# IEA Wind Energy Task 37 System Engineering Aerodynamic Optimization Case Study

## Preprint

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# IEA Wind Energy Task 37 System Engineering Aerodynamic Optimization Case Study

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**This paper presents the results from the first IEA Wind Task 37 aerodynamic optimization case study. Eight participants applied their optimization tools to a purely aerodynamic problem and the results were compared. Overall, the different tools produced widely different designs, while there was better agreement in the improvement achieved. This highlights the fact that further investigation is needed to try to understand these differences and develop best practices. There were too many differences between the different analysis and optimization codes to determine the sources of these differences. However, several potential sources of discrepancy were identified for further investigation. One hypothesis is that one potential source of discrepancy is that the design problem itself is relatively flat in design directions of constant loading. This flatness would impact the convergence in the optimization, and at the same time mean that discrepancies in the design itself would be less severe.**

## I. Introduction

Numerical optimization is a powerful tool for design. Within the literature, there are many examples of highly advanced multi-disciplinary design optimization frameworks performing aero-servo-elastic design simultaneously [1–10]. Many of the different authors apply different techniques and optimization framework structures to achieve these results. One of the objectives of the IEA Wind Task 37 on system engineering is to conduct focused studies on numerical design optimization to obtain a better understanding of these methods and to develop a set of best practices. To make these investigations tractable, the first test case is a purely aerodynamic optimization test case that is described here along with the results. Eight institutes participated and applied their tools in this investigation. This paper shows the results of this comparison and includes a discussion.

This paper is organized into four sections. First, the description of the test case is given in Section II. For the purpose of reproduction, all the details of this test case are given in the appendix. Each participant was asked to report on the details of their analysis and optimization tools, along with amount of human and computational effort expended. These results are given in Section III. To quantify the differences in the analysis tools, each participant was asked to show the predicted performance of the initial design. These results are shown in Section IV. Finally, the participants were asked

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to optimize the turbine and share both the optimal designs and the predicted performance of that design. These results are shown in Section V. Finally, Section VI gives a discussion on these results, followed by conclusions in Section VII.

The exact source of all the results is kept anonymous for several reasons. First, the authors wants to avoid a comparison between the institutes themselves. Furthermore, the comparison is not completely fair. Some bias could have been introduced as the test case was developed by the tool chain of Participant 1. While the different participants have frameworks at different levels of maturity, developed for different purposes, thus some may be more or less suited for this particular test case. The focus of this study is on how all these differences in the analysis and optimization lead to different outcomes in the solutions.

## II. Test Case Description

The initial design for the test case is based loosely on the DTU 10-MW Reference Wind Turbine design [11]. Since this design was already optimized aerodynamically, there was some concern that this initial design would not give enough design freedom to allow for a meaningful comparison. To avoid this, the chord and twist distributions were scaled to reduce the overall loading and give a sub-optimal design. The modified chord and twist distributions are given in Figure 1 along with the data from the original DTU 10-MW Reference Wind Turbine design. The initial design was further modified to remove tilt, coning and pre-bend so that unsteady and 3D aerodynamic effects would not be relevant in the analysis.

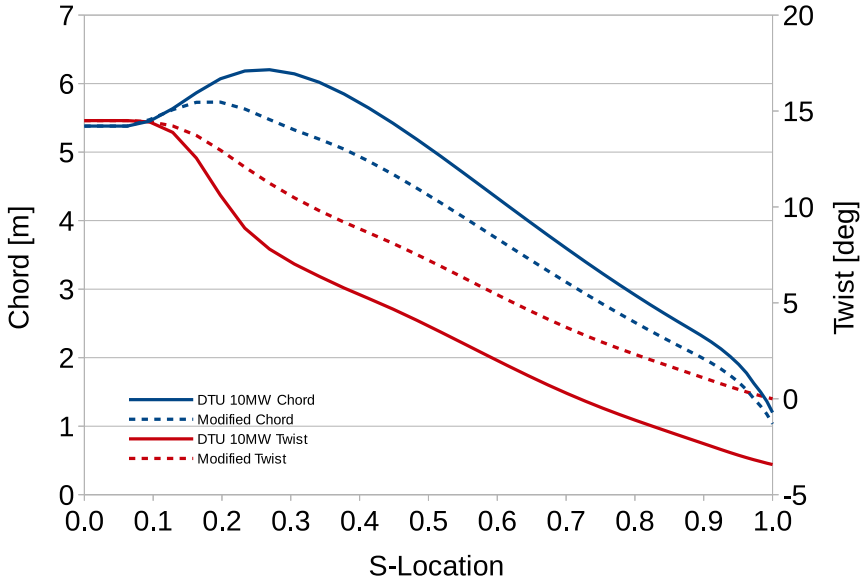


**Fig. 1 The chord and twist of the initial design compared with the DTU 10-MW Reference Wind Turbine**

Along with the plan-form properties, the test case also provided the airfoil data for a cylindrical profile, FFA-W3-600, FFA-W3-480, FFA-W3-360, FFA-W3-301 and FFA-W3-241 profile. The test case provided 2D airfoil shapes, but these data were not used in any of the optimization results given here.

The objective was to maximize the Annual Energy Production (AEP) according to a given wind speed distribution loosely based on a Class I site. The wind was assumed to be steady and uniform with no shear, turbulence or yaw error. The air density was specified to be $1.225 kg/m^3$.

Structural aspects were incorporated into the optimization problem by specifying a minimum absolute thickness constraint shown in Figure 2. Furthermore, a rotor thrust and root flap-wise bending moment constraint was included to limit the loads. To avoid tool differences affecting these constraints, these load constraints were specified relative to the corresponding load of the initial design. The constraint limit allowed for a 14% increase in the thrust along with a 11% increase in the root flap-wise bending moment. A combination of these two constraints would prevent the optimization from moving towards maximum $C_P$ designs.

To remove the effects of different controllers, the regulation of the turbine was specified. The maximum mechanical power was 10.6383 MW. The minimum and maximum rotational speed was 6.0 and 9.6 RPM respectively. The turbine

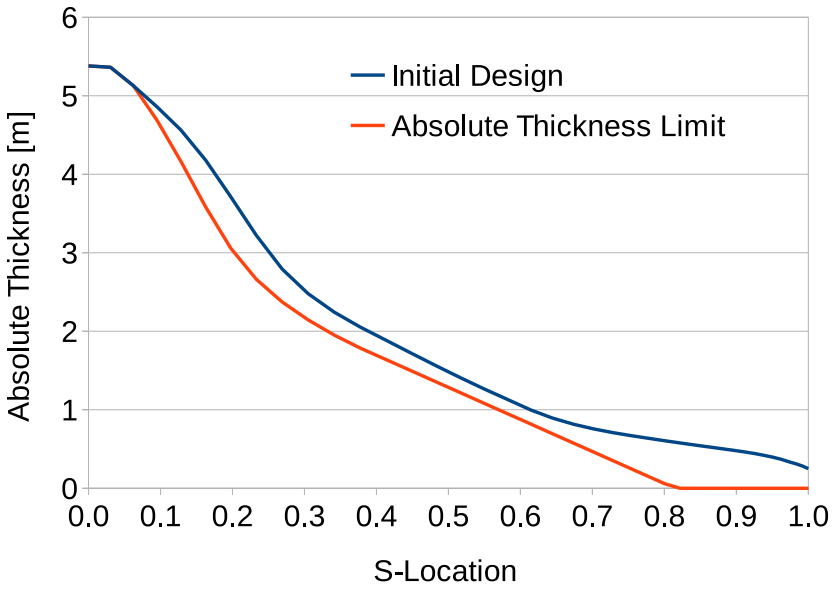


**Fig. 2 Minimum absolute thickness constraint**

is meant to operate as a standard pitch to feather regulated, variable speed machine. Within the variable speed region, the rotor should operate at a tip speed ratio of 7.8. Above rated conditions, the rotor should pitch to feather to avoid exceeding the maximum power constraint. For these high wind speeds, the participants were responsible for determining the required pitch according to their tools. The participants were free to use pitch to optimize performance when the rotor reached the minimum speed, but not in the variable speed region.

The design variable for this problem was the chord and twist distribution for the blade. Additionally, the participants could vary the airfoil distribution along the blade. The plan-form variables at the root of the blade are fixed in the optimization. Furthermore, the length of the blade nor any of the regulation parameters could be changed.

In summary, the optimization problem is given in Equation (1).

$$
\begin{aligned}
&\text{Maximize} && \text{Annual energy production} \\
&\text{Varying} && \\
& && \text{Chord} \\
& && \text{Twist} \\
& && \text{Relative thickness} \\
&\text{subject to} && \\
& && T \leq 1.14 \max T_0, \\
& && M \leq 1.11 \max M_0, \\
& && \text{Absolute thickness} \geq \text{limit} \\
& && \text{Basic regulation constraints}
\end{aligned}
\tag{1}
$$

For the purpose of reproduction, all the data for the test case is given in the Appendix A in tabular form.

It should be noted that the tool chain of Participant 1 was used to develop this test case. In developing the test case, multiple problems were solved with varying constraint limits. The test case here was chosen partly because it showed stable performance with this tool chain. Thus, there might be an unintentional bias in the results due to this.

## III. Participant Survey

To understand the tools behind the results, the participants were asked to fill out a survey on the analysis and optimization. Not all participants provided completed surveys furthermore, Participant 8 provided survey results and

no optimization results. However, to map these survey results to the optimization and analysis results, the participant numbering is kept consistent throughout the text. The participants were asked about whether they used various submodels (*e.g.* tip-loss, turbulent wake, *etc.*) within the analysis and to specify the versions of these submodels. Some users specified that they used a submodel but did not give any details; in these cases, the tables simply list "Yes".

Table 1 summarizes the results on the aerodynamic analysis. Overall, the participants were using standard Blade Element Momentum (BEM) models. Some users are using an unsteady BEM model typically these are corrected steady models, where the effect of the correction decays to zero in steady conditions. Another major difference is the use of a hub-loss model, which should affect the analysis towards the root portion of the blade. Some participants employ a yaw correction, but this should not be relevant in this test case.

**Table 1 Summary of the Aerodynamic Analysis Models**

| Participant | Aerodynamic Model | Angular Momentum | Tip Loss | Turbulent Wake | Other Sub-models |
|---|---|---|---|---|---|
| 1 | Steady BEM | Yes | Prandtl | Yes | Yaw |
| 2 | Unsteady BEM | N/A | N/A | No | N/A |
| 3 | Steady BEM | Yes | Prandtl | Burton | No |
| 4 | Unsteady BEM | Yes | Prandtl | Yes | Hub loss |
| 5 | Unsteady BEM | Yes | Prandtl | No | Hub loss |
| 6 | Steady BEM | Yes | Prandtl | Buhl | Yaw and reversed flow |
| 8 | Steady BEM | N/A | N/A | Yes | Hub loss |

The participants were asked about special features that may or may not be relevant to this test case. Participants 2, 3, 4 and 5 all reported using both a dynamic wake and a dynamic stall models in the analysis. The effect of these models should decay away in steady conditions. The analysis of participant 5 is based on the aero-structural model of FAST 8. The test case specifies that the blades should be rigid; thus, it is assumed that elastic deformation was not modeled. The model of Participant 3 is capable of solving the equations in the frequency domain; however this analysis is not relevant for this test case.

Table 2 summarizes the parameterization of the design variables and the optimization algorithms used by the participants. All the participants used splines in the parameterization; typically approximately 15 design variables were used in total, with one participant using 36 design variables. A wide variety of optimization algorithms and stopping criteria were used in the case study. All the optimization algorithms were gradient based, with the exception of participant 5 who used a genetic algorithm. Furthermore, a wide range of stopping conditions were employed. The wide variance in the optimization algorithms could be a source of differences in the optimal solutions.

**Table 2 Summary of the Parameterization and Optimization Algorithms**

| Participant | Parameterization | Number of Design Variables | Optimization Algorithm | Stopping Criteria |
|---|---|---|---|---|
| 1 | Bezier free form deformation splines | 13 | IPOPT [12] | $10^{-6}$ tolerance |
| 2 | $1^{st}$ order splines | 39 | MATLAB fmincon | $10^{-8}$ tolerance |
| 3 | Splines | 18 | Octave SQP | No decreasing direction |
| 4 | Splines | 15 | SQP | Iterative convergence |
| 5 | Splines | 15 | Genetic (NSGA-II [13]) | Number of generations |
| 6 | Splines | 15 | SQP | $10^{-6}$ tolerance |
| 8 | Akima | 14 | Sequential least squares | 200 Iterations Max |

The participants were asked to give additional details on the optimization. Participant 5 reported the following settings for the optimization: probability of mating two individuals CXPB = 0.9; crowding degree of mutation eta_mutate = 20; and crowding degree of cross over, eta_mate = 20. Participant 3 reported the use of penalty functions for the constraints. Participant 3 also reported that within the optimization only performed analysis below rated conditions. Participant 3 also normalized both the objective and the constraint values so that they would be close to unity.

Table 3 summarizes the coupling code and the gradient algorithms that were used in their framework. The coupling code describes how the analysis was coupled to the optimization. Most users used pure Python or MATLAB, while two participants use the dedicated package OpenMDAO. A majority of the participants used some form of finite differencing for the gradient calculations. However, Participant 6 used analytic gradients via Tapenade, which should improve the convergence in the optimization. Additionally, Participant 3 used complex step gradient algorithms, which should also improve the accuracy of the gradients and subsequently the convergence in the optimization as well. However, complex step methods may still have errors relative to analytic gradients. Participant 2 reported that they performed an extensive study to tune the gradient step-size.

**Table 3 Summary of the Coupling and Gradient Calculation**

| Participant | Coupling Code | Gradient Algorithm | Gradient Step Size |
|---|---|---|---|
| 1 | OpenMDAO | First order forward Euler | $10^{-2}$ |
| 2 | MATLAB | Finite difference | $10^{-6}$ |
| 3 | N/A | Complex step with some finite difference | $10^{-4}$ |
| 4 | N/A | Central differencing | 4% of bounds |
| 5 | OpenMDAO | N/A | N/A |
| 6 | Python | Reverse automatic differentiation with Tapenade | N/A |
| 8 | MATLAB and Python | Central differencing | $10^{-2}$ |

The participants were asked to describe additional special features of their optimization framework. Participant 5 reported that their tool is primarily used for offshore wind turbines.

The participants were asked to report on both the human effort and the computational effort that was required to obtain the results. Participant 2 described their human effort as follows: "A couple of days tuning the optimization. Unspecified amount of time on making the code sufficiently robust. 2 hours wrapping the simulation code for the optimization. A couple hours creating some post-processing functionality. 4-5 hours validating the optimization, more time could be needed. 3-4 hours setting up the input files. 1-2 hours fixing problems. 2 hours tuning the gradients. 1 day performing exploration studies. Minutes Setting up a single optimization." Participant 3 described their human effort as follows: "2 hours verifying complex step. 1 hour trying to speed up the code. 4 hours implementing a getAEP function. 2 hours writing post-processing functions. 4 hours validating the optimization. 2 hours creating a custom input for this problem."

Participant 2 described the computational effort for the problem as follows. They report both the calculation time for fmincon and a GA because they tried both in their preliminary studies, but they submitted the final results from fmincon. "With a I7-4790 CPU at 3.6 GHz with 16 GB RAM it took 10 hours calculation for all the studies, 1-2 hours for fmincon, 5 hours for GA." Participant 3 described the computational effort as follows: "With a serial CPU 200 hours of calculation for everything and 12 hours for a single calculation."

Overall, the survey results show that most of the users used a standard BEM model with the standard correction models. Thus, it is unlikely that the analysis should provide significant differences in the results. The biggest differences between the frameworks used by all the participants is in the set-up of the optimization. Most users employed a gradient-based algorithm with spline parameterization and finite difference gradients. A notable difference is Participant 5 who used genetic algorithms and Participant 6 who used analytic gradients.

## IV. Initial Design Performance

The first study in the comparison was of the performance of the initial design. There are two main sources of differences in comparing optimization results: the differences in the analysis for the same design and the difference in the optimal design. So, differences in the optimization results can be a result of both types of error. So The purpose of this study was to assess the level of agreement in the analysis and show the first main source of error.

Figure 3 shows the power predicted of the initial design by each of the participants. The figure shows that the differences in power can be seen, but they is small. Figures 4 and 5 shows the predicted values of the load constraints. The predicted thrust shows slightly larger differences, but it is the predicted root flap-wise bending moment that shows the greatest differences in the tools. Figure 6 shows the predicted pitch needed to maintain constant power above rated conditions. The figure shows only small differences, but the predicted pitch would have no impact on the AEP and no impact on the loads. Overall, if the differences in the optimization results are due to the analysis, they are first likely differences in the predicted bending moment, followed by thrust.

