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Revisiting the Temporal Leontief Inverse: New Insights on the Analysis of Regional Technological Economic Change

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ABSTRACT

The current availability of longer series of input-output tables, as well as the release of global input-output databases, has fostered a growing literature analyzing changes in the economic structure and their drivers. In this paper, we take advantage of these time-series by proposing a methodology designed to trace the contribution of different drivers of the change in interindustrial relationships over time. Based on the Temporal Leontief Inverse (TLI), the Extended TLI (ETLI) decomposes the economy-wide effects of changes in direct interindustrial links between years, isolating the impact of different determinants of economic (environmental, energy, etc.) spillovers according to the interests of the researcher. For example, one can explore how the multipliers of a particular industry were affected by changes in technology of other sectors and in the own sector; by changes in trade patterns in specific countries; by indirect changes in intraregional production chains in foreign nations; etc. The ETLI is illustrated by uncovering certain hidden effects not captured in a previous application of the original TLI to the Chicago region between 1980-1997.

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1. Introduction

Firms produce goods and services by combining raw materials, industrial inputs and labor according to a production function, which in the input-output (IO) framework is portrayed by a column in the technical coefficient matrix.[1] This column represents the direct linkages between sectors, i.e., the required inputs from all industries (local and imported) to produce a dollar of output in a particular sector. Since production in one sector demands production in other sectors, there is a ripple effect in the economy that multiplies the initial impact. When the economy is in equilibrium, this set of production functions results in a complex network of interindustrial linkages that reflect direct and indirect connections through various production chains. In the IO framework, this equilibrium is shown in the Leontief Inverse matrix, which columns contain the resulting multipliers for a given sector.

Over time, however, production requirements change due to several reasons as: innovations in production processes, entry and exit of firms, new output mix, factors' productivity, relative prices of factors and inputs, trade policies, etc., resulting in a new structure of inputs demand and consequently a new equilibrium for the year. Since the inception of input-output tables (IOTs), there has been interest in the analysis of such structural changes and identification of their drivers. Early works by Leontief (1941) and Leontief *et al.* (1953) highlighted structural changes between the 1919, 1929 and 1939 United States (US) tables comparing different measures of coefficients and linkages between these years. Due to the general equilibrium nature of the IO framework, studies in the 1960s began exploring decomposition techniques that isolated the impact of changes in composition and level of industries' final demand in addition to technology between any two periods. Skolka (1977) formally introduced the structural decomposition analysis (SDA) methodology, a comparative static exercise based on splitting identities to account for different drivers of change. The

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[1] Due to the assumption of perfect complementary inputs, i.e. a Leontief production function.

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procedure decomposes the total change in a sector's output into several components. Early papers relied on a simple three component analysis (technology, mix and level)[2], while modern decompositions include capital, labor, energy and materials. The simplicity of the SDA and its flexibility in incorporating environmental indicators, employment multipliers, energy requirements, etc., led to a vast literature exploring structural changes at different regional levels (e.g., Alcántara and Duarte, 2004 (energy), Peters *et al.*, 2007 (greenhouse gases); Zhang *et al.*, 2012 (water); Carrascal-Incera, 2017 (employment)).

Until recently, the availability of IOTs was limited to incomplete time-series, with Statistical Offices publishing data in 3-5 years gaps and oftentimes changing sectorial classification or accounting system between releases.[3] Since the minimum requirements to perform a SDA are two harmonized IOTs with any gap in-between,[4] this methodology is able to extract the most information from this restricted data environment. In the last decade, efforts to increase the timeliness of IOTs have resulted in several harmonized time-series at regional (IMPLAN, 2017), national (OECD, 2017) and global levels (Eora (Lenzen *et al.*, 2013), World IO Database (Timmer *et al.*, 2015), EXIOBASE (Stadler *et al.*, 2018)). In lieu of these time-series, several papers have applied the SDA methodology sequentially, the so called chained or chaining-SDA, to highlight the drivers of changes observed between each pair of years in the series (e.g., de Haan, 2001; Su and Ang, 2012; Carrascal-Incera, 2017).

