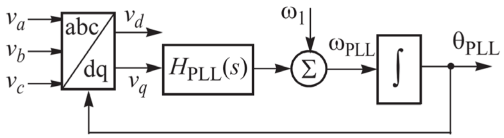
# Frequency Coupling Effects

- Three-phase VSC:



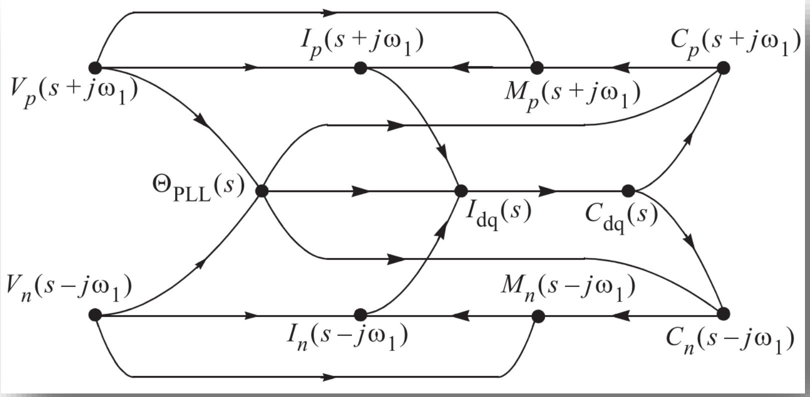




Current control




PLL

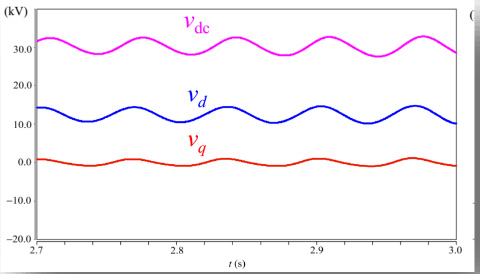
- Flow of perturbations:



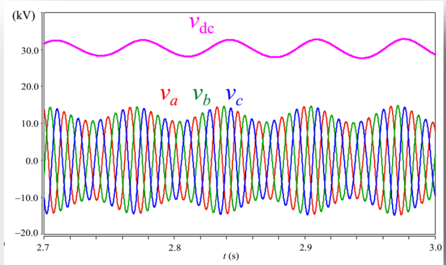

*All active devices—inverters, wind turbines, FACTS/HVDC, synchronous generators—have frequency coupling effects*

# Perturbation in DQ and Sequence

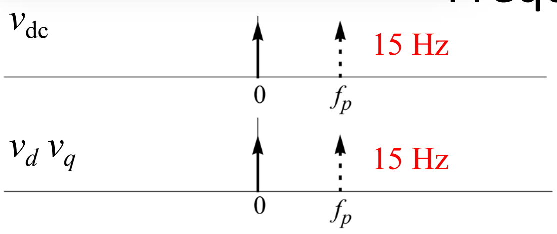
- DQ domain:

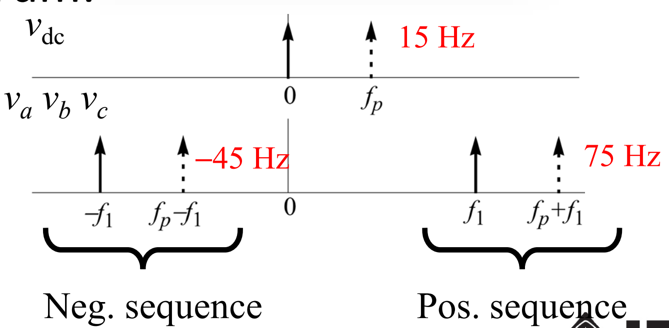



- Sequence domain:




Frequency spectrum:




**Note**: Mirror image perturbation components are not shown for clarity.

# Correct Definition of Sequence Admittance

- Sequence domain:

$$v_a = \overbrace{V_1 \cos(2\pi f_1 t + \phi_{v1}) + \hat{V}_p \cos[2\pi(f_p + f_1)t + \phi_{vp}]}^{\text{Positive Sequence}} + \underbrace{\hat{V}_n \cos[2\pi(f_p - f_1)t + \phi_{vn}]}_{\text{Negative Sequence}}$$

$$\begin{bmatrix} I_p(s + j\omega_1) \\ I_n(s - j\omega_1) \end{bmatrix} = \begin{bmatrix} Y_{pp}(s) & Y_{pn}(s) \\ Y_{np}(s) & Y_{nn}(s) \end{bmatrix} \begin{bmatrix} V_p(s + j\omega_1) \\ V_n(s - j\omega_1) \end{bmatrix}$$

  - Frequency coupling effects

- DQ domain:

$$v_d = V_1 \cos(\phi_{v1}) + \hat{V}_p \cos[2\pi f_p t + \phi_{vp}] + \hat{V}_n \cos[2\pi f_p t + \phi_{vn}]$$

$$v_q = V_1 \sin(\phi_{v1}) + \hat{V}_p \sin[2\pi f_p t + \phi_{vp}] - \hat{V}_n \sin[2\pi f_p t + \phi_{vn}]$$

$$\begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} = \begin{bmatrix} Y_{dd}(s) & Y_{dq}(s) \\ Y_{qd}(s) & Y_{qq}(s) \end{bmatrix} \begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix}$$

  - Relationship

$$\begin{bmatrix} \hat{V}_d(s) \\ \hat{V}_q(s) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} \hat{V}_p(s + j\omega_1) \\ \hat{V}_n(s - j\omega_1) \end{bmatrix} \quad \mathbf{Y}_{DQ} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \mathbf{Y}_{PN} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix}^{-1}$$

# Representations of Sequence Admittance

1. Transfer matrix:

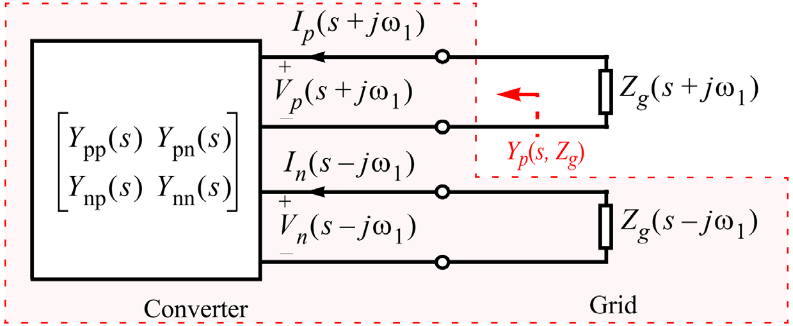
$$\begin{bmatrix} I_p(s+j\omega_1) \\ I_n(s-j\omega_1) \end{bmatrix} = \begin{bmatrix} Y_{pp}(s) & Y_{pn}(s) \\ Y_{np}(s) & Y_{nn}(s) \end{bmatrix} \begin{bmatrix} V_p(s+j\omega_1) \\ V_n(s-j\omega_1) \end{bmatrix}$$

2. SISO transfer functions:

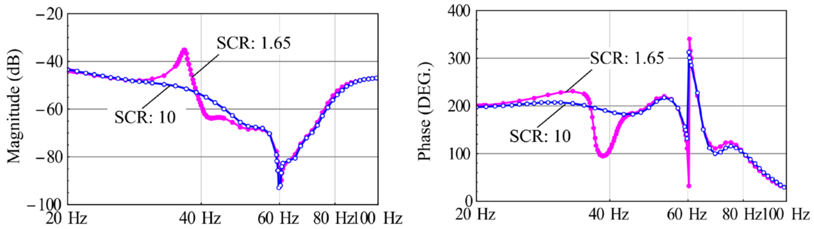
$$\left.\begin{aligned} Y_p(s) &= \frac{I_p(s)}{V_p(s)} \\ Y_{cp}(s) &= \frac{I_n(s-j2\omega_1)}{V_p(s)} \end{aligned}\right\} \quad \text{while } V_n(s-j2\omega_1) = 0$$

$$\left.\begin{aligned} Y_n(s) &= \frac{I_n(s)}{V_n(s)} \\ Y_{cn}(s) &= \frac{I_p(s+j2\omega_1)}{V_n(s)} \end{aligned}\right\} \quad \text{while } V_p(s+j2\omega_1) = 0$$