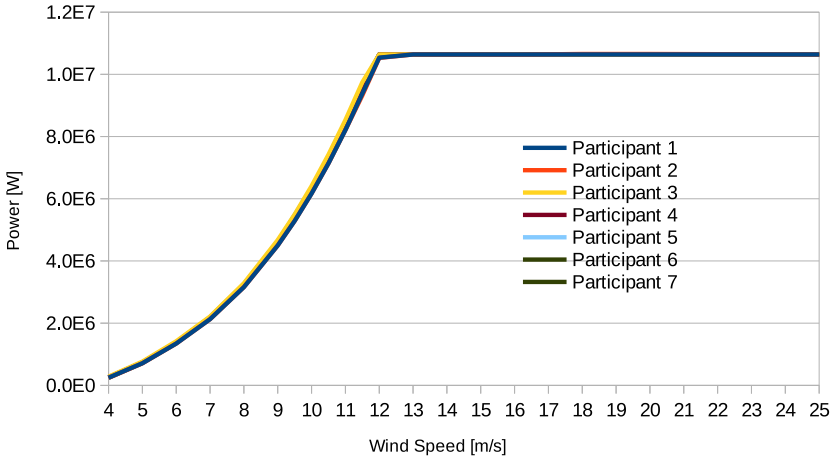


**Fig. 3 The power of the initial power**

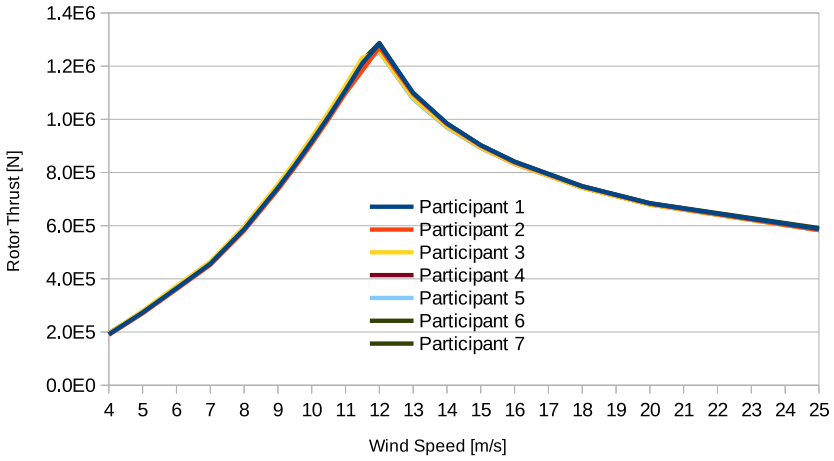


**Fig. 4 The power of the initial root flap-wise moment**

## V. Blind Results Comparison

All the participants were given the design problem without seeing the results of another participant. This prevented a user from tuning the optimization to acquire results in better agreement. Thus, the results presented here show the full range of different solutions one could be expect to obtain using the different optimization methods and analysis tools used in this study.

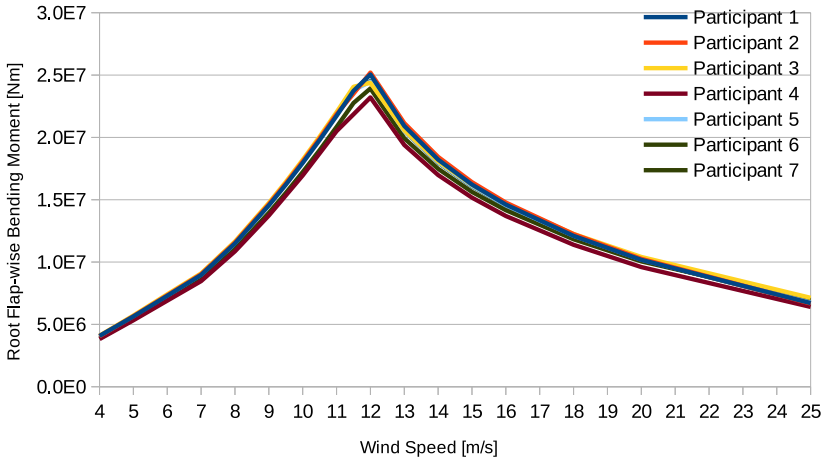


**Fig. 5 The power of the initial root flap-wise moment**

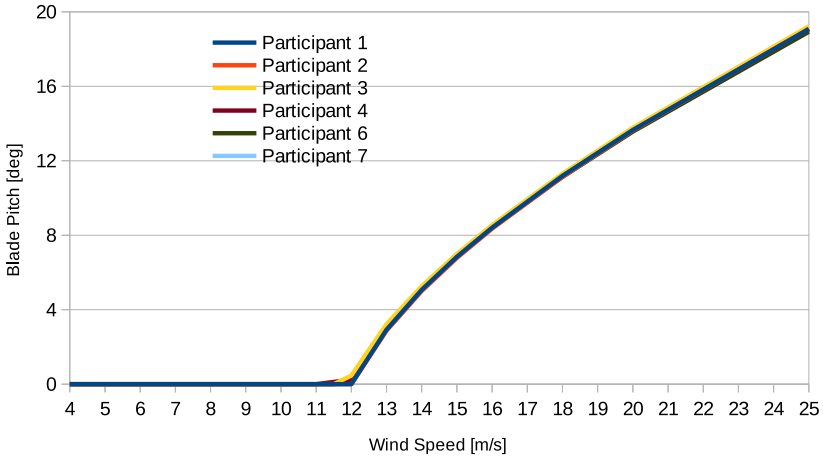


**Fig. 6 The power of the initial root blade pitch**

This section is divided into two sections. The first (Section V.A) shows the designs obtained from each of the different participants in the optimization. The second (Section V.B) gives the performance of the optimal designs as reported by the different participants.

Participant 3 provided a second set of results that did use pitch in the variable speed region for peak-shaving; because of the increased design freedom, these results are not directly comparable to the other results. For the sake of interest, these results will still be presented. They are differentiated by the tag "fix" or "free", where the fixed results had fixed pitch in the variable speed region and free allowed for peak shaving; only the fixed results are consistent with the results of the other participants.

### A. Optimal Design

Figures 7 through 9 show the optimal planform design, as predicted by the different participants. All the optimization results showed large increases in the chord, especially towards the root of the blade. These designs are not realistic, since they would exceed the maximum chord for industrial design. A maximum chord constraint was not included intentionally so that the design is purely a result of the optimization and not due to an a-priori constraint limit value. Furthermore, since this is a purely aerodynamic optimization, Figure 9 is showing that the different optimizations are driving towards thinner and more efficient airfoil sections.

Overall, the different designs do not show a strong agreement, with the largest differences occurring towards the root of the blade and again showing divergence towards the tip of the blade. Yet in the mid-span of the blades, the different results showed better agreement. It should be noted that Section V.B will show that participant 2 did not achieve full convergence, so differences between Participant 2 and the others may be attributed to a lack of convergence. Further discussion of these differences will be given in Section VI.

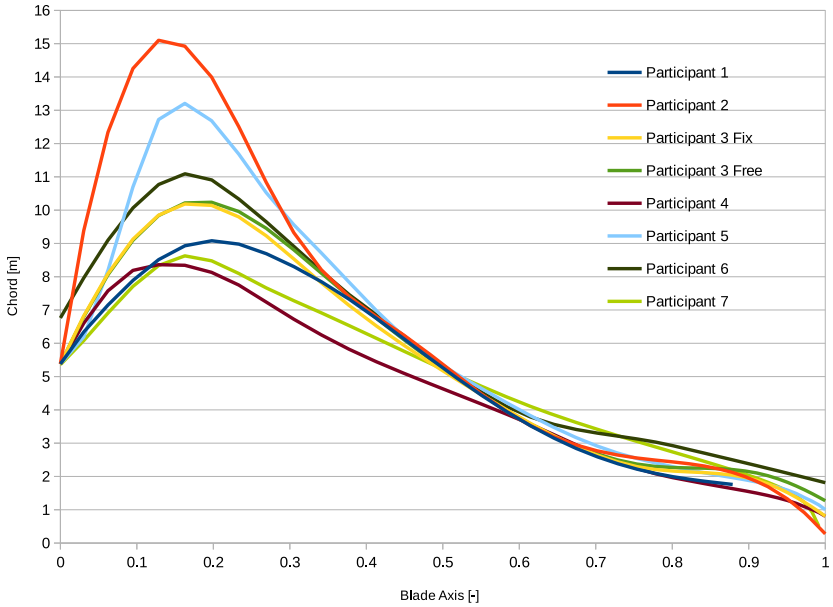


**Fig. 7 The chord of the optimal design**

Figure 10 shows the absolute thickness for each of the optimal designs. Most of the results respected the absolute thickness limit. However, Participant 2 showed some minor violations of this constraint for both the "fixed" and "free" results. This constraint is active in the mid-span for most participants. This could explain why better agreement in both chord and twist was achieved in this region, as one aspect of the design is specified by this limit.

Figure 11 shows the pitch schedule of all the different optimization results. Overall, all the participants pitched the blade to feather in the lower wind speeds to lower the thrust to avoid turbulent wake state. Then later all the participants used pitch to maintain constant power above rated. In these two regions, most of the results are within 2° of each other.

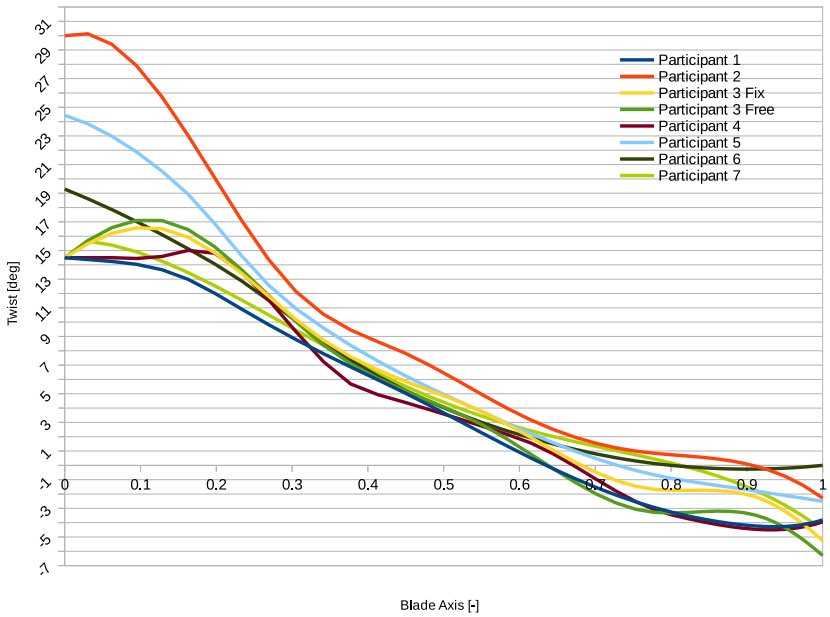


**Fig. 8 The twist of the optimal design**

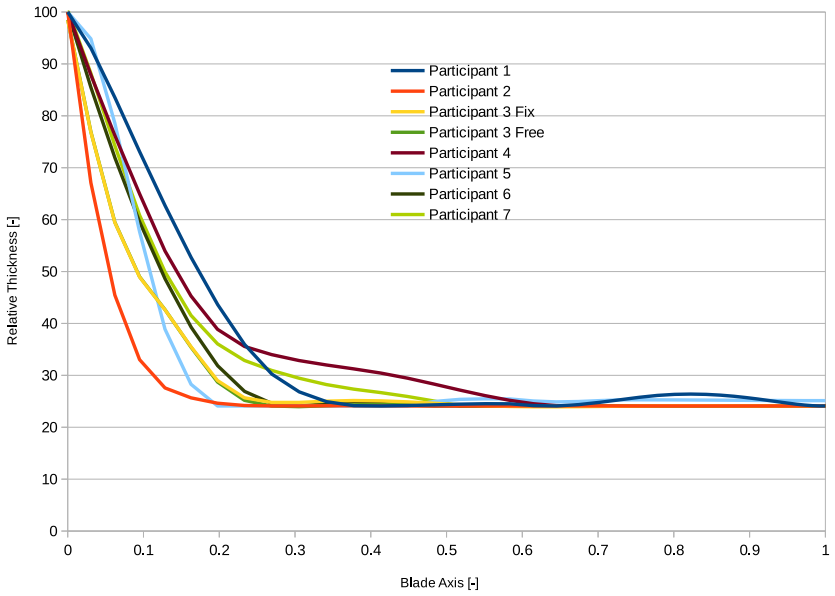


**Fig. 9 The relative thickness of the optimal design**

### B. Optimal Performance

Table 4 shows all but one of the participants were able to achieve an improvement between 10 and 12.5% in the AEP. The table shows that neither load constraint is active for Participant 2, while their improvement in AEP is below that of the other participants. This shows that it is likely that their optimization was not able to achieve full convergence. For the remaining participants, the root flap-wise bending moment constraint is active, and Participant 5 is exceeding this constraint. The thrust constraint is only active for Participant 1.

Figure 12 shows the optimal power curve for all the participants. As expected, all the participants improved the power below rated conditions. Figure 13 shows the increase in AEP achieved at each of the different wind speeds, while

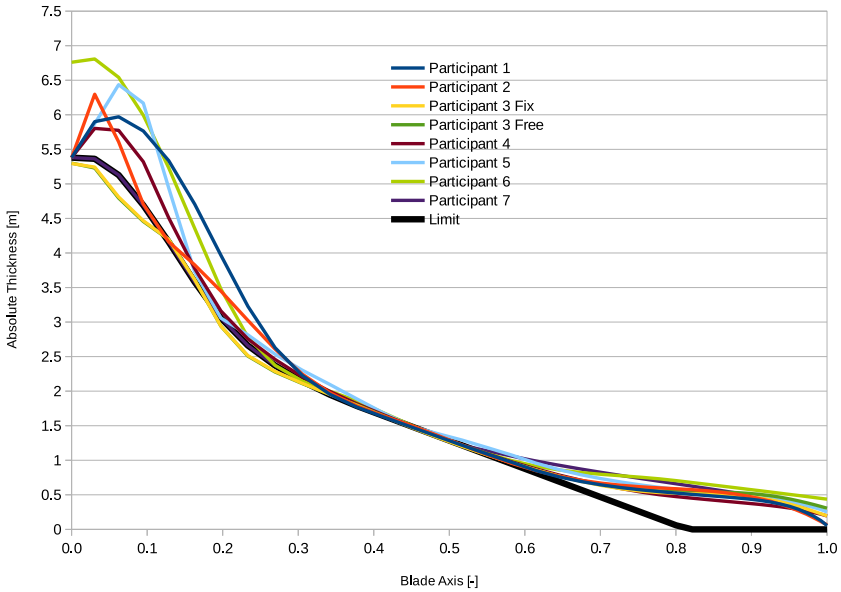


**Fig. 10 The absolute thickness of the optimal design**

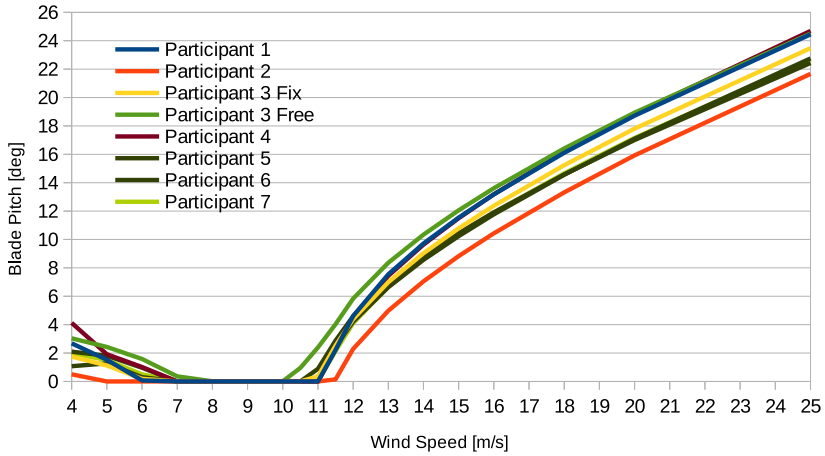


**Fig. 11 The pitch of the optimal design**

Figure 14 shows the ratio of power versus the power of the initial design. The results show that most of the improved production is occurring in the variable speed region. Furthermore, the different optimization frameworks are showing good agreement in power in this region. The greatest differences in power occur when the minimum rotor speed is reached and the optimization can use pitch to avoid turbulent wake state. Here the different frameworks achieved between no improvement and 36%.

Figure 15 shows the thrust curves of the optimal designs, while Figure 16 shows the root flap-wise bending moment of the optimal designs. As expected for all optimization results, the thrust and root flap-wise bending moment increased up until rated conditions and then decreased once the blades pitched to feather. Again, the free results from participants 3 are not comparable, while the Participant 2 optimization is likely not converged. Aside from these two results, the thrust shows good agreement between the participants. However, the bending moment results show the greatest disagreement between all the participants. This is consistent with the initial design results.