Alternatively to the SDA, Sonis and Hewings (1998) proposed an intertemporal approach to study the contribution of technological change over time inspired by Leontief (1970)'s dynamic inverse. Specifically designed for IOT time-series, the Temporal Leontief Inverse (TLI) is a compromise between comparative statics approaches (e.g. a SDA) and full dynamic models (e.g. the Dynamic Leontief model) that focuses exclusively on the effect of yearly changes in production requirements on total linkages between sectors (i.e., ignoring changes in final demand). The TLI traces the evolution of total linkages in the time-series by considering the impact of perturbations in direct input requirements that translates into changes in interindustrial linkages and their accumulation over time. This way, it works as a year-to-year analysis of technological change and, in contrast to a SDA, it has the advantage of being non-chained, i.e., keeping the reference year fixed. The first application of the model was performed by Okuyama *et al.* (2006) to assess the hollowing-out effect in Chicago from 1980-1997. Next, Firmansyah and Oktavilia (2015) applied the same steps to examine the structural changes underwent by the manufacturing industries in Indonesia during the period 1975-2005.

Nevertheless, the original TLI formulation is unable to reveal the specific drivers of such yearly changes, since the model reflects the compounding impact of variations in the input structure of all industries simultaneously. Therefore, although one can measure the contribution of a given year to the total linkages at the end of the time-series, one cannot isolate the impact of a particular sector (or set thereof) to more precisely identify the main sources of its variation by using the original version of the TLI.

To address the latter, in this paper we extend the original TLI methodology and devise a linear decomposition of the annual change. The extended TLI (ETLI) allows splitting the changes in direct input requirements in a given year into several partitions that isolate the temporal cumulative contribution of a particular driver of interest in a *ceteris paribus* fashion. This way, one can study how the contribution of a subset of direct linkages evolves and influences the resulting total linkages at the end of the time-series. For example, one can isolate the effects of technology changes by sectors or group of sectors; study the contribution of trade and evolving production chains to a particular industry' spillovers; and quantify the effects of hollowing-out, outsourcing, clustering, shift in production chains and trade patterns to economic and environmental multipliers. Although the ETLI has been once applied to reveal the main drivers of the evolving changes in greenhouse gases emissions spillovers of US households (Franco-Solís *et al.*, 2020), the methodology and its usefulness in structural change research has never been thoroughly documented.

A description of the TLI approach and the extension proposed is presented in the next section. Section 3 exemplifies how the framework can be applied to analyze issues like hollowing-out, outsourcing and trade effects. We illustrate our extended methodology in Section 4 by uncovering some hidden effects not captured in the application of the original TLI to Chicago done by Okuyama *et al.* (2006). Conclusions follow.

2. Methodology

Consider an economy with n industries and let $\mathbf{A}_t$ be its domestic/local coefficients matrix[5] at time t, which columns denote the value in dollars of inputs supplied by sector i and used to produce one dollar's worth of sector j's output.[6] Also, denote by $\mathbf{E}_t$ the matrix of changes in direct input requirement coefficients between time $t-1$ and t, such that $\mathbf{A}_t = \mathbf{A}_{t-1} + \mathbf{E}_t$. Hence, we can relate the total requirements matrix, i.e. the Leontief Inverse, ($\mathbf{B}_t$) for this economy with past direct input requirement in two ways:

$$\mathbf{B}_t = (\mathbf{I}-\mathbf{A}_t)^{-1} = (\mathbf{I}-\mathbf{A}_{t-1}-\mathbf{E}_t)^{-1} = [(\mathbf{I}-\mathbf{A}_{t-1})(\mathbf{I}-\mathbf{B}_{t-1}\mathbf{E}_t)]^{-1} = \mathbf{M}_t^{\mathrm{L}}\mathbf{B}_{t-1} \tag{1}$$

$$\mathbf{B}_t = (\mathbf{I}-\mathbf{A}_t)^{-1} = (\mathbf{I}-\mathbf{A}_{t-1}-\mathbf{E}_t)^{-1} = [(\mathbf{I}-\mathbf{E}_t\mathbf{B}_{t-1})(\mathbf{I}-\mathbf{A}_{t-1})]^{-1} = \mathbf{B}_{t-1}\mathbf{M}_t^{\mathrm{R}} \tag{2}$$

where,

$$\mathbf{M}_t^{\mathrm{L}} = (\mathbf{I}-\mathbf{B}_{t-1}\mathbf{E}_t)^{-1} \tag{3}$$

$$\mathbf{M}_t^{\mathrm{R}} = (\mathbf{I}-\mathbf{E}_t\mathbf{B}_{t-1})^{-1} \tag{4}$$