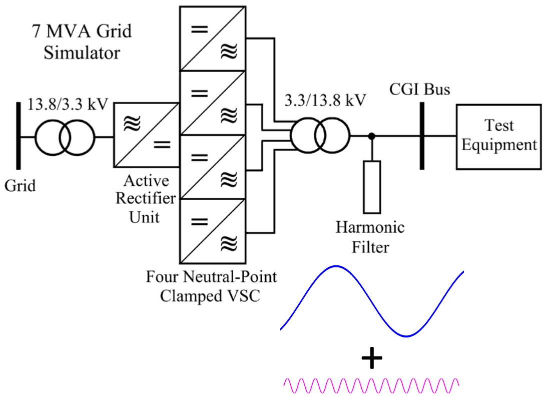
3. Grid-dependent impedance:




*Ex.: Positive-sequence admittance of a wind turbine:*



# Impedance Measurement at NREL



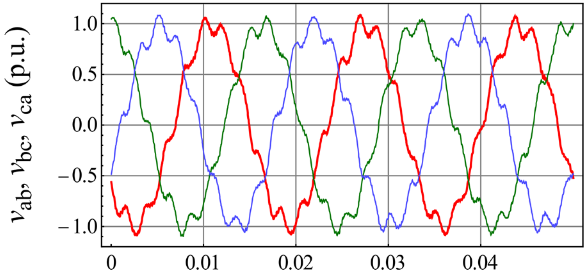

4-MW wind turbine drive

*Photo by NREL*

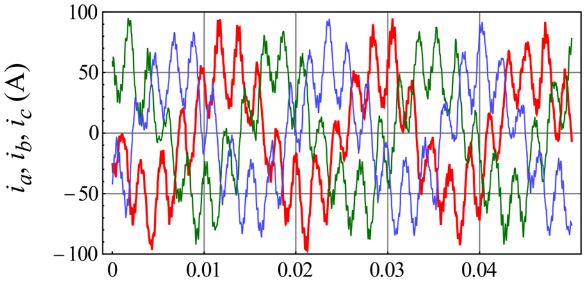
*Photo by NREL*

7-MVA grid simulator