**Table 4 Improvement in AEP**

| Participant | Initial AEP | Optimal AEP | AEP Increase | Thrust Increase | Flapwise Bending Moment Increase |
|---|---|---|---|---|---|
| 1 | 28.4 GWh | 31.9 GWh | 12.44% | 13.98% | 10.98% |
| 2 | 28.7 GWh | 31.3 GWh | 9.20% | 6.03% | 1.63% |
| 3 Fix | 29.5 GWh | 32.8 GWh | 11.11% | 12.65% | 11.00% |
| 3 Free | 29.5 GWh | 33.0 GWh | 11.71% | 9.95% | 11.00% |
| 4 | 29.2 GWh | 32.3 GWh | 10.49% | 13.11% | 10.95% |
| 5 | 29.3 GWh | 32.8 GWh | 11.99% | 13.78% | 11.41% |
| 6 | 29.3 GWh | 32.2 GWh | 10.11% | 12.86% | 11.00% |
| 7 | 29.0 GWh | 32.3 GWh | 11.36% | 12.86% | 10.40% |

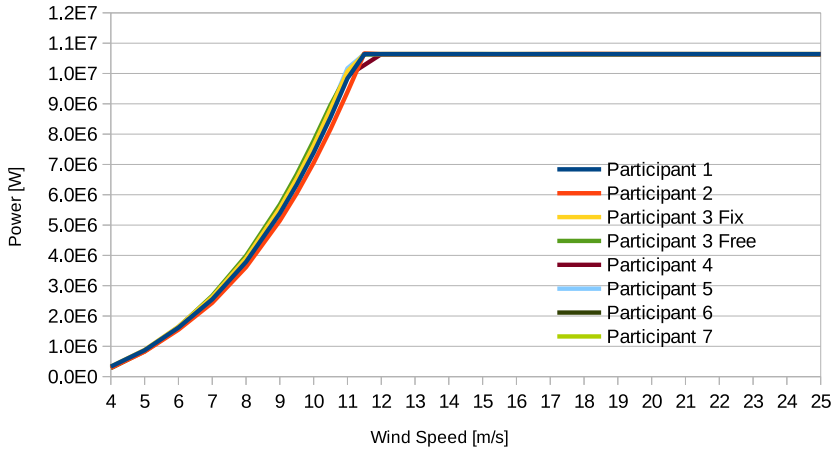


**Fig. 12 The power of the optimal design**

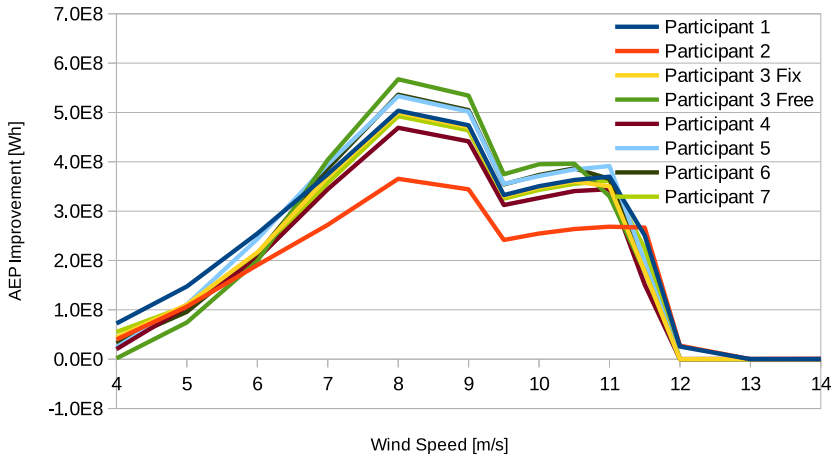


**Fig. 13 The relative AEP of the optimal design**

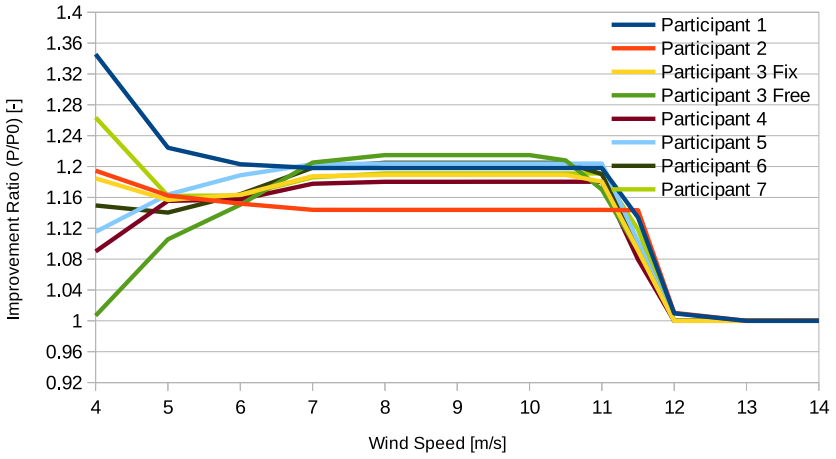


**Fig. 14 The relative power of the optimal design**

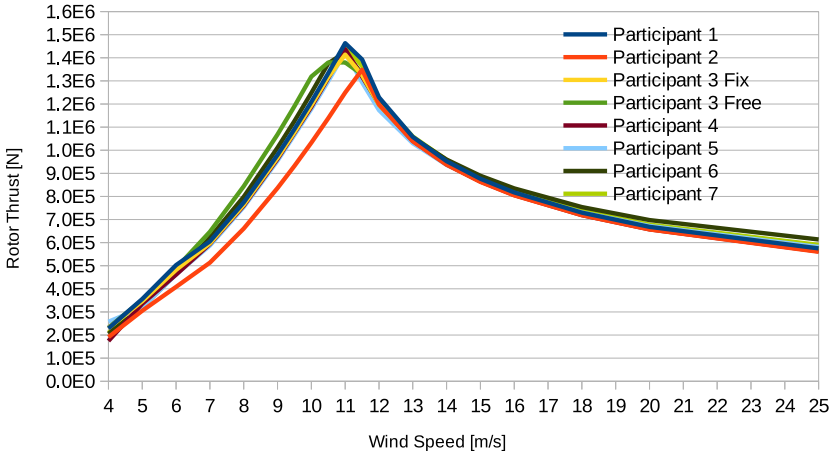


**Fig. 15 The thrust of the optimal design**

## VI. Discussion

The ultimate goal of design optimization is to determine the design configuration that would result in the best performance. The fact that there was a wide range of differences in the design configurations shows that further work is needed to advance the field of numerical aerodynamic optimization for wind turbines. These differences can be explained by the test case itself, as the participants were free to choose their tools and optimization setup. This opens up a lot of different variables that could explain the differences seen here. However, this also shows the range of outcomes one could achieve using the different tools that are available, which is an indication of the maturity of aerodynamic design optimization in the research community. This further shows that more effort is needed to develop a better understanding of aerodynamic optimization and what leads to the differences shown here.

Optimization test cases are different from analysis test cases because there are three ways that the final results can vary. First, common to both types of test cases, is the differences in the predicted performance of different analysis codes for a common design. This will be referred to as Type I discrepancies. The analysis tool can introduce additional discrepancies, since it also defines the optimal design point that is being sought by the optimization. Thus, perfectly converged optimization can arrive to different points based on analysis differences as well. This will be referred to as a Type II discrepancy. The third set of differences is in the optimization algorithms employed by the different users, ultimately this is a matter of the degree of convergence that was achieved and where in the design space the optimization explored. This is referred to as a Type III discrepancy. These differences are shown schematically in Figure 17.

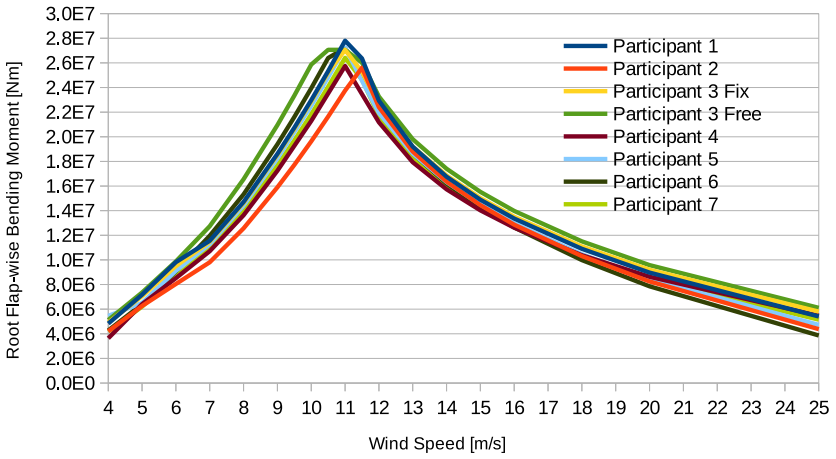


**Fig. 16 The root flap-wise bending moment of the optimal design**

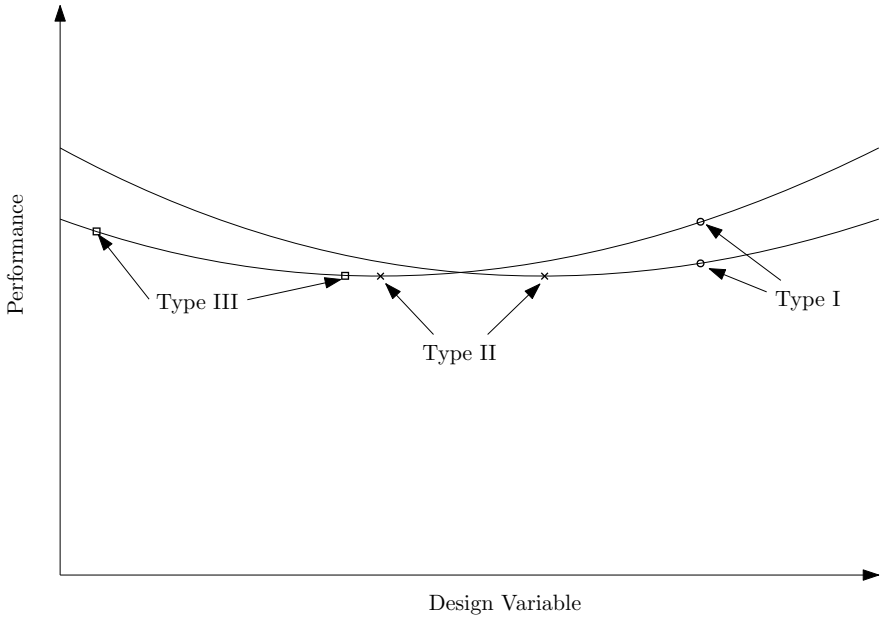


**Fig. 17 Example of the types of discrepancies in the optimization test case**

The relatively good agreement in the initial results (see Section IV) shows that the differences in the optimal designs cannot be attributed to Type I discrepancies alone. Yet, the freedom in the optimization test case makes it difficult to separate out Type II and Type III differences. Thus, we can only speculate on the sources of these differences and suggest future studies. It is likely that there are Type II discrepancies, as that the thrust constraint was only active for Participant I.

Type II discrepancies can potentially be explained by the BEM equations. The optimal power in the BEM equations is strongly defined by the overall loading [14]. This loading can be increased by increasing the chord and decreasing the twist. The results in Section V.A show that a high optimal chord corresponds to a high twist and vice versa. Thus, the different optimization solutions could be arriving at very similar loading levels. Once the loading level is fixed, the optimization is only varying the operating angle of attack. In this regime, the power is only affected by small changes in the lift to drag ratio (i.e. aerodynamic efficiency). Thus, the problem likely has a high degree of variation in design directions corresponding to changes in loading, while it has small variations in other directions. So the design problem may be flat in the direction of constant loading compared to directions that vary the loading. Figure 17 shows how

flatness could lead to large Type II discrepancies, despite Type I discrepancies being small. Thus, the relative flatness between these two design directions could make the BEM equations inherently sensitive to Type II discrepancies. When the design space is flat, large differences in the designs have a smaller impact on the performance and thrust. In this case, the differences shown in Section V.A are more acceptable.

Type I and II errors are explained by the analysis codes. These differences may be due to the different turbulent wake models, different tip loss functions and possibly different corrections to the airfoil data that were used by the participants. The impact of these different submodels on the optimal design has not been quantified, however the results here show some anecdotal evidence that differences in the analysis could explain some of the differences. For example, the wide variation in power improvement in the low wind speeds can be attributed to differences in the turbulent wake model. The large variations in design towards the tip and root of the blade could be attributed to different hub and tip loss models.

The effects of Type III discrepancies can manifest in two ways. First, the optimization problem could have multiple local minimum, and thus, different algorithms, perfectly converged, could lead to these different solutions. The optimization algorithm used by Participant 5 is less sensitive to local minimum; their optimal designs shows a larger chord and twist distributions and thinner airfoil sections than most of the other participants. At the same time, Participant 5 achieved a better performance than most participants. Furthermore, the design of Participant 5 is further from the initial design in the design space, thus, it is a possibility that the gradient-based results are getting trapped in a local minimum. Finally, the results of Participant 2 show that failing to converge can sacrifice 1–2% of potential improvement in AEP. Since the convergence of smooth convex optimization problems are based on the gradients of the problem, a very flat problem can also lead to large discrepancies due to different convergence tolerance and errors in the gradients.

Further information on Type I and II errors could be found by performing a cross comparison, where each of the participants simulated the solutions of the other participants. These calculations could not be completed in the time frame for this study.

Another source of discrepancy is in the relative maturity of the different optimization frameworks. Some of the participants in this study have a long history in numerical wind turbine design optimization, while others were actively developing their optimization tools during the study.

## VII. Conclusions

An aerodynamic optimization test case was developed loosely based on the DTU 10-MW Reference Wind Turbine. The test case is purely aerodynamic, but thrust, moment and absolute thickness constraints indirectly introduced some structural aspects. To focus on the performance of the optimization, a maximum chord constraint was not used, leading to designs with chords too large to be relevant to industry.

Multiple institutes participated in this test case. A survey of the tools and optimization set-up showed that the greatest differences were more in the optimization algorithms than in the analysis. Yet, there were still differences in the analysis codes. The analysis of a fixed initial design did show some small differences in the predicted performance between the different analysis codes. Thus, differences in the analysis is a factor in these comparisons.

The different optimizations produced widely different optimal designs. This wide differences in the designs is problematic, since it shows that there is a large uncertainty in the solutions. The differences in the optimal performance was not as stark though. One explanation for this is that the design problem itself may be flat and that flatness could explain the differences. If the design problem is flat, the impact of large design differences is not as severe.

Since the users were free to choose any combination of analysis and optimization methods, there are many possible sources for these differences. Thus, it is difficult for this study to explain the differences in the results definitively. Neither differences in the analysis nor the optimization can be ruled out. The paper discusses three potential types of discrepancies and gives examples of how they could have manifested in this study. The different sources of discrepancies should be the focus of future investigations of numerical aerodynamic design optimization of wind turbines.

# A. Test Case Data

To enable future research to use this test case, all the data for the test case are given here in tabular form. These data are given in Tables 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 and 21.

**Table 5 Initial planform design**

| *S* | *X* | *Y* | *Z* | Twist [deg] | Chord [m] | Lateral Shift [-] |
|---|---|---|---|---|---|---|
| 0.00000 | 0.0 | 0.0 | 0.000 | 14.500 | 5.380 | 0.500 |
| 0.03057 | 0.0 | 0.0 | 2.643 | 14.500 | 5.380 | 0.500 |
| 0.06222 | 0.0 | 0.0 | 5.380 | 14.500 | 5.380 | 0.500 |
| 0.09487 | 0.0 | 0.0 | 8.203 | 14.445 | 5.474 | 0.494 |
| 0.12841 | 0.0 | 0.0 | 11.103 | 14.222 | 5.620 | 0.472 |
| 0.16274 | 0.0 | 0.0 | 14.071 | 13.727 | 5.727 | 0.440 |
| 0.19772 | 0.0 | 0.0 | 17.095 | 12.959 | 5.730 | 0.412 |
| 0.23321 | 0.0 | 0.0 | 20.164 | 12.078 | 5.630 | 0.391 |
| 0.26907 | 0.0 | 0.0 | 23.265 | 11.236 | 5.475 | 0.376 |
| 0.30515 | 0.0 | 0.0 | 26.384 | 10.476 | 5.325 | 0.365 |
| 0.34128 | 0.0 | 0.0 | 29.508 | 9.806 | 5.190 | 0.358 |
| 0.37731 | 0.0 | 0.0 | 32.623 | 9.206 | 5.044 | 0.353 |
| 0.41309 | 0.0 | 0.0 | 35.716 | 8.656 | 4.869 | 0.350 |
| 0.44846 | 0.0 | 0.0 | 38.773 | 8.102 | 4.676 | 0.350 |
| 0.48328 | 0.0 | 0.0 | 41.782 | 7.517 | 4.473 | 0.350 |
| 0.51741 | 0.0 | 0.0 | 44.732 | 6.916 | 4.263 | 0.350 |
| 0.55073 | 0.0 | 0.0 | 47.611 | 6.315 | 4.052 | 0.350 |
| 0.58312 | 0.0 | 0.0 | 50.410 | 5.727 | 3.844 | 0.350 |
| 0.61449 | 0.0 | 0.0 | 53.120 | 5.164 | 3.642 | 0.350 |
| 0.64476 | 0.0 | 0.0 | 55.734 | 4.635 | 3.449 | 0.350 |
| 0.67385 | 0.0 | 0.0 | 58.247 | 4.147 | 3.265 | 0.350 |
| 0.70172 | 0.0 | 0.0 | 60.653 | 3.701 | 3.093 | 0.350 |
| 0.72833 | 0.0 | 0.0 | 62.950 | 3.298 | 2.931 | 0.350 |
| 0.75364 | 0.0 | 0.0 | 65.135 | 2.935 | 2.780 | 0.350 |
| 0.77766 | 0.0 | 0.0 | 67.208 | 2.608 | 2.641 | 0.350 |
| 0.80038 | 0.0 | 0.0 | 69.167 | 2.314 | 2.512 | 0.350 |
| 0.82181 | 0.0 | 0.0 | 71.016 | 2.045 | 2.395 | 0.350 |
| 0.84197 | 0.0 | 0.0 | 72.755 | 1.797 | 2.286 | 0.350 |
| 0.86089 | 0.0 | 0.0 | 74.386 | 1.566 | 2.187 | 0.350 |
| 0.87861 | 0.0 | 0.0 | 75.913 | 1.350 | 2.097 | 0.350 |
| 0.89517 | 0.0 | 0.0 | 77.340 | 1.148 | 2.011 | 0.350 |
| 0.91061 | 0.0 | 0.0 | 78.671 | 0.959 | 1.925 | 0.350 |
| 0.92498 | 0.0 | 0.0 | 79.908 | 0.784 | 1.836 | 0.350 |
| 0.93833 | 0.0 | 0.0 | 81.059 | 0.625 | 1.741 | 0.350 |
| 0.95072 | 0.0 | 0.0 | 82.125 | 0.483 | 1.642 | 0.350 |
| 0.96220 | 0.0 | 0.0 | 83.113 | 0.358 | 1.532 | 0.350 |
| 0.97281 | 0.0 | 0.0 | 84.026 | 0.251 | 1.402 | 0.350 |
| 0.98262 | 0.0 | 0.0 | 84.870 | 0.157 | 1.294 | 0.350 |
| 0.99166 | 0.0 | 0.0 | 85.649 | 0.070 | 1.176 | 0.350 |
| 1.00000 | 0.0 | 0.0 | 86.366 | 0.000 | 1.035 | 0.350 |