$\mathbf{M}_t^{\mathrm{L}}$ and $\mathbf{M}_t^{\mathrm{R}}$ are the left and right temporal multipliers first appeared in Sonis and Hewings (1998). These temporal multipliers denote the change in the economy's fields of influence[7] between periods due to the structural change $\mathbf{E}_t$ (see section 5 of

[2] Commonly used additive decompositions generate interaction terms that reflect the combined change between the elements. As highlighted by Rose and Casler (1996), these "residuals" present a challenge regarding their economic interpretation and the treatment of this information, which ultimately means that the specification of the decomposition is not necessarily unique. Several strategies have been proposed in the literature to distribute such joint effects among the different components (Casler, 2001). Dietzenbacher and Los (1998) proposed a way to overcome the "non-uniqueness problem" by using the average of the polar decompositions and the usage of midpoint weights, which has been widely applied in the literature.

[3] Exemption to this are IOTs for Denmark, which have been consistently published yearly since 1966 in a standard convention (Statistics Denmark, 2017).

[4] Preferably at constant prices to control for changes in prices as one determinant of the possible change in coefficients.

[5] The **A** matrix that is used in this paper is essentially a regional requirements matrix (domestic/local purchases), not a total requirements matrix (local plus imports). So, we shall use the term "domestic coefficients" and not the traditional "technical coefficients". This allows us to further explore the concept of "hollowing out" as those cases where the entries in columns of **A** decreased reflecting a substitution of non-local for local inputs.

[6] The standard input-output notation is used in this paper. Moreover, matrices are named in bold capital letters, vectors in bold lower-case letters and scalars in italic lower-case letters. The matrix **I** is an identity matrix of appropriate dimensions.

[7] The concept of a "field of influence" was developed by Sonis and Hewings (1991) to provide a formal, general tool to measure the analytical impact of changes

Sonis and Hewings (1998) for the derivation), moving the total requirements matrix from one period to another. In fact, $\mathbf{M}_t^{\mathrm{L}}$ is a generalization of the Sherman-Morrison formula for a multi-element change in an inverse matrix (Sonis and Hewings, 1989).

Therefore, the Leontief Inverse can be rewritten as:

$$\mathbf{B}_t = \mathbf{M}_t^{\mathrm{L}}\mathbf{B}_{t-1} = \mathbf{B}_{t-1} + \left(\mathbf{M}_t^{\mathrm{L}} - \mathbf{I}\right)\mathbf{B}_{t-1} \tag{5}$$

$$\mathbf{B}_t = \mathbf{B}_{t-1}\mathbf{M}_t^{\mathrm{R}} = \mathbf{B}_{t-1} + \mathbf{B}_{t-1}\left(\mathbf{M}_t^{\mathrm{R}} - \mathbf{I}\right) \tag{6}$$

where the corresponding second terms of Eqs. 5 and 6 can be denoted as the temporal increment matrix $\mathbf{D}_t$, such that:

$$\mathbf{B}_t = \mathbf{B}_{t-1} + \mathbf{D}_t \tag{7}$$

Then, both the additive and the multiplicative temporal decomposition of the Leontief inverse can be derived as:

$$\mathbf{B}_t = \mathbf{B}_{t-1} + \mathbf{D}_t = \mathbf{B}_{t-2} + \mathbf{D}_{t-1} + \mathbf{D}_t = \ldots = \mathbf{B}_0 + \mathbf{D}_1 + \mathbf{D}_2 + \ldots + \mathbf{D}_t \tag{8}$$

$$\mathbf{B}_t = \mathbf{M}_t^{\mathrm{L}}\mathbf{B}_{t-1} = \mathbf{M}_t^{\mathrm{L}}\mathbf{M}_{t-1}^{\mathrm{L}}\mathbf{B}_{t-2} = \ldots = \mathbf{M}_t^{\mathrm{L}}\mathbf{M}_{t-1}^{\mathrm{L}}\ldots\mathbf{M}_2^{\mathrm{L}}\mathbf{M}_1^{\mathrm{L}}\mathbf{B}_0 \tag{9}$$