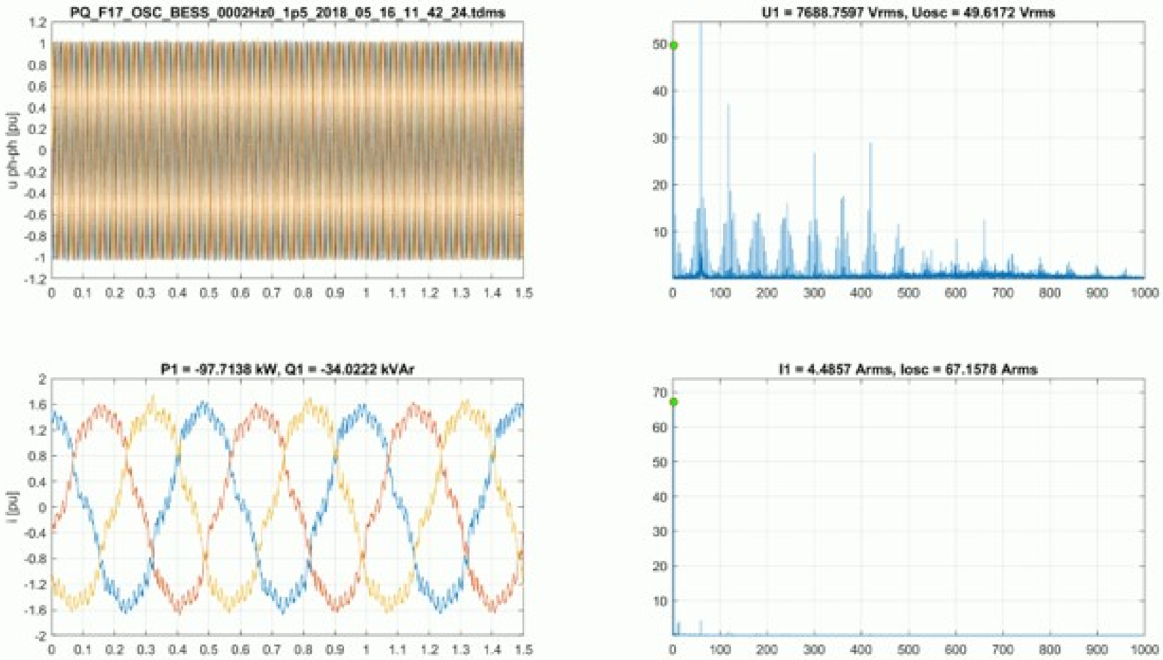
- Perturbed voltages:




- Response currents:



# Impedance Sweep




*Credit:*
*P. Koralewicz*



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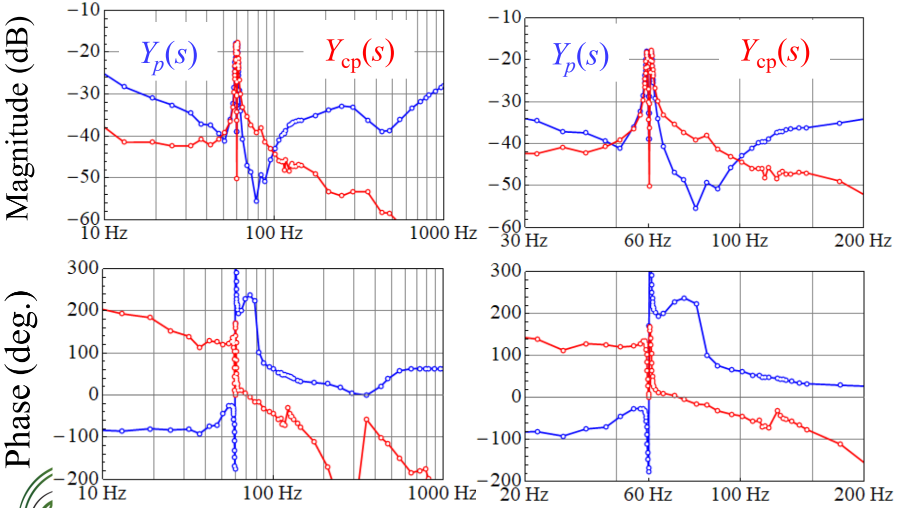
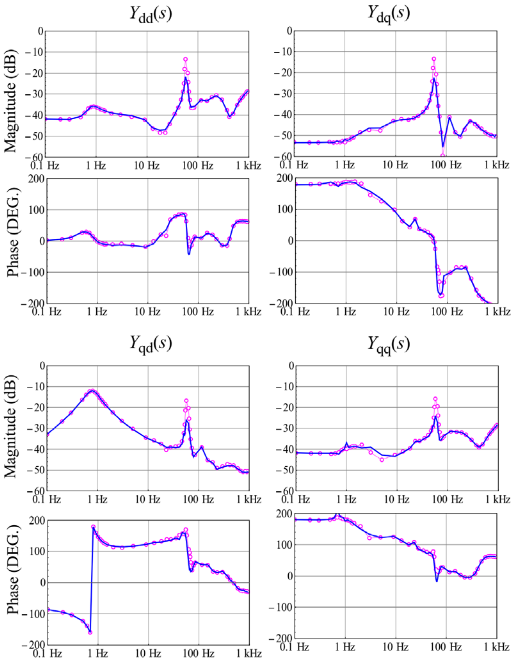
# Admittance of a 1.9-MW Wind Turbine