**Table 6 The blending weights for the airfoil coefficients**

| $S$ | Cylinder | FFA-W3-600 | FFA-W3-480 | FFA-W3-360 | FFA-W3-301 | FFA-W3-241 |
|---|---|---|---|---|---|---|
| 0.00000 | 1.000 | | | | | |
| 0.03057 | 0.992 | 0.008 | | | | |
| 0.06222 | 0.883 | 0.117 | | | | |
| 0.09487 | 0.720 | 0.280 | | | | |
| 0.12841 | 0.530 | 0.470 | | | | |
| 0.16274 | 0.325 | 0.675 | | | | |
| 0.19772 | 0.119 | 0.881 | | | | |
| 0.23321 | | 0.768 | 0.232 | | | |
| 0.26907 | | 0.250 | 0.750 | | | |
| 0.30515 | | | 0.879 | 0.121 | | |
| 0.34128 | | | 0.602 | 0.398 | | |
| 0.37731 | | | 0.393 | 0.607 | | |
| 0.41309 | | | 0.228 | 0.772 | | |
| 0.44846 | | | 0.065 | 0.935 | | |
| 0.48328 | | | | 0.805 | 0.195 | |
| 0.51741 | | | | 0.484 | 0.516 | |
| 0.55073 | | | | 0.171 | 0.829 | |
| 0.58312 | | | | | 0.870 | 0.130 |
| 0.61449 | | | | | 0.545 | 0.455 |
| 0.64476 | | | | | 0.301 | 0.699 |
| 0.67385 | | | | | 0.146 | 0.854 |
| 0.70172 | | | | | 0.057 | 0.943 |
| 0.72833 | | | | | 0.014 | 0.986 |
| 0.75364 | | | | | | 1.000 |
| 0.77766 | | | | | | 1.000 |
| 0.80038 | | | | | | 1.000 |
| 0.82181 | | | | | | 1.000 |
| 0.84197 | | | | | | 1.000 |
| 0.86089 | | | | | | 1.000 |
| 0.87861 | | | | | | 1.000 |
| 0.89517 | | | | | | 1.000 |
| 0.91061 | | | | | | 1.000 |
| 0.92498 | | | | | | 1.000 |
| 0.93833 | | | | | | 1.000 |
| 0.95072 | | | | | | 1.000 |
| 0.96220 | | | | | | 1.000 |
| 0.97281 | | | | | | 1.000 |
| 0.98262 | | | | | | 1.000 |
| 0.99166 | | | | | | 1.000 |
| 1.00000 | | | | | | 1.000 |

**Table 7 The blending weights for the cross-section shape**

| $S$ | Cylinder | TC72 | FFA-W3-480 | FFA-W3-360 | FFA-W3-301 | FFA-W3-241 |
|---|---|---|---|---|---|---|
| 0.00000 | 1.000 | | | | | |
| 0.03057 | 0.988 | 0.012 | | | | |
| 0.06222 | 0.832 | 0.168 | | | | |
| 0.09487 | 0.597 | 0.403 | | | | |
| 0.12841 | 0.323 | 0.677 | | | | |
| 0.16274 | 0.028 | 0.972 | | | | |
| 0.19772 | | 0.693 | 0.307 | | | |
| 0.23321 | | 0.381 | 0.619 | | | |
| 0.26907 | | 0.124 | 0.876 | | | |
| 0.30515 | | | 0.879 | 0.121 | | |
| 0.34128 | | | 0.602 | 0.398 | | |
| 0.37731 | | | 0.393 | 0.607 | | |
| 0.41309 | | | 0.228 | 0.772 | | |
| 0.44846 | | | 0.065 | 0.935 | | |
| 0.48328 | | | | 0.805 | 0.195 | |
| 0.51741 | | | | 0.484 | 0.516 | |
| 0.55073 | | | | 0.171 | 0.829 | |
| 0.58312 | | | | | 0.870 | 0.131 |
| 0.61449 | | | | | 0.545 | 0.455 |
| 0.64476 | | | | | 0.301 | 0.699 |
| 0.67385 | | | | | 0.146 | 0.854 |
| 0.70172 | | | | | 0.057 | 0.943 |
| 0.72833 | | | | | 0.014 | 0.986 |
| 0.75364 | | | | | | 1.000 |
| 0.77766 | | | | | | 1.000 |
| 0.80038 | | | | | | 1.000 |
| 0.82181 | | | | | | 1.000 |
| 0.84197 | | | | | | 1.000 |
| 0.86089 | | | | | | 1.000 |
| 0.87861 | | | | | | 1.000 |
| 0.89517 | | | | | | 1.000 |
| 0.91061 | | | | | | 1.000 |
| 0.92498 | | | | | | 1.000 |
| 0.93833 | | | | | | 1.000 |
| 0.95072 | | | | | | 1.000 |
| 0.96220 | | | | | | 1.000 |
| 0.97281 | | | | | | 1.000 |
| 0.98262 | | | | | | 1.000 |
| 0.99166 | | | | | | 1.000 |
| 1.00000 | | | | | | 1.000 |

**Table 8 The relative thickness of profiles shapes**

| Airfoil | Relative Thickness |
|---|---|
| FFA-W3-241 | 24.1% |
| FFA-W3-301 | 30.1% |
| FFA-W3-360 | 36.0% |
| FFA-W3-480 | 48.0% |
| FFA-W3-600 | 60.0% |
| TC72 | 72.0% |
| Cylinder | 100.0% |

**Table 9 Airfoil coefficients for thin airfoils at negative angles of attack**

| | FFA-W3-241 | | | FFA-W3-301 | | | FFA-W3-360 | | |
|---|---|---|---|---|---|---|---|---|---|
| AoA [deg] | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ |
| -180 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| -175 | 0.1736 | 0.0114 | 0.0218 | 0.1736 | 0.0099 | 0.0218 | 0.1736 | 0.0099 | 0.0218 |
| -170 | 0.3420 | 0.0452 | 0.0434 | 0.3420 | 0.0392 | 0.0434 | 0.3420 | 0.0392 | 0.0434 |
| -165 | 0.5000 | 0.1005 | 0.0647 | 0.5000 | 0.0871 | 0.0647 | 0.5000 | 0.0871 | 0.0647 |
| -160 | 0.6428 | 0.1755 | 0.0855 | 0.6428 | 0.1521 | 0.0855 | 0.6428 | 0.1521 | 0.0855 |
| -155 | 0.7660 | 0.2679 | 0.1057 | 0.7660 | 0.2322 | 0.1057 | 0.7660 | 0.2322 | 0.1057 |
| -150 | 0.8660 | 0.3750 | 0.1250 | 0.8660 | 0.3250 | 0.1250 | 0.8660 | 0.3250 | 0.1250 |
| -145 | 0.9397 | 0.4935 | 0.1434 | 0.9397 | 0.4277 | 0.1434 | 0.9397 | 0.4277 | 0.1434 |
| -140 | 0.9848 | 0.6197 | 0.1607 | 0.9848 | 0.5371 | 0.1607 | 0.9848 | 0.5371 | 0.1607 |
| -135 | 1.0000 | 0.7500 | 0.1768 | 1.0000 | 0.6500 | 0.1768 | 1.0000 | 0.6500 | 0.1768 |
| -130 | 0.9848 | 0.8803 | 0.1915 | 0.9848 | 0.7629 | 0.1915 | 0.9848 | 0.7629 | 0.1915 |
| -125 | 0.9397 | 1.0065 | 0.2048 | 0.9397 | 0.8723 | 0.2048 | 0.9397 | 0.8723 | 0.2048 |
| -120 | 0.8660 | 1.1250 | 0.2165 | 0.8660 | 0.9750 | 0.2165 | 0.8660 | 0.9750 | 0.2165 |
| -115 | 0.7660 | 1.2321 | 0.2266 | 0.7660 | 1.0678 | 0.2266 | 0.7660 | 1.0678 | 0.2266 |
| -110 | 0.6428 | 1.3245 | 0.2349 | 0.6428 | 1.1479 | 0.2349 | 0.6428 | 1.1479 | 0.2349 |
| -105 | 0.5000 | 1.3995 | 0.2415 | 0.5000 | 1.2129 | 0.2415 | 0.5000 | 1.2129 | 0.2415 |
| -100 | 0.3420 | 1.4548 | 0.2462 | 0.3420 | 1.2608 | 0.2462 | 0.3420 | 1.2608 | 0.2462 |
| -95 | 0.1736 | 1.4886 | 0.2490 | 0.1736 | 1.2901 | 0.2490 | 0.1736 | 1.2901 | 0.2490 |
| -90 | 0.0000 | 1.5000 | 0.2500 | 0.0000 | 1.3000 | 0.2500 | 0.0000 | 1.3000 | 0.2500 |
| -85 | -0.1736 | 1.4886 | 0.2490 | -0.1736 | 1.2901 | 0.2490 | -0.1736 | 1.2901 | 0.2490 |
| -80 | -0.3420 | 1.4548 | 0.2462 | -0.3420 | 1.2608 | 0.2462 | -0.3420 | 1.2608 | 0.2462 |
| -75 | -0.5000 | 1.3995 | 0.2415 | -0.5000 | 1.2129 | 0.2415 | -0.5000 | 1.2129 | 0.2415 |
| -70 | -0.6428 | 1.3245 | 0.2349 | -0.6428 | 1.1479 | 0.2349 | -0.6428 | 1.1479 | 0.2349 |
| -65 | -0.7660 | 1.2321 | 0.2266 | -0.7660 | 1.0678 | 0.2266 | -0.7660 | 1.0678 | 0.2266 |
| -60 | -0.8660 | 1.1250 | 0.2165 | -0.8660 | 0.9750 | 0.2165 | -0.8660 | 0.9750 | 0.2165 |
| -55 | -0.9397 | 1.0065 | 0.2048 | -0.9397 | 0.8723 | 0.2048 | -0.9397 | 0.8723 | 0.2048 |
| -50 | -0.9848 | 0.8603 | 0.1915 | -0.9848 | 0.7629 | 0.1915 | -0.9848 | 0.7629 | 0.1915 |
| -45 | -1.0120 | 0.7120 | 0.1708 | -1.0000 | 0.6500 | 0.1768 | -1.0000 | 0.6500 | 0.1768 |
| -40 | -1.0376 | 0.5475 | 0.1416 | -1.0216 | 0.5352 | 0.1416 | -0.9716 | 0.5352 | 0.1416 |
| -39 | -1.0419 | 0.5165 | 0.1346 | -1.0239 | 0.5066 | 0.1346 | -0.9639 | 0.5136 | 0.1346 |
| -38 | -1.0462 | 0.4816 | 0.1276 | -1.0162 | 0.4851 | 0.1276 | -0.9462 | 0.4851 | 0.1276 |
| -37 | -1.0525 | 0.4487 | 0.1185 | -1.0185 | 0.4565 | 0.1205 | -0.9285 | 0.4635 | 0.1205 |
| -36 | -1.0568 | 0.4177 | 0.1095 | -1.0108 | 0.4279 | 0.1135 | -0.9108 | 0.4349 | 0.1135 |
| -35 | -1.0611 | 0.3848 | 0.1005 | -1.0031 | 0.3994 | 0.0965 | -0.8931 | 0.4064 | 0.0965 |
| -34 | -1.0654 | 0.3558 | 0.0894 | -0.9954 | 0.3708 | 0.0794 | -0.8854 | 0.3778 | 0.0794 |
| -33 | -1.0717 | 0.3289 | 0.0824 | -0.9877 | 0.3353 | 0.0624 | -0.8677 | 0.3523 | 0.0624 |
| -32 | -1.0765 | 0.3031 | 0.0678 | -0.9835 | 0.3097 | 0.0515 | -0.8564 | 0.3315 | 0.0447 |
| -30 | -1.0889 | 0.2560 | 0.0508 | -0.9703 | 0.2663 | 0.0389 | -0.8378 | 0.2777 | 0.0327 |
| -28 | -1.0993 | 0.2090 | 0.0337 | -0.9672 | 0.2229 | 0.0263 | -0.8191 | 0.2338 | 0.0208 |
| -26 | -1.1168 | 0.1756 | 0.0206 | -0.9441 | 0.1941 | 0.0154 | -0.7924 | 0.2045 | 0.0102 |
| -24 | -1.1282 | 0.1423 | 0.0075 | -0.9310 | 0.1654 | 0.0045 | -0.7756 | 0.1751 | -0.0004 |
| -22 | -1.1215 | 0.1183 | -0.0008 | -0.9047 | 0.1417 | -0.0039 | -0.7441 | 0.1513 | -0.0089 |
| -20 | -1.1148 | 0.0943 | -0.0091 | -0.8784 | 0.1181 | -0.0123 | -0.7126 | 0.1274 | -0.0174 |
| -18 | -1.0919 | 0.0765 | -0.0123 | -0.8459 | 0.0986 | -0.0175 | -0.6678 | 0.1085 | -0.0229 |
| -16 | -1.0691 | 0.0587 | -0.0156 | -0.8134 | 0.0792 | -0.0227 | -0.6231 | 0.0896 | -0.0285 |
| -14 | -1.0379 | 0.0454 | -0.0156 | -0.7728 | 0.0643 | -0.0235 | -0.5742 | 0.0748 | -0.0300 |
| -12 | -1.0067 | 0.0321 | -0.0155 | -0.7322 | 0.0495 | -0.0244 | -0.5252 | 0.0600 | -0.0314 |
| -10 | -0.8479 | 0.0230 | -0.0318 | -0.6935 | 0.0381 | -0.0227 | -0.4827 | 0.0485 | -0.0279 |
| -8 | -0.6892 | 0.0138 | -0.0480 | -0.6547 | 0.0267 | -0.0210 | -0.4402 | 0.0370 | -0.0244 |
| -6 | -0.4278 | 0.0118 | -0.0611 | -0.4507 | 0.0204 | -0.0389 | -0.2983 | 0.0294 | -0.0359 |
| -4 | -0.1665 | 0.0098 | -0.0742 | -0.2467 | 0.0140 | -0.0569 | -0.1564 | 0.0219 | -0.0474 |
| -2 | 0.0863 | 0.0095 | -0.0811 | 0.0295 | 0.0129 | -0.0717 | 0.1744 | 0.0203 | -0.0782 |