If we now combine both decompositions, we construct a formula with an intuitive interpretation:

$$\begin{aligned}\mathbf{B}_t = {} & \mathbf{I} + (\mathbf{B}_0 - \mathbf{I}) + \left(\mathbf{M}_1^{\mathrm{L}} - \mathbf{I}\right)\mathbf{B}_0 + \left(\mathbf{M}_2^{\mathrm{L}} - \mathbf{I}\right)\mathbf{M}_1^{\mathrm{L}}\mathbf{B}_0 + \ldots \\ & + \left(\mathbf{M}_t^{\mathrm{L}} - \mathbf{I}\right)\mathbf{M}_{t-1}^{\mathrm{L}}\ldots\mathbf{M}_2^{\mathrm{L}}\mathbf{M}_1^{\mathrm{L}}\mathbf{B}_0\end{aligned} \tag{10}$$

Using Eq. 6 instead, we can also obtain the equivalent multiplicative and additive TLI decompositions that involve the right temporal multipliers:

$$\mathbf{B}_t = \mathbf{B}_{t-1}\mathbf{M}_t^{\mathrm{R}} = \mathbf{B}_{t-2}\mathbf{M}_t^{\mathrm{R}}\mathbf{M}_{t-1}^{\mathrm{R}} = \ldots = \mathbf{B}_0\mathbf{M}_t^{\mathrm{R}}\mathbf{M}_{t-1}^{\mathrm{R}}\ldots\mathbf{M}_2^{\mathrm{R}}\mathbf{M}_1^{\mathrm{R}} \tag{11}$$

Each element in Eq. 10 quantifies the contribution of the changes in direct input requirements in a particular year to the total linkages observed in the last period. This represents a temporal decomposition of change, where one can trace the evolutionary path of the Leontief inverse's elements and measure the contribution of each period to the current state of the matrix.

This derivation is useful when one is interested in analyzing the full structural change between periods. However, isolating parts of the structural change is fundamental when assessing the contribution of particular sectors/clusters/countries to the overall structural change (i.e., changes of economy-wide interindustrial linkages). In a temporal context, it reflects the evolutionary path of its contribution. For instance, in a multi-regional scenario, a researcher might be interested in studying the indirect impact of one country' structural change in the output multiplier of another country's industry. In a single-region perspective, one might trace the contribution of a technological change in one sector on the total connectivity change of the economy.

in the technical coefficient **A** matrix on the associated Leontief inverse **B**. Assume that one element of $\mathbf{A}_t$ in row i and column j is changed, i.e., $\mathrm{a}_{ij}^* = \mathrm{a}_{ij} + \Delta\mathrm{a}_{ij}$, producing $\mathbf{A}^* = \mathbf{A} + \Delta\mathbf{A}$. Then, the n x n matrix showing the change in each element of **B** caused by that change in a_{ij} $(\Delta\mathrm{a}_{ij})$ is defined by Sonis and Hewings as: $\Delta\mathbf{B}_{(ij)} = \mathbf{F}_{(ij)}k_{ij} = \mathbf{b}_{.i}\mathbf{b}_{j.}\left(\frac{\Delta\mathrm{a}_{ij}}{1-\mathrm{b}_{ji}\Delta\mathrm{a}_{ij}}\right) = \begin{bmatrix}\mathrm{b}_{1i}\\ \mathrm{b}_{2i}\\ \vdots\\ \mathrm{b}_{ni}\end{bmatrix}\begin{bmatrix}\mathrm{b}_{j1} & \mathrm{b}_{j2} & \cdots & \mathrm{b}_{jn}\end{bmatrix}\left(\frac{\Delta\mathrm{a}_{ij}}{1-\mathrm{b}_{ji}\Delta\mathrm{a}_{ij}}\right)$ where $\mathbf{F}_{(ij)}$ is the first order (direct) field of influence matrix of the incremental change a_{ij}. By using increasingly complex mathematical representations, Sonis and Hewings also propose higher-order fields of influence when two or more coefficients change (Sonis and Hewings, 1991, 1992).