- Seq. admittance measurement:

$$\left.\begin{aligned} Y_p(s) &= \frac{I_p(s)}{V_p(s)} \\ Y_{cp}(s) &= \frac{I_n(s-j2\omega_1)}{V_p(s)} \end{aligned}\right\} \quad \text{while } V_n(s-j2\omega_1) = 0$$




- DQ admittance measurement




*Blue lines: derived from sequence measurements*

*Pink lines: direct DQ measurements*

# Reference Frame of Sequence Admittance

Positive-sequence admittance:

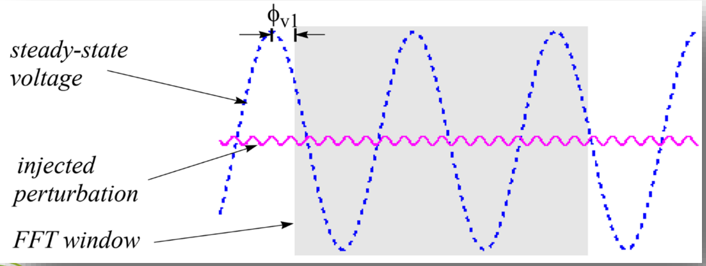
$$Y_p(s) = \frac{I_p(s)}{V_p(s)}$$

Coupling admittance:

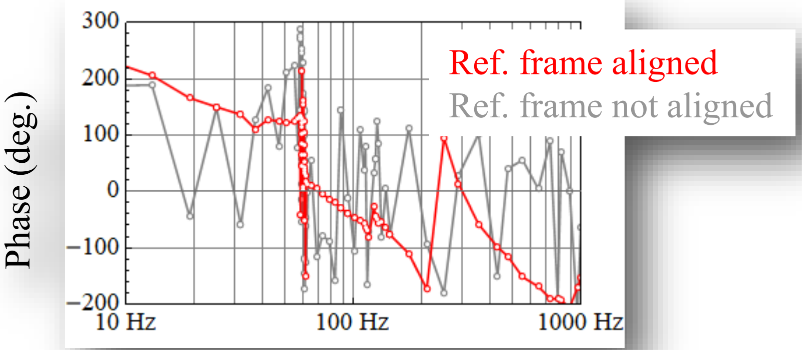
$$Y_c(s) = \frac{I_n(s - j2\omega_1)}{V_p(s)}$$

The reference frame of the sequence impedance is defined by the starting point of the data window used for fast Fourier transform analysis with respect to the fundamental trajectory of voltages.

$$\angle Y_c(s)\big|_{\phi_{v1}=0} = \angle Y_c(s)\big|_{\phi_{v1}=\alpha} + 2\alpha$$




Phase response of $Y_c(s)$



# Phasor Domain Impedance

- Perturbations in phasor domain:

$$v_a = [V_1 + \hat{V}_m \cos(2\pi f_p t + \phi_{vm})]\cos[2\pi f_1 t + \phi_{v1} + \hat{V}_\theta \cos(2\pi f_p t + \phi_{v\theta})]$$

$$v_b = [V_1 + \hat{V}_m \cos(2\pi f_p t + \phi_{vm})]\cos[2\pi f_1 t + \phi_{v1} - 2\pi/3 + \hat{V}_\theta \cos(2\pi f_p t + \phi_{v\theta})]$$

$$v_c = [V_1 + \hat{V}_m \cos(2\pi f_p t + \phi_{vm})]\cos[2\pi f_1 t + \phi_{v1} + 2\pi/3 + \hat{V}_\theta \cos(2\pi f_p t + \phi_{v\theta})]$$