**Table 10 Airfoil coefficients for thin airfoils at positive angles of attack**

| | FFA-W3-241 | | | FFA-W3-301 | | | FFA-W3-360 | | |
|---|---|---|---|---|---|---|---|---|---|
| AoA [deg] | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ |
| 0 | 0.3391 | 0.0092 | -0.0880 | 0.3056 | 0.0118 | -0.0865 | 0.5053 | 0.0187 | -0.1090 |
| 2 | 0.5867 | 0.0094 | -0.0933 | 0.5670 | 0.0119 | -0.0954 | 0.8241 | 0.0188 | -0.1329 |
| 4 | 0.8301 | 0.0099 | -0.0977 | 0.8199 | 0.0125 | -0.1024 | 1.1209 | 0.0196 | -0.1510 |
| 6 | 1.0656 | 0.0109 | -0.1008 | 1.0614 | 0.0136 | -0.1071 | 1.3897 | 0.0213 | -0.16298 |
| 8 | 1.2914 | 0.0124 | -0.1026 | 1.2874 | 0.0152 | -0.1094 | 1.6254 | 0.0240 | -0.16908 |
| 10 | 1.5012 | 0.0144 | -0.1024 | 1.4840 | 0.0180 | -0.10796 | 1.8109 | 0.0279 | -0.16832 |
| 12 | 1.6886 | 0.0173 | -0.0998 | 1.6388 | 0.0224 | -0.10256 | 1.8589 | 0.0365 | -0.15848 |
| 14 | 1.8103 | 0.0226 | -0.0941 | 1.7327 | 0.0303 | -0.09524 | 1.8159 | 0.0760 | -0.15576 |
| 16 | 1.8139 | 0.0354 | -0.0874 | 1.7142 | 0.0539 | -0.09104 | 1.7786 | 0.1165 | -0.16696 |
| 18 | 1.7545 | 0.0647 | -0.0850 | 1.6828 | 0.0954 | -0.09794 | 1.7560 | 0.1571 | -0.18094 |
| 20 | 1.6071 | 0.1035 | -0.0913 | 1.6567 | 0.1435 | -0.1103 | 1.7630 | 0.2063 | -0.19598 |
| 22 | 1.5257 | 0.1437 | -0.1026 | 1.6444 | 0.2280 | -0.12996 | 1.8002 | 0.3057 | -0.21428 |
| 24 | 1.4428 | 0.1841 | -0.1140 | 1.6329 | 0.3148 | -0.15332 | 1.8495 | 0.4153 | -0.23648 |
| 26 | 1.3826 | 0.2290 | -0.1282 | 1.6333 | 0.3926 | -0.1760 | 1.8775 | 0.5163 | -0.25882 |
| 28 | 1.3218 | 0.2738 | -0.1423 | 1.6175 | 0.4623 | -0.19658 | 1.8828 | 0.6069 | -0.27966 |
| 30 | 1.2583 | 0.3278 | -0.1582 | 1.5975 | 0.5266 | -0.2149 | 1.8689 | 0.6892 | -0.2983 |
| 32 | 1.1944 | 0.3816 | -0.1739 | 1.5708 | 0.5855 | -0.2313 | 1.8439 | 0.7625 | -0.31466 |
| 33 | 1.1734 | 0.4209 | -0.1790 | 1.5758 | 0.6215 | -0.2366 | 1.8349 | 0.7954 | -0.3166 |
| 34 | 1.1568 | 0.4504 | -0.1831 | 1.5571 | 0.6562 | -0.2406 | 1.8297 | 0.8232 | -0.3106 |
| 35 | 1.1379 | 0.4815 | -0.1873 | 1.5475 | 0.6897 | -0.2447 | 1.8136 | 0.8446 | -0.3047 |
| 36 | 1.1248 | 0.5105 | -0.1914 | 1.5268 | 0.7148 | -0.2487 | 1.7854 | 0.8684 | -0.2987 |
| 37 | 1.1177 | 0.5450 | -0.1953 | 1.5259 | 0.7464 | -0.2496 | 1.7669 | 0.8912 | -0.2926 |
| 38 | 1.1024 | 0.5737 | -0.1993 | 1.5139 | 0.7762 | -0.2455 | 1.7269 | 0.9066 | -0.2865 |
| 39 | 1.0891 | 0.5999 | -0.2032 | 1.4918 | 0.7982 | -0.2444 | 1.6864 | 0.9255 | -0.2844 |
| 40 | 1.0755 | 0.6280 | -0.2071 | 1.4596 | 0.8197 | -0.2423 | 1.6458 | 0.9385 | -0.2803 |
| 45 | 1.0175 | 0.7578 | -0.2161 | 1.2951 | 0.9122 | -0.2373 | 1.4473 | 1.0290 | -0.2633 |
| 50 | 0.9716 | 0.8820 | -0.2184 | 1.1256 | 0.9665 | -0.23537 | 1.1356 | 1.0715 | -0.25437 |
| 55 | 0.9268 | 1.0104 | -0.2214 | 0.9808 | 0.9957 | -0.23243 | 0.9808 | 1.0857 | -0.24543 |
| 60 | 0.8660 | 1.1250 | -0.2255 | 0.8660 | 1.0350 | -0.2335 | 0.8660 | 1.1000 | -0.2365 |
| 65 | 0.7660 | 1.2321 | -0.2306 | 0.7660 | 1.0828 | -0.2336 | 0.7660 | 1.1278 | -0.2366 |
| 70 | 0.6428 | 1.3245 | -0.2349 | 0.6428 | 1.1479 | -0.2349 | 0.6428 | 1.1629 | -0.2389 |
| 75 | 0.5000 | 1.3995 | -0.2415 | 0.5000 | 1.2129 | -0.2415 | 0.5000 | 1.2129 | -0.2415 |
| 80 | 0.3420 | 1.4548 | -0.2462 | 0.3420 | 1.2608 | -0.2462 | 0.3420 | 1.2608 | -0.2462 |
| 85 | 0.1736 | 1.4886 | -0.2490 | 0.1736 | 1.2901 | -0.2490 | 0.1736 | 1.2901 | -0.2490 |
| 90 | 0.0000 | 1.5000 | -0.2500 | 0.0000 | 1.3000 | -0.2500 | 0.0000 | 1.3000 | -0.2500 |
| 95 | -0.1736 | 1.4886 | -0.2490 | -0.1736 | 1.2901 | -0.2490 | -0.1736 | 1.2901 | -0.2490 |
| 100 | -0.3420 | 1.4548 | -0.2462 | -0.3420 | 1.2608 | -0.2462 | -0.3420 | 1.2608 | -0.2462 |
| 105 | -0.5000 | 1.3995 | -0.2415 | -0.5000 | 1.2129 | -0.2415 | -0.5000 | 1.2129 | -0.2415 |
| 110 | -0.6428 | 1.3245 | -0.2349 | -0.6428 | 1.1479 | -0.2349 | -0.6428 | 1.1479 | -0.2349 |
| 115 | -0.7660 | 1.2321 | -0.2266 | -0.7660 | 1.0678 | -0.2266 | -0.7660 | 1.0678 | -0.2266 |
| 120 | -0.8660 | 1.1250 | -0.2165 | -0.8660 | 0.9750 | -0.2165 | -0.8660 | 0.9750 | -0.2165 |
| 125 | -0.9397 | 1.0065 | -0.2048 | -0.9397 | 0.8723 | -0.2048 | -0.9397 | 0.8723 | -0.2048 |
| 130 | -0.9848 | 0.8803 | -0.1915 | -0.9848 | 0.7629 | -0.1915 | -0.9848 | 0.7629 | -0.1915 |
| 135 | -1.0000 | 0.7500 | -0.1768 | -1.0000 | 0.6500 | -0.1768 | -1.0000 | 0.6500 | -0.1768 |
| 140 | -0.9848 | 0.6197 | -0.1607 | -0.9848 | 0.5371 | -0.1607 | -0.9848 | 0.5371 | -0.1607 |
| 145 | -0.9397 | 0.4935 | -0.1434 | -0.9397 | 0.4277 | -0.1434 | -0.9397 | 0.4277 | -0.1434 |
| 150 | -0.8660 | 0.3750 | -0.1250 | -0.8660 | 0.3250 | -0.1250 | -0.8660 | 0.3250 | -0.1250 |
| 155 | -0.7660 | 0.2679 | -0.1057 | -0.7660 | 0.2322 | -0.1057 | -0.7660 | 0.2322 | -0.1057 |
| 160 | -0.6428 | 0.1755 | -0.0855 | -0.6428 | 0.1521 | -0.0855 | -0.6428 | 0.1521 | -0.0855 |
| 165 | -0.5000 | 0.1005 | -0.0647 | -0.5000 | 0.0871 | -0.0647 | -0.5000 | 0.0871 | -0.0647 |
| 170 | -0.3420 | 0.0452 | -0.0434 | -0.3420 | 0.0392 | -0.0434 | -0.3420 | 0.0392 | -0.0434 |
| 175 | -0.1736 | 0.0114 | -0.0218 | -0.1736 | 0.0099 | -0.0218 | -0.1736 | 0.0099 | -0.0218 |
| 180 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

**Table 11 Airfoil coefficients for thick airfoils at negative angles of attack**

| | FFA-W3-480 | | | FFA-W3-600 | | | Cylinder | | |
|---|---|---|---|---|---|---|---|---|---|
| AoA [deg] | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ |
| -180 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6000 | 0.0000 |
| -175 | 0.1736 | 0.0099 | 0.0218 | 0.1736 | 0.0099 | 0.0218 | 0.0000 | 0.6000 | 0.0000 |
| -170 | 0.3420 | 0.0392 | 0.0434 | 0.3420 | 0.0392 | 0.0434 | 0.0000 | 0.6000 | 0.0000 |
| -165 | 0.5000 | 0.0871 | 0.0647 | 0.5000 | 0.0871 | 0.0647 | 0.0000 | 0.6000 | 0.0000 |
| -160 | 0.6428 | 0.1521 | 0.0855 | 0.6428 | 0.1521 | 0.0855 | 0.0000 | 0.6000 | 0.0000 |
| -155 | 0.7660 | 0.2322 | 0.1057 | 0.7660 | 0.2322 | 0.1057 | 0.0000 | 0.6000 | 0.0000 |
| -150 | 0.8660 | 0.3250 | 0.1250 | 0.8660 | 0.3250 | 0.1250 | 0.0000 | 0.6000 | 0.0000 |
| -145 | 0.9397 | 0.4277 | 0.1434 | 0.9397 | 0.4277 | 0.1434 | 0.0000 | 0.6000 | 0.0000 |
| -140 | 0.9848 | 0.5371 | 0.1607 | 0.9848 | 0.5371 | 0.1607 | 0.0000 | 0.6000 | 0.0000 |
| -135 | 1.0000 | 0.6500 | 0.1768 | 1.0000 | 0.6500 | 0.1768 | 0.0000 | 0.6000 | 0.0000 |
| -130 | 0.9848 | 0.7629 | 0.1915 | 0.9848 | 0.7629 | 0.1915 | 0.0000 | 0.6000 | 0.0000 |
| -125 | 0.9397 | 0.8723 | 0.2048 | 0.9397 | 0.8723 | 0.2048 | 0.0000 | 0.6000 | 0.0000 |
| -120 | 0.8660 | 0.9750 | 0.2165 | 0.8660 | 0.9750 | 0.2165 | 0.0000 | 0.6000 | 0.0000 |
| -115 | 0.7660 | 1.0678 | 0.2266 | 0.7660 | 1.0678 | 0.2266 | 0.0000 | 0.6000 | 0.0000 |
| -110 | 0.6428 | 1.1479 | 0.2349 | 0.6428 | 1.1479 | 0.2349 | 0.0000 | 0.6000 | 0.0000 |
| -105 | 0.5000 | 1.2129 | 0.2415 | 0.5000 | 1.2129 | 0.2415 | 0.0000 | 0.6000 | 0.0000 |
| -100 | 0.3420 | 1.2608 | 0.2462 | 0.3420 | 1.2608 | 0.2462 | 0.0000 | 0.6000 | 0.0000 |
| -95 | 0.1736 | 1.2901 | 0.2490 | 0.1736 | 1.2901 | 0.2490 | 0.0000 | 0.6000 | 0.0000 |
| -90 | 0.0000 | 1.3000 | 0.2500 | 0.0000 | 1.3000 | 0.2500 | 0.0000 | 0.6000 | 0.0000 |
| -85 | -0.1736 | 1.2901 | 0.2490 | -0.1736 | 1.2901 | 0.2490 | 0.0000 | 0.6000 | 0.0000 |
| -80 | -0.3420 | 1.2608 | 0.2462 | -0.3420 | 1.2608 | 0.2462 | 0.0000 | 0.6000 | 0.0000 |
| -75 | -0.5000 | 1.2129 | 0.2415 | -0.5000 | 1.2129 | 0.2415 | 0.0000 | 0.6000 | 0.0000 |
| -70 | -0.6428 | 1.1479 | 0.2349 | -0.6428 | 1.1479 | 0.2349 | 0.0000 | 0.6000 | 0.0000 |
| -65 | -0.7660 | 1.0678 | 0.2266 | -0.7660 | 1.0678 | 0.2266 | 0.0000 | 0.6000 | 0.0000 |
| -60 | -0.8660 | 0.9750 | 0.2165 | -0.8660 | 0.9750 | 0.2165 | 0.0000 | 0.6000 | 0.0000 |
| -55 | -0.9397 | 0.8723 | 0.1978 | -0.9397 | 0.8723 | 0.1978 | 0.0000 | 0.6000 | 0.0000 |
| -50 | -0.9848 | 0.7629 | 0.1775 | -0.9848 | 0.7629 | 0.1775 | 0.0000 | 0.6000 | 0.0000 |
| -45 | -1.0000 | 0.6500 | 0.1558 | -1.0000 | 0.6500 | 0.1488 | 0.0000 | 0.6000 | 0.0000 |
| -40 | -0.9816 | 0.5352 | 0.1246 | -0.9516 | 0.5282 | 0.1176 | 0.0000 | 0.6000 | 0.0000 |
| -39 | -0.9539 | 0.5136 | 0.1106 | -0.9039 | 0.5026 | 0.1076 | 0.0000 | 0.6000 | 0.0000 |
| -38 | -0.9262 | 0.4851 | 0.0966 | -0.8262 | 0.4671 | 0.0906 | 0.0000 | 0.6000 | 0.0000 |
| -37 | -0.8885 | 0.4565 | 0.0825 | -0.7185 | 0.4345 | 0.0805 | 0.0000 | 0.6000 | 0.0000 |
| -36 | -0.8508 | 0.4279 | 0.0655 | -0.6208 | 0.3989 | 0.0665 | 0.0000 | 0.6000 | 0.0000 |
| -35 | -0.8231 | 0.3924 | 0.0585 | -0.5231 | 0.3634 | 0.0595 | 0.0000 | 0.6000 | 0.0000 |
| -34 | -0.7854 | 0.3638 | 0.0484 | -0.4454 | 0.3278 | 0.0524 | 0.0000 | 0.6000 | 0.0000 |
| -33 | -0.7477 | 0.3383 | 0.0384 | -0.3477 | 0.3013 | 0.0454 | 0.0000 | 0.6000 | 0.0000 |
| -32 | -0.7011 | 0.3123 | 0.0305 | -0.2571 | 0.2729 | 0.04699 | 0.0000 | 0.6000 | 0.0000 |
| -30 | -0.6208 | 0.2663 | 0.0200 | -0.1574 | 0.2332 | 0.0606 | 0.0000 | 0.6000 | 0.0000 |
| -28 | -0.5406 | 0.2402 | 0.0094 | -0.0776 | 0.2034 | 0.0742 | 0.0000 | 0.6000 | 0.0000 |
| -26 | -0.4694 | 0.2169 | -0.0006 | 0.0115 | 0.1858 | 0.08749 | 0.0000 | 0.6000 | 0.0000 |
| -24 | -0.3881 | 0.1936 | -0.0106 | 0.1007 | 0.1681 | 0.10079 | 0.0000 | 0.6000 | 0.0000 |
| -22 | -0.3161 | 0.1732 | -0.0197 | 0.1962 | 0.1526 | 0.11329 | 0.0000 | 0.6000 | 0.0000 |
| -20 | -0.2442 | 0.1529 | -0.0289 | 0.2918 | 0.1370 | 0.12579 | 0.0000 | 0.6000 | 0.0000 |
| -18 | -0.1641 | 0.1355 | -0.0368 | 0.3932 | 0.1240 | 0.1370 | 0.0000 | 0.6000 | 0.0000 |
| -16 | -0.0841 | 0.1180 | -0.0448 | 0.4947 | 0.1109 | 0.14822 | 0.0000 | 0.6000 | 0.0000 |
| -14 | 0.0021 | 0.1035 | -0.0510 | 0.6004 | 0.1009 | 0.15711 | 0.0000 | 0.6000 | 0.0000 |
| -12 | 0.0883 | 0.0890 | -0.0572 | 0.7061 | 0.0908 | 0.1660 | 0.0000 | 0.6000 | 0.0000 |
| -10 | 0.1722 | 0.0773 | -0.0604 | 0.7880 | 0.0836 | 0.16771 | 0.0000 | 0.6000 | 0.0000 |
| -8 | 0.2561 | 0.0656 | -0.0636 | 0.8700 | 0.0765 | 0.16942 | 0.0000 | 0.6000 | 0.0000 |
| -6 | 0.3179 | 0.0563 | -0.0605 | 0.8358 | 0.0748 | -0.1470 | 0.0000 | 0.6000 | 0.0000 |
| -4 | 0.3798 | 0.0470 | -0.0574 | 0.8162 | 0.07311 | -0.12588 | 0.0000 | 0.6000 | 0.0000 |
| -2 | 0.3573 | 0.0405 | -0.0351 | 0.6603 | 0.07553 | -0.08764 | 0.0000 | 0.6000 | 0.0000 |

**Table 12 Airfoil coefficients for thick airfoils at positive angles of attack**

| | FFA-W3-480 | | | FFA-W3-600 | | | Cylinder | | |
|---|---|---|---|---|---|---|---|---|---|
| AoA [deg] | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ | $C_l$ | $C_d$ | $C_m$ |
| 0 | 0.3348 | 0.0341 | -0.0128 | 0.5199 | 0.07795 | -0.05062 | 0.0000 | 0.6000 | 0.0000 |
| 2 | 0.5652 | 0.0316 | -0.0494 | 0.2636 | 0.07795 | -0.00078 | 0.0000 | 0.6000 | 0.0000 |
| 4 | 0.8769 | 0.0343 | -0.0894 | 0.2146 | 0.07082 | 0.06962 | 0.0000 | 0.6000 | 0.0000 |
| 6 | 1.0425 | 0.0451 | -0.11168 | 0.1656 | 0.07483 | 0.01974 | 0.0000 | 0.6000 | 0.0000 |
| 8 | 0.9487 | 0.0700 | -0.12084 | 0.4475 | 0.08587 | -0.02718 | 0.0000 | 0.6000 | 0.0000 |
| 10 | 0.9088 | 0.0886 | -0.1376 | 0.7071 | 0.0999 | -0.06712 | 0.0000 | 0.6000 | 0.0000 |
| 12 | 0.9761 | 0.0993 | -0.15942 | 0.9540 | 0.11148 | -0.10334 | 0.0000 | 0.6000 | 0.0000 |
| 14 | 1.1130 | 0.1070 | -0.18234 | 1.1891 | 0.12072 | -0.13694 | 0.0000 | 0.6000 | 0.0000 |
| 16 | 1.3065 | 0.1163 | -0.20664 | 1.4183 | 0.13678 | -0.16866 | 0.0000 | 0.6000 | 0.0000 |
| 18 | 1.5414 | 0.1317 | -0.2315 | 1.6392 | 0.14998 | -0.19886 | 0.0000 | 0.6000 | 0.0000 |
| 20 | 1.8049 | 0.1570 | -0.25662 | 1.8216 | 0.16894 | -0.22524 | 0.0000 | 0.6000 | 0.0000 |
| 22 | 2.0020 | 0.2757 | -0.28058 | 1.9837 | 0.20657 | -0.25094 | 0.0000 | 0.6000 | 0.0000 |
| 24 | 2.1216 | 0.4224 | -0.30574 | 2.0985 | 0.30919 | -0.27474 | 0.0000 | 0.6000 | 0.0000 |
| 26 | 2.1916 | 0.5610 | -0.3294 | 2.1904 | 0.4344 | -0.29658 | 0.0000 | 0.6000 | 0.0000 |
| 28 | 2.2291 | 0.6861 | -0.35198 | 2.2541 | 0.5512 | -0.31712 | 0.0000 | 0.6000 | 0.0000 |
| 30 | 2.2322 | 0.7958 | -0.3710 | 2.2731 | 0.6732 | -0.33352 | 0.0000 | 0.6000 | 0.0000 |
| 32 | 2.2188 | 0.8915 | -0.38784 | 2.2740 | 0.7756 | -0.34854 | 0.0000 | 0.6000 | 0.0000 |
| 33 | 2.2092 | 0.9229 | -0.3845 | 2.2780 | 0.8209 | -0.3466 | 0.0000 | 0.6000 | 0.0000 |
| 34 | 2.1927 | 0.9472 | -0.3806 | 2.2658 | 0.8551 | -0.3476 | 0.0000 | 0.6000 | 0.0000 |
| 35 | 2.1625 | 0.9664 | -0.3747 | 2.2408 | 0.8862 | -0.3487 | 0.0000 | 0.6000 | 0.0000 |
| 36 | 2.1313 | 0.9889 | -0.3687 | 2.2331 | 0.9141 | -0.3427 | 0.0000 | 0.6000 | 0.0000 |
| 37 | 2.0878 | 1.0073 | -0.3626 | 2.2134 | 0.9390 | -0.3366 | 0.0000 | 0.6000 | 0.0000 |
| 38 | 2.0435 | 1.0284 | -0.3565 | 2.1820 | 0.9709 | -0.3305 | 0.0000 | 0.6000 | 0.0000 |
| 39 | 1.9984 | 1.0521 | -0.3504 | 2.1391 | 0.9996 | -0.3244 | 0.0000 | 0.6000 | 0.0000 |
| 40 | 1.9418 | 1.0769 | -0.3443 | 2.0951 | 1.0254 | -0.3183 | 0.0000 | 0.6000 | 0.0000 |
| 45 | 1.6535 | 1.1697 | -0.3233 | 1.8370 | 1.1419 | -0.3003 | 0.0000 | 0.6000 | 0.0000 |
| 50 | 1.3016 | 1.1965 | -0.30437 | 1.5156 | 1.1615 | -0.29137 | 0.0000 | 0.6000 | 0.0000 |
| 55 | 1.0308 | 1.1957 | -0.29543 | 1.2008 | 1.1657 | -0.28243 | 0.0000 | 0.6000 | 0.0000 |
| 60 | 0.8660 | 1.1900 | -0.2865 | 0.9660 | 1.1750 | -0.2735 | 0.0000 | 0.6000 | 0.0000 |
| 65 | 0.7660 | 1.1928 | -0.2766 | 0.7660 | 1.1778 | -0.2636 | 0.0000 | 0.6000 | 0.0000 |
| 70 | 0.6428 | 1.2029 | -0.2649 | 0.6428 | 1.1979 | -0.2549 | 0.0000 | 0.6000 | 0.0000 |
| 75 | 0.5000 | 1.2129 | -0.2515 | 0.5000 | 1.2129 | -0.2515 | 0.0000 | 0.6000 | 0.0000 |
| 80 | 0.3420 | 1.2608 | -0.2462 | 0.3420 | 1.2608 | -0.2462 | 0.0000 | 0.6000 | 0.0000 |
| 85 | 0.1736 | 1.2901 | -0.2490 | 0.1736 | 1.2901 | -0.2490 | 0.0000 | 0.6000 | 0.0000 |
| 90 | 0.0000 | 1.3000 | -0.2500 | 0.0000 | 1.3000 | -0.2500 | 0.0000 | 0.6000 | 0.0000 |
| 95 | -0.1736 | 1.2901 | -0.2490 | -0.1736 | 1.2901 | -0.2490 | 0.0000 | 0.6000 | 0.0000 |
| 100 | -0.3420 | 1.2608 | -0.2462 | -0.3420 | 1.2608 | -0.2462 | 0.0000 | 0.6000 | 0.0000 |
| 105 | -0.5000 | 1.2129 | -0.2415 | -0.5000 | 1.2129 | -0.2415 | 0.0000 | 0.6000 | 0.0000 |
| 110 | -0.6428 | 1.1479 | -0.2349 | -0.6428 | 1.1479 | -0.2349 | 0.0000 | 0.6000 | 0.0000 |
| 115 | -0.7660 | 1.0678 | -0.2266 | -0.7660 | 1.0678 | -0.2266 | 0.0000 | 0.6000 | 0.0000 |
| 120 | -0.8660 | 0.9750 | -0.2165 | -0.8660 | 0.9750 | -0.2165 | 0.0000 | 0.6000 | 0.0000 |
| 125 | -0.9397 | 0.8723 | -0.2048 | -0.9397 | 0.8723 | -0.2048 | 0.0000 | 0.6000 | 0.0000 |
| 130 | -0.9848 | 0.7629 | -0.1915 | -0.9848 | 0.7629 | -0.1915 | 0.0000 | 0.6000 | 0.0000 |
| 135 | -1.0000 | 0.6500 | -0.1768 | -1.0000 | 0.6500 | -0.1768 | 0.0000 | 0.6000 | 0.0000 |
| 140 | -0.9848 | 0.5371 | -0.1607 | -0.9848 | 0.5371 | -0.1607 | 0.0000 | 0.6000 | 0.0000 |
| 145 | -0.9397 | 0.4277 | -0.1434 | -0.9397 | 0.4277 | -0.1434 | 0.0000 | 0.6000 | 0.0000 |
| 150 | -0.8660 | 0.3250 | -0.1250 | -0.8660 | 0.3250 | -0.1250 | 0.0000 | 0.6000 | 0.0000 |
| 155 | -0.7660 | 0.2322 | -0.1057 | -0.7660 | 0.2322 | -0.1057 | 0.0000 | 0.6000 | 0.0000 |
| 160 | -0.6428 | 0.1521 | -0.0855 | -0.6428 | 0.1521 | -0.0855 | 0.0000 | 0.6000 | 0.0000 |
| 165 | -0.5000 | 0.0871 | -0.0647 | -0.5000 | 0.0871 | -0.0647 | 0.0000 | 0.6000 | 0.0000 |
| 170 | -0.3420 | 0.0392 | -0.0434 | -0.3420 | 0.0392 | -0.0434 | 0.0000 | 0.6000 | 0.0000 |
| 175 | -0.1736 | 0.0099 | -0.0218 | -0.1736 | 0.0099 | -0.0218 | 0.0000 | 0.6000 | 0.0000 |
| 180 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6000 | 0.0000 |

**Table 13 FFA-W3-241 cross section shape coordinates**

| ID | $s$ | $x$ | $y$ | | ID | $s$ | $x$ | $y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 1.00000 | -0.00360 | | 41 | 0.50369 | 0.00130 | 0.00956 |
| 2 | 0.00794 | 0.98338 | -0.00140 | | 42 | 0.50897 | 0.00507 | 0.02005 |
| 3 | 0.01692 | 0.96457 | 0.00096 | | 43 | 0.51468 | 0.01133 | 0.03033 |
| 4 | 0.02689 | 0.94365 | 0.00311 | | 44 | 0.52116 | 0.02006 | 0.04087 |
| 5 | 0.03778 | 0.92072 | 0.00466 | | 45 | 0.52841 | 0.03122 | 0.05135 |
| 6 | 0.04954 | 0.89589 | 0.00528 | | 46 | 0.53648 | 0.04474 | 0.06170 |
| 7 | 0.06215 | 0.86928 | 0.00473 | | 47 | 0.54536 | 0.06056 | 0.07176 |
| 8 | 0.07557 | 0.84102 | 0.00291 | | 48 | 0.55506 | 0.07861 | 0.08139 |
| 9 | 0.08976 | 0.81124 | -0.00030 | | 49 | 0.56554 | 0.09880 | 0.09044 |
| 10 | 0.10469 | 0.78010 | -0.00514 | | 50 | 0.57678 | 0.12104 | 0.09873 |
| 11 | 0.12036 | 0.74772 | -0.01183 | | 51 | 0.58876 | 0.14522 | 0.10613 |
| 12 | 0.13669 | 0.71428 | -0.02024 | | 52 | 0.60144 | 0.17121 | 0.11247 |
| 13 | 0.15363 | 0.67994 | -0.03015 | | 53 | 0.61479 | 0.19891 | 0.11764 |
| 14 | 0.17107 | 0.64485 | -0.04127 | | 54 | 0.62877 | 0.22817 | 0.12153 |
| 15 | 0.18887 | 0.60918 | -0.05308 | | 55 | 0.64336 | 0.25885 | 0.12409 |
| 16 | 0.20687 | 0.57311 | -0.06502 | | 56 | 0.65851 | 0.29081 | 0.12526 |
| 17 | 0.22491 | 0.53681 | -0.07651 | | 57 | 0.67418 | 0.32389 | 0.12504 |
| 18 | 0.24285 | 0.50045 | -0.08706 | | 58 | 0.69033 | 0.35794 | 0.12350 |
| 19 | 0.26055 | 0.46422 | -0.09623 | | 59 | 0.70690 | 0.39279 | 0.12076 |
| 20 | 0.27795 | 0.42827 | -0.10370 | | 60 | 0.72380 | 0.42827 | 0.11696 |
| 21 | 0.29497 | 0.39279 | -0.10938 | | 61 | 0.74098 | 0.46422 | 0.11223 |
| 22 | 0.31158 | 0.35794 | -0.11324 | | 62 | 0.75834 | 0.50045 | 0.10670 |
| 23 | 0.32775 | 0.32389 | -0.11531 | | 63 | 0.77582 | 0.53681 | 0.10052 |
| 24 | 0.34342 | 0.29081 | -0.11567 | | 64 | 0.79331 | 0.57311 | 0.09382 |
| 25 | 0.35857 | 0.25885 | -0.11441 | | 65 | 0.81072 | 0.60918 | 0.08672 |
| 26 | 0.37317 | 0.22817 | -0.11167 | | 66 | 0.82798 | 0.64485 | 0.07935 |
| 27 | 0.38717 | 0.19891 | -0.10754 | | 67 | 0.84498 | 0.67994 | 0.07187 |
| 28 | 0.40054 | 0.17121 | -0.10213 | | 68 | 0.86163 | 0.71428 | 0.06441 |
| 29 | 0.41324 | 0.14522 | -0.09560 | | 69 | 0.87785 | 0.74772 | 0.05707 |
| 30 | 0.42523 | 0.12104 | -0.08811 | | 70 | 0.89355 | 0.78010 | 0.04996 |
| 31 | 0.43647 | 0.09880 | -0.07985 | | 71 | 0.90866 | 0.81124 | 0.04316 |
| 32 | 0.44691 | 0.07861 | -0.07104 | | 72 | 0.92309 | 0.84102 | 0.03673 |
| 33 | 0.45650 | 0.06056 | -0.06188 | | 73 | 0.93678 | 0.86928 | 0.03070 |
| 34 | 0.46519 | 0.04474 | -0.05258 | | 74 | 0.94966 | 0.89589 | 0.02510 |
| 35 | 0.47295 | 0.03122 | -0.04334 | | 75 | 0.96168 | 0.92072 | 0.01996 |
| 36 | 0.47976 | 0.02006 | -0.03427 | | 76 | 0.97276 | 0.94365 | 0.01529 |
| 37 | 0.48562 | 0.01133 | -0.02550 | | 77 | 0.98287 | 0.96457 | 0.01107 |
| 38 | 0.49063 | 0.00507 | -0.01700 | | 78 | 0.99197 | 0.98338 | 0.00727 |
| 39 | 0.49480 | 0.00130 | -0.00903 | | 79 | 1.00000 | 1.00000 | 0.00391 |
| 40 | 0.49987 | 0.00004 | 0.00160 | | | | | |

**Table 14 FFA-W3-301 cross section shape coordinates**

| ID | $s$ | $x$ | $y$ | | ID | $s$ | $x$ | $y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 1.00000 | -0.00891 | | 41 | 0.50771 | 0.00126 | 0.01326 |
| 2 | 0.00783 | 0.98338 | -0.00558 | | 42 | 0.51414 | 0.00503 | 0.02667 |
| 3 | 0.01662 | 0.96457 | -0.00261 | | 43 | 0.52095 | 0.01130 | 0.04001 |
| 4 | 0.02634 | 0.94364 | -0.00051 | | 44 | 0.52833 | 0.02003 | 0.05339 |
| 5 | 0.03694 | 0.92071 | 0.00058 | | 45 | 0.53629 | 0.03118 | 0.06655 |
| 6 | 0.04841 | 0.89588 | 0.00040 | | 46 | 0.54490 | 0.04470 | 0.07938 |
| 7 | 0.06072 | 0.86927 | -0.00134 | | 47 | 0.55415 | 0.06052 | 0.09167 |
| 8 | 0.07387 | 0.84101 | -0.00479 | | 48 | 0.56406 | 0.07858 | 0.10323 |
| 9 | 0.08783 | 0.81124 | -0.01002 | | 49 | 0.57459 | 0.09877 | 0.11385 |
| 10 | 0.10259 | 0.78009 | -0.01715 | | 50 | 0.58576 | 0.12101 | 0.12332 |
| 11 | 0.11813 | 0.74772 | -0.02631 | | 51 | 0.59754 | 0.14518 | 0.13146 |
| 12 | 0.13438 | 0.71428 | -0.03722 | | 52 | 0.60994 | 0.17118 | 0.13814 |
| 13 | 0.15125 | 0.67993 | -0.04966 | | 53 | 0.62294 | 0.19888 | 0.14325 |
| 14 | 0.16862 | 0.64484 | -0.06318 | | 54 | 0.63655 | 0.22813 | 0.14673 |
| 15 | 0.18632 | 0.60917 | -0.07722 | | 55 | 0.65075 | 0.25882 | 0.14866 |
| 16 | 0.20418 | 0.57310 | -0.09116 | | 56 | 0.66551 | 0.29078 | 0.14908 |
| 17 | 0.22203 | 0.53680 | -0.10442 | | 57 | 0.68080 | 0.32386 | 0.14807 |
| 18 | 0.23973 | 0.50044 | -0.11652 | | 58 | 0.69656 | 0.35791 | 0.14578 |
| 19 | 0.25716 | 0.46420 | -0.12710 | | 59 | 0.71274 | 0.39276 | 0.14230 |
| 20 | 0.27426 | 0.42824 | -0.13591 | | 60 | 0.72926 | 0.42824 | 0.13776 |
| 21 | 0.29095 | 0.39276 | -0.14283 | | 61 | 0.74605 | 0.46420 | 0.13229 |
| 22 | 0.30721 | 0.35791 | -0.14780 | | 62 | 0.76304 | 0.50044 | 0.12601 |
| 23 | 0.32300 | 0.32386 | -0.15080 | | 63 | 0.78013 | 0.53680 | 0.11909 |
| 24 | 0.33829 | 0.29078 | -0.15188 | | 64 | 0.79724 | 0.57310 | 0.11169 |
| 25 | 0.35305 | 0.25882 | -0.15108 | | 65 | 0.81428 | 0.60917 | 0.10391 |
| 26 | 0.36728 | 0.22813 | -0.14847 | | 66 | 0.83117 | 0.64484 | 0.09587 |
| 27 | 0.38093 | 0.19888 | -0.14411 | | 67 | 0.84782 | 0.67993 | 0.08766 |
| 28 | 0.39403 | 0.17118 | -0.13810 | | 68 | 0.86413 | 0.71428 | 0.07939 |
| 29 | 0.40653 | 0.14518 | -0.13052 | | 69 | 0.88004 | 0.74772 | 0.07116 |
| 30 | 0.41844 | 0.12101 | -0.12155 | | 70 | 0.89545 | 0.78009 | 0.06309 |
| 31 | 0.42974 | 0.09877 | -0.11136 | | 71 | 0.91028 | 0.81124 | 0.05528 |
| 32 | 0.44040 | 0.07858 | -0.10018 | | 72 | 0.92445 | 0.84101 | 0.04780 |
| 33 | 0.45039 | 0.06052 | -0.08825 | | 73 | 0.93791 | 0.86927 | 0.04074 |
| 34 | 0.45969 | 0.04470 | -0.07582 | | 74 | 0.95057 | 0.89588 | 0.03417 |
| 35 | 0.46825 | 0.03118 | -0.06312 | | 75 | 0.96237 | 0.92071 | 0.02812 |
| 36 | 0.47608 | 0.02003 | -0.05035 | | 76 | 0.97326 | 0.94364 | 0.02266 |
| 37 | 0.48320 | 0.01130 | -0.03764 | | 77 | 0.98319 | 0.96457 | 0.01775 |
| 38 | 0.48971 | 0.00503 | -0.02502 | | 78 | 0.99211 | 0.98338 | 0.01333 |
| 39 | 0.49573 | 0.00126 | -0.01256 | | 79 | 1.00000 | 1.00000 | 0.00937 |
| 40 | 0.50101 | 0.00000 | -0.00119 | | | | | |