- Phasor impedance:

$$\begin{bmatrix} V_m(s) \\ V_\theta(s) \end{bmatrix} = \begin{bmatrix} Z_{mm}(s) & Z_{m\theta}(s) \\ Z_{\theta m}(s) & Z_{\theta\theta}(s) \end{bmatrix} \begin{bmatrix} I_m(s) \\ I_\theta(s) \end{bmatrix}$$

*Maps magnitude and angle perturbations between voltages to currents*

- Relation to DQ and sequence:

$$\begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix} = \begin{bmatrix} \cos\phi_{v1} & -V_1 \sin\phi_{v1} \\ \sin\phi_{v1} & V_1 \cos\phi_{v1} \end{bmatrix} \begin{bmatrix} V_m(s) \\ V_\theta(s) \end{bmatrix}$$

$$\begin{bmatrix} V_p(s + j\omega_1) \\ V_n(s - j\omega_1) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^{j\phi_{v1}} & jV_1 e^{j\phi_{v1}} \\ e^{-j\phi_{v1}} & -jV_1 e^{-j\phi_{v1}} \end{bmatrix} \begin{bmatrix} V_m(s) \\ V_\theta(s) \end{bmatrix}$$



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# Phasor Impedance of Basic Elements

- Three-phase R-L branch:
  - Sequence impedance:

$$Z_p(s) = Z_n(s) = R + sL$$

  - Phasor impedance:

$$\mathbf{Z}_{\mathrm{M}\Theta} = \begin{bmatrix} |R + j\omega_1 L| + sL\cos\phi_{i1} & -sLI_1\cos\phi_{i1} \\ \dfrac{sL\sin\phi_{i1}}{V_1} & 1 + \dfrac{sL\cos\phi_{i1}}{|R + j\omega_1 L|} \end{bmatrix}$$

  - Phasor impedance in steady-state ($s$ = 0):

$$\begin{bmatrix} V_m(s) \\ V_\theta(s) \end{bmatrix} = \begin{bmatrix} |R + j\omega_1 L| & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_m(s) \\ I_\theta(s) \end{bmatrix}$$

- Constant power AC load:

$$\mathbf{S} = P + jQ = 3\mathbf{VI}^* = 3\left(\frac{V_1}{\sqrt{2}} e^{j\phi_{v1}}\right)\left(\frac{I_1}{\sqrt{2}} e^{j\phi_{i1}}\right)^*$$

  - Separating mag. and angles:

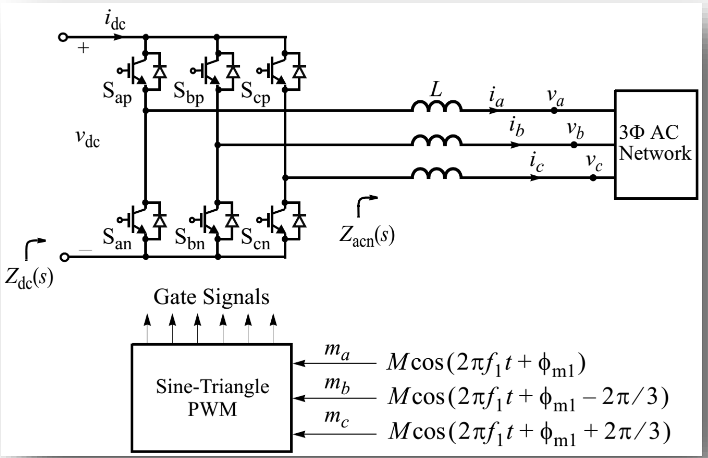
$$|\mathbf{S}| = \frac{3}{2} V_1 \cdot I_1 \qquad \arg[\mathbf{S}] = \phi_{v1} - \phi_{i1}$$

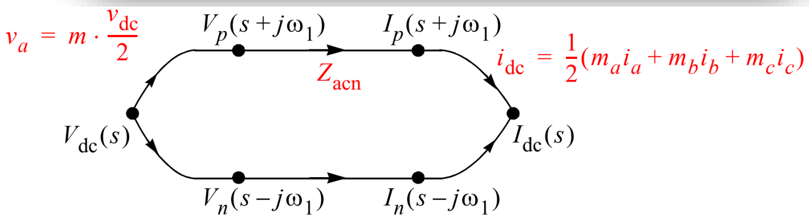
  - Taking differential:

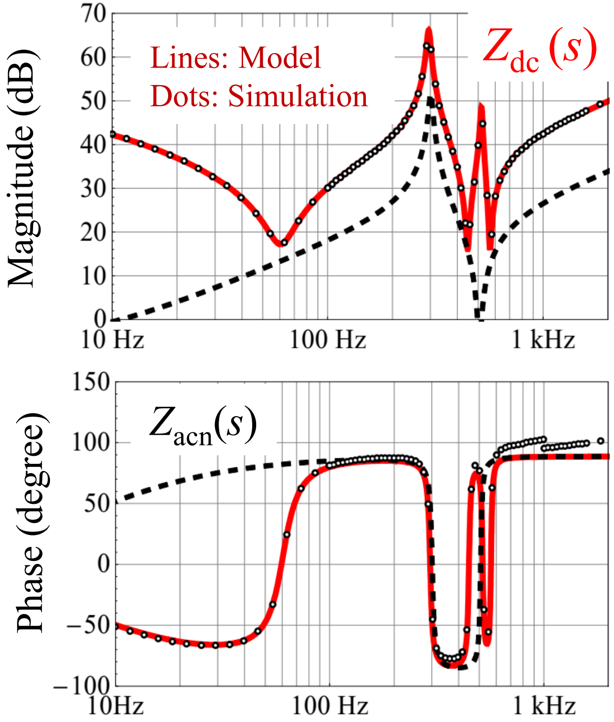
$$\frac{3}{2}\left(V_1 \cdot dI_1 + dV_1 \cdot I_1\right) = 0 \qquad d\phi_{v1} - d\phi_{i1} = 0$$

$$\begin{bmatrix} V_m(s) \\ V_\theta(s) \end{bmatrix} = \begin{bmatrix} -\dfrac{V_1}{I_1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_m(s) \\ I_\theta(s) \end{bmatrix}$$

# Coupling of AC and DC Power Systems

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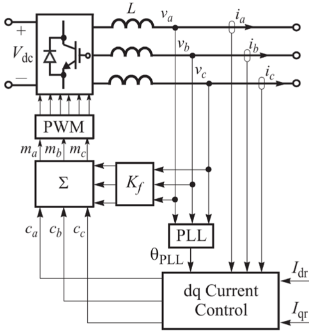
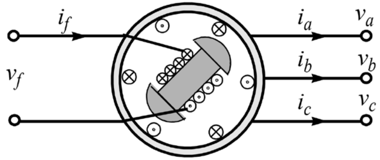
# Three-Port Model for AC/DC Coupling

- Inverters/HVDC:




$$\begin{bmatrix} I_{dc}(s) \\ I_p(s+j\omega_1) \\ I_n(s-j\omega_1) \end{bmatrix} = \begin{bmatrix} Y_{ss}(s) & Y_{sp}(s) & Y_{sn}(s) \\ Y_{ps}(s) & Y_{pp}(s) & Y_{pn}(s) \\ Y_{ns}(s) & Y_{np}(s) & Y_{nn}(s) \end{bmatrix} \begin{bmatrix} V_{dc}(s) \\ V_p(s+j\omega_1) \\ V_n(s-j\omega_1) \end{bmatrix}$$