**Table 15 FFA-W3-360 cross section shape coordinates**

| ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 1.00000 | -0.01393 | | 41 | 0.50753 | 0.00138 | 0.01755 |
| 2 | 0.00773 | 0.98339 | -0.00928 | | 42 | 0.51835 | 0.00515 | 0.04140 |
| 3 | 0.01633 | 0.96457 | -0.00555 | | 43 | 0.52693 | 0.01141 | 0.05950 |
| 4 | 0.02575 | 0.94365 | -0.00342 | | 44 | 0.53576 | 0.02014 | 0.07716 |
| 5 | 0.03603 | 0.92072 | -0.00273 | | 45 | 0.54482 | 0.03126 | 0.09405 |
| 6 | 0.04716 | 0.89589 | -0.00361 | | 46 | 0.55420 | 0.04481 | 0.11001 |
| 7 | 0.05914 | 0.86929 | -0.00636 | | 47 | 0.56389 | 0.06063 | 0.12474 |
| 8 | 0.07199 | 0.84103 | -0.01120 | | 48 | 0.57393 | 0.07868 | 0.13804 |
| 9 | 0.08569 | 0.81126 | -0.01816 | | 49 | 0.58441 | 0.09887 | 0.14982 |
| 10 | 0.10022 | 0.78011 | -0.02713 | | 50 | 0.59536 | 0.12111 | 0.15996 |
| 11 | 0.11553 | 0.74774 | -0.03807 | | 51 | 0.60684 | 0.14528 | 0.16844 |
| 12 | 0.13156 | 0.71431 | -0.05083 | | 52 | 0.61887 | 0.17128 | 0.17512 |
| 13 | 0.14821 | 0.67996 | -0.06500 | | 53 | 0.63146 | 0.19897 | 0.17994 |
| 14 | 0.16531 | 0.64487 | -0.08000 | | 54 | 0.64464 | 0.22823 | 0.18291 |
| 15 | 0.18269 | 0.60921 | -0.09522 | | 55 | 0.65840 | 0.25891 | 0.18414 |
| 16 | 0.20017 | 0.57314 | -0.11009 | | 56 | 0.67272 | 0.29086 | 0.18370 |
| 17 | 0.21761 | 0.53685 | -0.12416 | | 57 | 0.68757 | 0.32394 | 0.18171 |
| 18 | 0.23490 | 0.50049 | -0.13705 | | 58 | 0.70290 | 0.35799 | 0.17825 |
| 19 | 0.25192 | 0.46426 | -0.14848 | | 59 | 0.71866 | 0.39283 | 0.17347 |
| 20 | 0.26860 | 0.42831 | -0.15816 | | 60 | 0.73478 | 0.42831 | 0.16749 |
| 21 | 0.28486 | 0.39283 | -0.16578 | | 61 | 0.75120 | 0.46426 | 0.16048 |
| 22 | 0.30067 | 0.35799 | -0.17134 | | 62 | 0.76781 | 0.50049 | 0.15262 |
| 23 | 0.31601 | 0.32394 | -0.17488 | | 63 | 0.78454 | 0.53685 | 0.14408 |
| 24 | 0.33085 | 0.29086 | -0.17643 | | 64 | 0.80130 | 0.57314 | 0.13505 |
| 25 | 0.34517 | 0.25891 | -0.17603 | | 65 | 0.81800 | 0.60921 | 0.12568 |
| 26 | 0.35896 | 0.22823 | -0.17375 | | 66 | 0.83454 | 0.64487 | 0.11611 |
| 27 | 0.37219 | 0.19897 | -0.16970 | | 67 | 0.85085 | 0.67996 | 0.10647 |
| 28 | 0.38486 | 0.17128 | -0.16402 | | 68 | 0.86683 | 0.71431 | 0.09687 |
| 29 | 0.39695 | 0.14528 | -0.15683 | | 69 | 0.88240 | 0.74774 | 0.08740 |
| 30 | 0.40845 | 0.12111 | -0.14818 | | 70 | 0.89749 | 0.78011 | 0.07815 |
| 31 | 0.41937 | 0.09887 | -0.13821 | | 71 | 0.91201 | 0.81126 | 0.06920 |
| 32 | 0.42973 | 0.07868 | -0.12695 | | 72 | 0.92589 | 0.84103 | 0.06063 |
| 33 | 0.43958 | 0.06063 | -0.11440 | | 73 | 0.93907 | 0.86929 | 0.05251 |
| 34 | 0.44898 | 0.04481 | -0.10064 | | 74 | 0.95147 | 0.89589 | 0.04489 |
| 35 | 0.45796 | 0.03126 | -0.08587 | | 75 | 0.96303 | 0.92072 | 0.03783 |
| 36 | 0.46662 | 0.02014 | -0.07006 | | 76 | 0.97371 | 0.94365 | 0.03137 |
| 37 | 0.47509 | 0.01141 | -0.05330 | | 77 | 0.98345 | 0.96457 | 0.02543 |
| 38 | 0.48323 | 0.00515 | -0.03624 | | 78 | 0.99223 | 0.98339 | 0.01997 |
| 39 | 0.49308 | 0.00138 | -0.01459 | | 79 | 1.00000 | 1.00000 | 0.01503 |
| 40 | 0.50237 | 0.00012 | 0.00612 | | | | | |

**Table 16 FFA-W3-480 cross section shape coordinates**

| ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 1.00000 | -0.01857 | | 41 | 0.51000 | 0.00138 | 0.02340 |
| 2 | 0.00752 | 0.98339 | -0.01237 | | 42 | 0.52358 | 0.00515 | 0.05520 |
| 3 | 0.01577 | 0.96457 | -0.00740 | | 43 | 0.53415 | 0.01141 | 0.07933 |
| 4 | 0.02472 | 0.94365 | -0.00456 | | 44 | 0.54479 | 0.02014 | 0.10288 |
| 5 | 0.03445 | 0.92072 | -0.00364 | | 45 | 0.55544 | 0.03126 | 0.12540 |
| 6 | 0.04499 | 0.89589 | -0.00481 | | 46 | 0.56614 | 0.04481 | 0.14668 |
| 7 | 0.05637 | 0.86929 | -0.00848 | | 47 | 0.57683 | 0.06063 | 0.16632 |
| 8 | 0.06866 | 0.84103 | -0.01493 | | 48 | 0.58756 | 0.07868 | 0.18405 |
| 9 | 0.08188 | 0.81126 | -0.02421 | | 49 | 0.59840 | 0.09887 | 0.19976 |
| 10 | 0.09603 | 0.78011 | -0.03617 | | 50 | 0.60944 | 0.12111 | 0.21328 |
| 11 | 0.11108 | 0.74774 | -0.05076 | | 51 | 0.62075 | 0.14528 | 0.22459 |
| 12 | 0.12698 | 0.71431 | -0.06777 | | 52 | 0.63240 | 0.17128 | 0.23349 |
| 13 | 0.14361 | 0.67996 | -0.08667 | | 53 | 0.64445 | 0.19897 | 0.23992 |
| 14 | 0.16073 | 0.64487 | -0.10667 | | 54 | 0.65697 | 0.22823 | 0.24388 |
| 15 | 0.17812 | 0.60921 | -0.12696 | | 55 | 0.67000 | 0.25891 | 0.24552 |
| 16 | 0.19557 | 0.57314 | -0.14679 | | 56 | 0.68354 | 0.29086 | 0.24493 |
| 17 | 0.21289 | 0.53685 | -0.16555 | | 57 | 0.69761 | 0.32394 | 0.24228 |
| 18 | 0.22994 | 0.50049 | -0.18273 | | 58 | 0.71218 | 0.35799 | 0.23767 |
| 19 | 0.24661 | 0.46426 | -0.19797 | | 59 | 0.72720 | 0.39283 | 0.23129 |
| 20 | 0.26280 | 0.42831 | -0.21088 | | 60 | 0.74261 | 0.42831 | 0.22332 |
| 21 | 0.27845 | 0.39283 | -0.22104 | | 61 | 0.75836 | 0.46426 | 0.21397 |
| 22 | 0.29355 | 0.35799 | -0.22845 | | 62 | 0.77435 | 0.50049 | 0.20349 |
| 23 | 0.30812 | 0.32394 | -0.23317 | | 63 | 0.79051 | 0.53685 | 0.19211 |
| 24 | 0.32218 | 0.29086 | -0.23524 | | 64 | 0.80672 | 0.57314 | 0.18007 |
| 25 | 0.33572 | 0.25891 | -0.23471 | | 65 | 0.82290 | 0.60921 | 0.16757 |
| 26 | 0.34880 | 0.22823 | -0.23167 | | 66 | 0.83896 | 0.64487 | 0.15481 |
| 27 | 0.36141 | 0.19897 | -0.22627 | | 67 | 0.85480 | 0.67996 | 0.14196 |
| 28 | 0.37358 | 0.17128 | -0.21869 | | 68 | 0.87034 | 0.71431 | 0.12916 |
| 29 | 0.38533 | 0.14528 | -0.20911 | | 69 | 0.88549 | 0.74774 | 0.11653 |
| 30 | 0.39668 | 0.12111 | -0.19757 | | 70 | 0.90018 | 0.78011 | 0.10420 |
| 31 | 0.40767 | 0.09887 | -0.18428 | | 71 | 0.91432 | 0.81126 | 0.09227 |
| 32 | 0.41834 | 0.07868 | -0.16927 | | 72 | 0.92784 | 0.84103 | 0.08084 |
| 33 | 0.42877 | 0.06063 | -0.15253 | | 73 | 0.94067 | 0.86929 | 0.07001 |
| 34 | 0.43904 | 0.04481 | -0.13419 | | 74 | 0.95274 | 0.89589 | 0.05985 |
| 35 | 0.44918 | 0.03126 | -0.11449 | | 75 | 0.96400 | 0.92072 | 0.05044 |
| 36 | 0.45928 | 0.02014 | -0.09341 | | 76 | 0.97439 | 0.94365 | 0.04183 |
| 37 | 0.46945 | 0.01141 | -0.07107 | | 77 | 0.98387 | 0.96457 | 0.03391 |
| 38 | 0.47945 | 0.00515 | -0.04832 | | 78 | 0.99242 | 0.98339 | 0.02663 |
| 39 | 0.49180 | 0.00138 | -0.01945 | | 79 | 1.00000 | 1.00000 | 0.02004 |
| 40 | 0.50352 | 0.00012 | 0.00816 | | | | | |

**Table 17 TC-72 cross section shape coordinates, Part 1**

| ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 0.99937 | -0.02908 | | 51 | 0.24835 | 0.44713 | -0.35336 | | 101 | 0.44384 | 0.02643 | -0.13282 |
| 2 | 0.00491 | 0.98757 | -0.03411 | | 52 | 0.25292 | 0.43529 | -0.35489 | | 102 | 0.44693 | 0.02343 | -0.12533 |
| 3 | 0.00986 | 0.97569 | -0.03922 | | 53 | 0.25746 | 0.42347 | -0.35608 | | 103 | 0.44998 | 0.02065 | -0.11786 |
| 4 | 0.01487 | 0.96410 | -0.04534 | | 54 | 0.26199 | 0.41168 | -0.35694 | | 104 | 0.45300 | 0.01807 | -0.11041 |
| 5 | 0.01992 | 0.95277 | -0.05209 | | 55 | 0.26649 | 0.39993 | -0.35747 | | 105 | 0.45599 | 0.01569 | -0.10298 |
| 6 | 0.02500 | 0.94160 | -0.05923 | | 56 | 0.27097 | 0.38822 | -0.35768 | | 106 | 0.45894 | 0.01350 | -0.09560 |
| 7 | 0.03009 | 0.93059 | -0.06672 | | 57 | 0.27543 | 0.37658 | -0.35756 | | 107 | 0.46185 | 0.01149 | -0.08825 |
| 8 | 0.03521 | 0.91973 | -0.07449 | | 58 | 0.27986 | 0.36500 | -0.35714 | | 108 | 0.46473 | 0.00964 | -0.08095 |
| 9 | 0.04033 | 0.90900 | -0.08251 | | 59 | 0.28427 | 0.35350 | -0.35642 | | 109 | 0.46758 | 0.00796 | -0.07371 |
| 10 | 0.04543 | 0.89851 | -0.09073 | | 60 | 0.28866 | 0.34208 | -0.35541 | | 110 | 0.47039 | 0.00646 | -0.06651 |
| 11 | 0.05048 | 0.88835 | -0.09911 | | 61 | 0.29302 | 0.33076 | -0.35411 | | 111 | 0.47317 | 0.00515 | -0.05936 |
| 12 | 0.05553 | 0.87826 | -0.10761 | | 62 | 0.29736 | 0.31954 | -0.35251 | | 112 | 0.47592 | 0.00404 | -0.05227 |
| 13 | 0.06058 | 0.86822 | -0.11619 | | 63 | 0.30167 | 0.30843 | -0.35063 | | 113 | 0.47864 | 0.00312 | -0.04524 |
| 14 | 0.06564 | 0.85821 | -0.12481 | | 64 | 0.30596 | 0.29744 | -0.34847 | | 114 | 0.48132 | 0.00234 | -0.03828 |
| 15 | 0.07070 | 0.84819 | -0.13345 | | 65 | 0.31023 | 0.28658 | -0.34602 | | 115 | 0.48397 | 0.00169 | -0.03139 |
| 16 | 0.07577 | 0.83815 | -0.14208 | | 66 | 0.31446 | 0.27585 | -0.34330 | | 116 | 0.48658 | 0.00115 | -0.02457 |
| 17 | 0.08084 | 0.82809 | -0.15069 | | 67 | 0.31867 | 0.26526 | -0.34031 | | 117 | 0.48917 | 0.00069 | -0.01783 |
| 18 | 0.08591 | 0.81800 | -0.15927 | | 68 | 0.32286 | 0.25482 | -0.33705 | | 118 | 0.49172 | 0.00033 | -0.01117 |
| 19 | 0.09098 | 0.80787 | -0.16782 | | 69 | 0.32702 | 0.24455 | -0.33354 | | 119 | 0.49425 | 0.00010 | -0.00458 |
| 20 | 0.09605 | 0.79770 | -0.17630 | | 70 | 0.33115 | 0.23444 | -0.32977 | | 120 | 0.49674 | 0.00002 | 0.00193 |
| 21 | 0.10112 | 0.78747 | -0.18472 | | 71 | 0.33525 | 0.22450 | -0.32576 | | 121 | 0.49924 | 0.00010 | 0.00845 |
| 22 | 0.10619 | 0.77719 | -0.19307 | | 72 | 0.33932 | 0.21473 | -0.32151 | | 122 | 0.50176 | 0.00038 | 0.01504 |
| 23 | 0.11126 | 0.76684 | -0.20131 | | 73 | 0.34337 | 0.20516 | -0.31703 | | 123 | 0.50431 | 0.00078 | 0.02168 |
| 24 | 0.11632 | 0.75641 | -0.20945 | | 74 | 0.34739 | 0.19576 | -0.31234 | | 124 | 0.50689 | 0.00125 | 0.02842 |
| 25 | 0.12138 | 0.74590 | -0.21747 | | 75 | 0.35141 | 0.18649 | -0.30743 | | 125 | 0.50951 | 0.00179 | 0.03524 |
| 26 | 0.12643 | 0.73532 | -0.22536 | | 76 | 0.35539 | 0.17742 | -0.30233 | | 126 | 0.51216 | 0.00241 | 0.04214 |
| 27 | 0.13148 | 0.72464 | -0.23310 | | 77 | 0.35934 | 0.16858 | -0.29702 | | 127 | 0.51485 | 0.00316 | 0.04911 |
| 28 | 0.13652 | 0.71388 | -0.24069 | | 78 | 0.36325 | 0.15995 | -0.29154 | | 128 | 0.51756 | 0.00405 | 0.05615 |
| 29 | 0.14156 | 0.70302 | -0.24812 | | 79 | 0.36713 | 0.15154 | -0.28588 | | 129 | 0.52031 | 0.00511 | 0.06325 |
| 30 | 0.14658 | 0.69207 | -0.25537 | | 80 | 0.37098 | 0.14335 | -0.28005 | | 130 | 0.52309 | 0.00636 | 0.07041 |
| 31 | 0.15160 | 0.68104 | -0.26244 | | 81 | 0.37479 | 0.13540 | -0.27406 | | 131 | 0.52591 | 0.00782 | 0.07762 |
| 32 | 0.15661 | 0.66991 | -0.26931 | | 82 | 0.37857 | 0.12768 | -0.26791 | | 132 | 0.52876 | 0.00945 | 0.08488 |
| 33 | 0.16161 | 0.65868 | -0.27599 | | 83 | 0.38231 | 0.12019 | -0.26161 | | 133 | 0.53164 | 0.01125 | 0.09219 |
| 34 | 0.16659 | 0.64737 | -0.28245 | | 84 | 0.38602 | 0.11294 | -0.25517 | | 134 | 0.53455 | 0.01321 | 0.09955 |
| 35 | 0.17157 | 0.63596 | -0.28870 | | 85 | 0.38970 | 0.10593 | -0.24861 | | 135 | 0.53750 | 0.01533 | 0.10696 |
| 36 | 0.17654 | 0.62447 | -0.29471 | | 86 | 0.39334 | 0.09915 | -0.24194 | | 136 | 0.54049 | 0.01764 | 0.11441 |
| 37 | 0.18149 | 0.61289 | -0.30050 | | 87 | 0.39695 | 0.09261 | -0.23514 | | 137 | 0.54351 | 0.02013 | 0.12189 |
| 38 | 0.18643 | 0.60124 | -0.30604 | | 88 | 0.40052 | 0.08632 | -0.22825 | | 138 | 0.54656 | 0.02282 | 0.12940 |
| 39 | 0.19136 | 0.58950 | -0.31134 | | 89 | 0.40406 | 0.08027 | -0.22125 | | 139 | 0.54965 | 0.02571 | 0.13693 |
| 40 | 0.19628 | 0.57768 | -0.31638 | | 90 | 0.40757 | 0.07446 | -0.21417 | | 140 | 0.55277 | 0.02880 | 0.14448 |
| 41 | 0.20118 | 0.56579 | -0.32116 | | 91 | 0.41104 | 0.06890 | -0.20700 | | 141 | 0.55593 | 0.03210 | 0.15204 |
| 42 | 0.20608 | 0.55382 | -0.32568 | | 92 | 0.41448 | 0.06359 | -0.19976 | | 142 | 0.55912 | 0.03561 | 0.15960 |
| 43 | 0.21097 | 0.54178 | -0.32992 | | 93 | 0.41788 | 0.05852 | -0.19246 | | 143 | 0.56235 | 0.03934 | 0.16717 |
| 44 | 0.21576 | 0.52990 | -0.33388 | | 94 | 0.42125 | 0.05370 | -0.18510 | | 144 | 0.56561 | 0.04329 | 0.17472 |
| 45 | 0.22048 | 0.51813 | -0.33755 | | 95 | 0.42458 | 0.04912 | -0.17770 | | 145 | 0.56891 | 0.04747 | 0.18225 |
| 46 | 0.22518 | 0.50633 | -0.34094 | | 96 | 0.42788 | 0.04477 | -0.17026 | | 146 | 0.57224 | 0.05188 | 0.18976 |
| 47 | 0.22985 | 0.49451 | -0.34404 | | 97 | 0.43114 | 0.04066 | -0.16279 | | 147 | 0.57561 | 0.05653 | 0.19723 |
| 48 | 0.23451 | 0.48268 | -0.34683 | | 98 | 0.43437 | 0.03677 | -0.15531 | | 148 | 0.57901 | 0.06142 | 0.20466 |
| 49 | 0.23914 | 0.47083 | -0.34932 | | 99 | 0.43756 | 0.03310 | -0.14782 | | 149 | 0.58245 | 0.06656 | 0.21202 |
| 50 | 0.24376 | 0.45898 | -0.35150 | | 100 | 0.44072 | 0.02966 | -0.14032 | | 150 | 0.58592 | 0.07195 | 0.21932 |