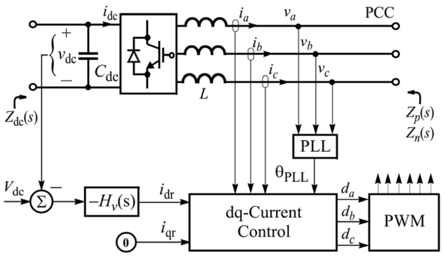
- Synchronous generators:




$$\begin{bmatrix} I_f(s) \\ I_p(s+j\omega_1) \\ I_n(s-j\omega_1) \end{bmatrix} = \begin{bmatrix} Y_{ss}(s) & Y_{sp}(s) & Y_{sn}(s) \\ Y_{ps}(s) & Y_{pp}(s) & Y_{pn}(s) \\ Y_{ns}(s) & Y_{np}(s) & Y_{nn}(s) \end{bmatrix} \begin{bmatrix} V_f(s) \\ V_p(s+j\omega_1) \\ V_n(s-j\omega_1) \end{bmatrix}$$

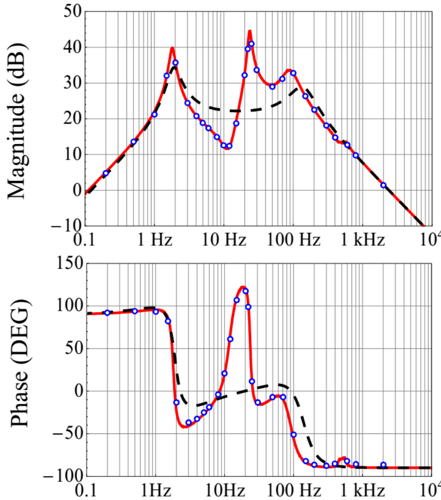
# AC and DC Impedance with Networks

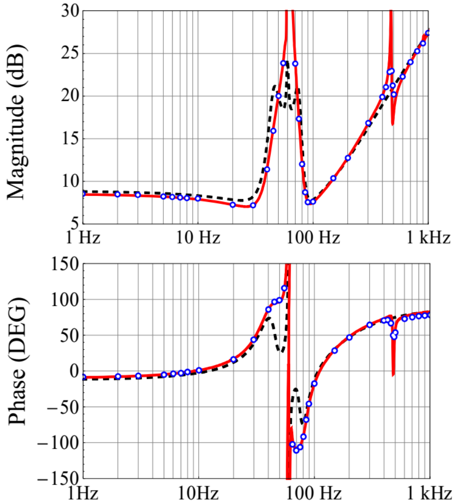
## HVDC converter:



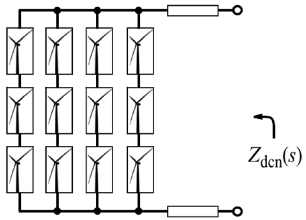

## DC impedance, $Z_{dc}(s)$:

## AC impedance, $Z_p(s)$:

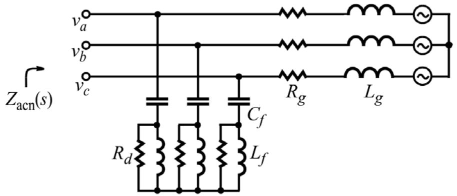



## DC network:




## AC network:




*Red lines: model considering network on other side*

*Black lines: model ignoring network on other side*

*Circles: simulations*



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# Summary

- Sequence impedance must account for frequency coupling effects.
- Sequence and DQ impedances are mathematically equivalent.
- Sequence impedance is better suited for stability analysis.
  - DQ impedance is sometimes better suited for modeling.
- Phasor impedance shows relationship with models used in transient stability programs.
- Three-port impedance models can be used for evaluating interactions between AC and DC power systems through HVDC converters and inverters.
- **Future development**: Use of correct definition of sequence impedance for stability and fault analysis.

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# Thank you!

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NREL/PR-5D00-77669

This work was authored by Alliance for Sustainable Energy, LLC, the manager and operator of the National Renewable Energy Laboratory for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. Funding provided by U.S. Department of Energy Office of Energy Efficiency and Renewable Energy Wind Energy Technologies Office. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

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