**Table 18 TC-72 cross section shape coordinates, Part 2**

| ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* |
|---|---|---|---|---|---|---|---|---|
| 151 | 0.58943 | 0.07759 | 0.22655 | | 201 | 0.80312 | 0.59151 | 0.33077 |
| 152 | 0.59297 | 0.08348 | 0.23369 | | 202 | 0.80805 | 0.60368 | 0.32658 |
| 153 | 0.59655 | 0.08962 | 0.24073 | | 203 | 0.81299 | 0.61582 | 0.32217 |
| 154 | 0.60016 | 0.09601 | 0.24767 | | 204 | 0.81795 | 0.62791 | 0.31753 |
| 155 | 0.60381 | 0.10267 | 0.25448 | | 205 | 0.82292 | 0.63995 | 0.31267 |
| 156 | 0.60748 | 0.10957 | 0.26116 | | 206 | 0.82790 | 0.65195 | 0.30759 |
| 157 | 0.61119 | 0.11672 | 0.26771 | | 207 | 0.83290 | 0.66388 | 0.30229 |
| 158 | 0.61494 | 0.12411 | 0.27411 | | 208 | 0.83791 | 0.67576 | 0.29678 |
| 159 | 0.61871 | 0.13174 | 0.28035 | | 209 | 0.84294 | 0.68757 | 0.29105 |
| 160 | 0.62252 | 0.13962 | 0.28643 | | 210 | 0.84797 | 0.69930 | 0.28513 |
| 161 | 0.62636 | 0.14775 | 0.29232 | | 211 | 0.85301 | 0.71097 | 0.27900 |
| 162 | 0.63023 | 0.15610 | 0.29802 | | 212 | 0.85807 | 0.72255 | 0.27267 |
| 163 | 0.63413 | 0.16468 | 0.30351 | | 213 | 0.86313 | 0.73406 | 0.26614 |
| 164 | 0.63806 | 0.17348 | 0.30880 | | 214 | 0.86820 | 0.74547 | 0.25942 |
| 165 | 0.64203 | 0.18250 | 0.31388 | | 215 | 0.87328 | 0.75679 | 0.25251 |
| 166 | 0.64602 | 0.19173 | 0.31873 | | 216 | 0.87836 | 0.76802 | 0.24541 |
| 167 | 0.65004 | 0.20117 | 0.32334 | | 217 | 0.88345 | 0.77914 | 0.23813 |
| 168 | 0.65408 | 0.21079 | 0.32773 | | 218 | 0.88854 | 0.79016 | 0.23067 |
| 169 | 0.65816 | 0.22060 | 0.33187 | | 219 | 0.89364 | 0.80107 | 0.22303 |
| 170 | 0.66226 | 0.23059 | 0.33578 | | 220 | 0.89874 | 0.81187 | 0.21523 |
| 171 | 0.66640 | 0.24074 | 0.33945 | | 221 | 0.90384 | 0.82255 | 0.20726 |
| 172 | 0.67056 | 0.25105 | 0.34287 | | 222 | 0.90894 | 0.83312 | 0.19912 |
| 173 | 0.67474 | 0.26152 | 0.34604 | | 223 | 0.91405 | 0.84356 | 0.19083 |
| 174 | 0.67895 | 0.27213 | 0.34895 | | 224 | 0.91915 | 0.85387 | 0.18238 |
| 175 | 0.68319 | 0.28288 | 0.35162 | | 225 | 0.92425 | 0.86406 | 0.17378 |
| 176 | 0.68746 | 0.29375 | 0.35404 | | 226 | 0.92935 | 0.87411 | 0.16504 |
| 177 | 0.69175 | 0.30475 | 0.35621 | | 227 | 0.93445 | 0.88404 | 0.15616 |
| 178 | 0.69606 | 0.31586 | 0.35812 | | 228 | 0.93954 | 0.89382 | 0.14715 |
| 179 | 0.70041 | 0.32708 | 0.35978 | | 229 | 0.94463 | 0.90347 | 0.13800 |
| 180 | 0.70477 | 0.33841 | 0.36119 | | 230 | 0.94971 | 0.91297 | 0.12874 |
| 181 | 0.70916 | 0.34982 | 0.36237 | | 231 | 0.95479 | 0.92233 | 0.11935 |
| 182 | 0.71358 | 0.36132 | 0.36327 | | 232 | 0.95985 | 0.93154 | 0.10984 |
| 183 | 0.71802 | 0.37291 | 0.36392 | | 233 | 0.96491 | 0.94061 | 0.10023 |
| 184 | 0.72249 | 0.38456 | 0.36429 | | 234 | 0.96996 | 0.94954 | 0.09052 |
| 185 | 0.72697 | 0.39629 | 0.36440 | | 235 | 0.97500 | 0.95831 | 0.08071 |
| 186 | 0.73149 | 0.40808 | 0.36424 | | 236 | 0.98002 | 0.96691 | 0.07079 |
| 187 | 0.73602 | 0.41992 | 0.36380 | | 237 | 0.98504 | 0.97534 | 0.06076 |
| 188 | 0.74060 | 0.43186 | 0.36311 | | 238 | 0.99004 | 0.98356 | 0.05061 |
| 189 | 0.74532 | 0.44415 | 0.36214 | | 239 | 0.99503 | 0.99158 | 0.04033 |
| 190 | 0.75005 | 0.45644 | 0.36092 | | 240 | 1.00000 | 0.99937 | 0.02993 |
| 191 | 0.75479 | 0.46874 | 0.35943 | | | | | |
| 192 | 0.75954 | 0.48104 | 0.35769 | | | | | |
| 193 | 0.76432 | 0.49334 | 0.35569 | | | | | |
| 194 | 0.76910 | 0.50565 | 0.35344 | | | | | |
| 195 | 0.77391 | 0.51795 | 0.35093 | | | | | |
| 196 | 0.77874 | 0.53025 | 0.34818 | | | | | |
| 197 | 0.78358 | 0.54254 | 0.34517 | | | | | |
| 198 | 0.78844 | 0.55482 | 0.34193 | | | | | |
| 199 | 0.79331 | 0.56707 | 0.33845 | | | | | |
| 200 | 0.79821 | 0.57931 | 0.33472 | | | | | |

**Table 19 Cylinder cross section shape coordinates**

| ID | *s* | *x* | *y* | | ID | *s* | *x* | *y* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 1.00000 | 0.00000 | | 52 | 0.51000 | 0.00099 | -0.03140 |
| 2 | 0.01000 | 0.99901 | 0.03140 | | 53 | 0.52000 | 0.00394 | -0.06267 |
| 3 | 0.02000 | 0.99606 | 0.06267 | | 54 | 0.53000 | 0.00886 | -0.09369 |
| 4 | 0.03000 | 0.99114 | 0.09369 | | 55 | 0.54000 | 0.01571 | -0.12434 |
| 5 | 0.04000 | 0.98429 | 0.12434 | | 56 | 0.55000 | 0.02447 | -0.15451 |
| 6 | 0.05000 | 0.97553 | 0.15451 | | 57 | 0.56000 | 0.03511 | -0.18406 |
| 7 | 0.06000 | 0.96489 | 0.18406 | | 58 | 0.57000 | 0.04759 | -0.21289 |
| 8 | 0.07000 | 0.95241 | 0.21289 | | 59 | 0.58000 | 0.06185 | -0.24088 |
| 9 | 0.08000 | 0.93815 | 0.24088 | | 60 | 0.59000 | 0.07784 | -0.26791 |
| 10 | 0.09000 | 0.92216 | 0.26791 | | 61 | 0.60000 | 0.09549 | -0.29389 |
| 11 | 0.10000 | 0.90451 | 0.29389 | | 62 | 0.61000 | 0.11474 | -0.31871 |
| 12 | 0.11000 | 0.88526 | 0.31871 | | 63 | 0.62000 | 0.13552 | -0.34227 |
| 13 | 0.12000 | 0.86448 | 0.34227 | | 64 | 0.63000 | 0.15773 | -0.36448 |
| 14 | 0.13000 | 0.84227 | 0.36448 | | 65 | 0.64000 | 0.18129 | -0.38526 |
| 15 | 0.14000 | 0.81871 | 0.38526 | | 66 | 0.65000 | 0.20611 | -0.40451 |
| 16 | 0.15000 | 0.79389 | 0.40451 | | 67 | 0.66000 | 0.23209 | -0.42216 |
| 17 | 0.16000 | 0.76791 | 0.42216 | | 68 | 0.67000 | 0.25912 | -0.43815 |
| 18 | 0.17000 | 0.74088 | 0.43815 | | 69 | 0.68000 | 0.28711 | -0.45241 |
| 19 | 0.18000 | 0.71289 | 0.45241 | | 70 | 0.69000 | 0.31594 | -0.46489 |
| 20 | 0.19000 | 0.68406 | 0.46489 | | 71 | 0.70000 | 0.34549 | -0.47553 |
| 21 | 0.20000 | 0.65451 | 0.47553 | | 72 | 0.71000 | 0.37566 | -0.48429 |
| 22 | 0.21000 | 0.62434 | 0.48429 | | 73 | 0.72000 | 0.40631 | -0.49114 |
| 23 | 0.22000 | 0.59369 | 0.49114 | | 74 | 0.73000 | 0.43733 | -0.49606 |
| 24 | 0.23000 | 0.56267 | 0.49606 | | 75 | 0.74000 | 0.46860 | -0.49901 |
| 25 | 0.24000 | 0.53140 | 0.49901 | | 76 | 0.75000 | 0.50000 | -0.50000 |
| 26 | 0.25000 | 0.50000 | 0.50000 | | 77 | 0.76000 | 0.53140 | -0.49901 |
| 27 | 0.26000 | 0.46860 | 0.49901 | | 78 | 0.77000 | 0.56267 | -0.49606 |
| 28 | 0.27000 | 0.43733 | 0.49606 | | 79 | 0.78000 | 0.59369 | -0.49114 |
| 29 | 0.28000 | 0.40631 | 0.49114 | | 80 | 0.79000 | 0.62434 | -0.48429 |
| 30 | 0.29000 | 0.37566 | 0.48429 | | 81 | 0.80000 | 0.65451 | -0.47553 |
| 31 | 0.30000 | 0.34549 | 0.47553 | | 82 | 0.81000 | 0.68406 | -0.46489 |
| 32 | 0.31000 | 0.31594 | 0.46489 | | 83 | 0.82000 | 0.71289 | -0.45241 |
| 33 | 0.32000 | 0.28711 | 0.45241 | | 84 | 0.83000 | 0.74088 | -0.43815 |
| 34 | 0.33000 | 0.25912 | 0.43815 | | 85 | 0.84000 | 0.76791 | -0.42216 |
| 35 | 0.34000 | 0.23209 | 0.42216 | | 86 | 0.85000 | 0.79389 | -0.40451 |
| 36 | 0.35000 | 0.20611 | 0.40451 | | 87 | 0.86000 | 0.81871 | -0.38526 |
| 37 | 0.36000 | 0.18129 | 0.38526 | | 88 | 0.87000 | 0.84227 | -0.36448 |
| 38 | 0.37000 | 0.15773 | 0.36448 | | 89 | 0.88000 | 0.86448 | -0.34227 |
| 39 | 0.38000 | 0.13552 | 0.34227 | | 90 | 0.89000 | 0.88526 | -0.31871 |
| 40 | 0.39000 | 0.11474 | 0.31871 | | 91 | 0.90000 | 0.90451 | -0.29389 |
| 41 | 0.40000 | 0.09549 | 0.29389 | | 92 | 0.91000 | 0.92216 | -0.26791 |
| 42 | 0.41000 | 0.07784 | 0.26791 | | 93 | 0.92000 | 0.93815 | -0.24088 |
| 43 | 0.42000 | 0.06185 | 0.24088 | | 94 | 0.93000 | 0.95241 | -0.21289 |
| 44 | 0.43000 | 0.04759 | 0.21289 | | 95 | 0.94000 | 0.96489 | -0.18406 |
| 45 | 0.44000 | 0.03511 | 0.18406 | | 96 | 0.95000 | 0.97553 | -0.15451 |
| 46 | 0.45000 | 0.02447 | 0.15451 | | 97 | 0.96000 | 0.98429 | -0.12434 |
| 47 | 0.46000 | 0.01571 | 0.12434 | | 98 | 0.97000 | 0.99114 | -0.09369 |
| 48 | 0.47000 | 0.00886 | 0.09369 | | 99 | 0.98000 | 0.99606 | -0.06267 |
| 49 | 0.48000 | 0.00394 | 0.06267 | | 100 | 0.99000 | 0.99901 | -0.03140 |
| 50 | 0.49000 | 0.00099 | 0.03140 | | 101 | 1.00000 | 1.00000 | 0.00000 |
| 51 | 0.50000 | 0.00000 | 0.00000 | | | | | |

**Table 20 Wind speed and frequency**

| Wind Speed $m/s$ | Cumulative Distribution |
|---|---|
| 4.0 | 0.09703 |
| 5.0 | 0.10542 |
| 6.0 | 0.10657 |
| 7.0 | 0.10153 |
| 8.0 | 0.09185 |
| 9.0 | 0.06073 |
| 9.5 | 0.03623 |
| 10.0 | 0.03275 |
| 10.5 | 0.02930 |
| 11.0 | 0.02596 |
| 11.5 | 0.02276 |
| 12.0 | 0.02861 |
| 13.0 | 0.02906 |
| 14.0 | 0.02055 |
| 15.0 | 0.01401 |
| 16.0 | 0.01249 |
| 18.0 | 0.00739 |
| 20.0 | 0.00318 |
| 25.0 | 0.00033 |

**Table 21 Absolute thickness constraint**

| $S$ | Absolute Thickness $[m]$ |
|---|---|
| 0.00000 | 5.38000 |
| 0.03057 | 5.36300 |
| 0.06222 | 5.12887 |
| 0.09487 | 4.69181 |
| 0.12841 | 4.16284 |
| 0.16274 | 3.58395 |
| 0.19772 | 3.05710 |
| 0.23321 | 2.66154 |
| 0.26907 | 2.37277 |
| 0.30515 | 2.14436 |
| 0.34128 | 1.95137 |
| 0.37731 | 1.78626 |
| 0.41309 | 1.64011 |
| 0.44846 | 1.49563 |
| 0.48328 | 1.35340 |
| 0.51741 | 1.21399 |
| 0.55073 | 1.07789 |
| 0.58312 | 0.94557 |
| 0.61449 | 0.81742 |
| 0.64476 | 0.69379 |
| 0.67385 | 0.57494 |
| 0.70172 | 0.46110 |
| 0.72833 | 0.35242 |
| 0.75364 | 0.24901 |
| 0.77766 | 0.15091 |
| 0.80038 | 0.05811 |

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## References

[1] Dykes, K., and Meadows, R., "Applications of Systems Engineering to the Research, Design, and Development of Wind Energy Systems," Tech. rep., National Renewable Energy Laboratories, 1617 Cole Boulevard, Golden Colorado, USA, December 2011. NREL/TP-5000-52616.

[2] Bottasso, C., Campagnolo, F., and Croce, A., "Multi-disciplinary constrained optimization of wind turbines," *Multibody System Dynamics*, Vol. 27, No. 1, 2012, pp. 21–53. URL http://dx.doi.org/10.1007/s11044-011-9271-x.

[3] McWilliam, M. K., "Towards Multidisciplinary Design Optimization Capability of Horizontal Axis Wind Turbines," Ph.D. thesis, University of Victoria, 2015.

[4] Zahle, F., Tibaldi, C., Pavese, C., McWilliam, M., Blasques, J. P. A. A., and Hansen, M. H., "Design of an aeroelastically tailored 10 MW wind turbine rotor," *Journal of Physics: Conference Series*, Vol. 753, 2016. https://doi.org/10.1088/1742-6596/753/6/062008.

[5] Ashuri, T., Martins, J. R. R. A., Zaaijer, M. B., van Kuik, G. A. M., and van Bussel, G. J. W., "Aeroservoelastic design definition of a 20 MW common research wind turbine model," *Wind Energy*, 2016.

[6] Scott, S., Capuzzi, M., Langston, D., Bossanyi, E., McCann, G., Weaver, P. M., and Pirrera, A., "Effects of aeroelastic tailoring on performance characteristics of wind turbine systems," *Renewable Energy*, Vol. 114, 2017, pp. 887 – 903. https://doi.org/https://doi.org/10.1016/j.renene.2017.06.048, URL http://www.sciencedirect.com/science/article/pii/S0960148117305530.

[7] Ning, A., Damiani, R., and Moriarty, P., "Objectives and Constraints for Wind Turbine Optimization," *Journal of Solar Energy Engineering*, Vol. 136, No. 4, 2014, p. 12 pages. https://doi.org/10.1115/1.4027693, URL http://dx.doi.org/10.1115/1.4027693.

[8] Gaertner, E., and Lackner, M. A., "Aero-elastic design optimization of floating offshore wind turbine blades," *2018 Wind Energy Symposium*, 2018.

[9] Macquart, T., Maes, V., Langston, D., Pirrera, A., and Weaver, P. M., "A New Optimisation Framework for Investigating Wind Turbine Blade Designs," *Advances in Structural and Multidisciplinary Optimization*, edited by A. Schumacher, T. Vietor, S. Fiebig, K.-U. Bletzinger, and K. Maute, Springer International Publishing, Cham, 2018, pp. 2044–2060.

[10] Merz, K. O., "Conceptual Design of a Stall-Regulated Rotor for a Deepwater Offshore Wind Turbine," Ph.D. thesis, NTNU, Department of Civil and Transport Engineering, Trondheim, Norway, 2011.

[11] Bak, C., Zahle, F., Bitsche, R., Kim, T., Yde, A., Henriksen, L. C., Natarajan, A., and Hansen, M. H., "Description of the DTU 10 MW Reference Wind Turbine," Tech. rep., Technical University of Denmark Institute for Wind Energy, 2013.

[12] Wächter, A., and Biegler, L. T., "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, Vol. 106, No. 1, 2006, pp. 25–57. https://doi.org/10.1007/s10107-004-0559-y, URL https://doi.org/10.1007/s10107-004-0559-y.

[13] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T., "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, Vol. 6, No. 2, 2002, pp. 182–197. https://doi.org/10.1109/4235.996017.

[14] Manwell, J. F., McGowan, J. G., and Rogers, A. L., *Wind Energy Explained, Theory, Design and Application*, John Wiley and Sons, Chichester, England, 2002.