To do so, we decompose the temporal multiplier $\mathbf{M}_t^{\mathrm{L}}$ into a linear sum of effects, by using the Woodbury Matrix Identity[8]. Starting from Eq. 3 and applying the identity:

$$\mathbf{M}_t^{\mathrm{L}} = (\mathbf{I} - \mathbf{B}_{t-1}\mathbf{E}_t)^{-1} = \mathbf{I} + \mathbf{B}_{t-1}(\mathbf{I} - \mathbf{E}_t\mathbf{B}_{t-1})^{-1}\mathbf{E}_t \tag{12}$$

$$\mathbf{M}_t^{\mathrm{L}} = \mathbf{I} + \mathbf{B}_{t-1}\mathbf{M}_t^{\mathrm{R}}\mathbf{E}_t \tag{13}$$

$$\mathbf{M}_t^{\mathrm{L}} = \mathbf{I} + \mathbf{B}_t\mathbf{E}_t \tag{14}$$

This derivation can be also extracted for the right temporal multiplier

$$\mathbf{M}_t^{\mathrm{R}} = \mathbf{I} + \mathbf{E}_t\mathbf{B}_t \tag{15}$$

Now, we can divide the E_t matrix into a sum of k specific partitions p $[p = 1, 2, \ldots, k]$ such that $\mathbf{E}_t = \sum_{p=1}^{k}\mathbf{E}_t^p$. Matrix $\mathbf{E}_t^p$ (n x n) is composed by the corresponding partition p of changes in direct input requirement coefficients and zeros elsewhere. For illustration, Figure 1 presents an example of a single-region $\mathbf{E}_t$ matrix with three $(k = 3)$ possible partitions p and its corresponding $\mathbf{E}_t^p$ matrices.

The left and the right temporal multipliers can be thus decomposed into a sum of partitions of the structural change:

$$\mathbf{M}_t^{\mathrm{L}} = \mathbf{I} + \mathbf{B}_t\mathbf{E}_t^1 + \mathbf{B}_t\mathbf{E}_t^2 + \ldots + \mathbf{B}_t\mathbf{E}_t^k \tag{16}$$

$$\mathbf{M}_t^{\mathrm{R}} = \mathbf{I} + \mathbf{E}_t^1\mathbf{B}_t + \mathbf{E}_t^2\mathbf{B}_t + \ldots + \mathbf{E}_t^k\mathbf{B}_t \tag{17}$$

This is in the same spirit as a hypothetical extraction method,[9] but instead of extracting sectors of the economy, the partitions extract parts of the changes in the domestic coefficients between periods. Hence, we can measure the contribution of each partition to the total change in linkages observed.

In terms of the temporal increment matrix $\mathbf{D}_t$ (Eq. 7), if we substitute $\mathbf{M}_t^{\mathrm{L}}$ and $\mathbf{M}_t^{\mathrm{R}}$ for $\mathbf{I} + \mathbf{B}_t\mathbf{E}_t$ (Eq. 14) and $\mathbf{I} + \mathbf{E}_t\mathbf{B}_t$ (Eq. 15), respectively, $\mathbf{D}_t$ can be expressed as:

$$\mathbf{D}_t = \mathbf{B}_t - \mathbf{B}_{t-1} = \left(\mathbf{M}_t^{\mathrm{L}} - \mathbf{I}\right)\mathbf{B}_{t-1} = (\mathbf{I} + \mathbf{B}_t\mathbf{E}_t - \mathbf{I})\mathbf{B}_{t-1} = \mathbf{B}_t\mathbf{E}_t\mathbf{B}_{t-1} \tag{18}$$

$$\mathbf{D}_t = \mathbf{B}_t - \mathbf{B}_{t-1} = \mathbf{B}_{t-1}\left(\mathbf{M}_t^{\mathrm{R}} - \mathbf{I}\right) = \mathbf{B}_{t-1}(\mathbf{I} + \mathbf{E}_t\mathbf{B}_t - \mathbf{I}) = \mathbf{B}_{t-1}\mathbf{E}_t\mathbf{B}_t \tag{19}$$

Then, if we apply the k partitions of $\mathbf{E}_t$, Eqs. 18 and 19 can be derived as:

$$\mathbf{D}_t = \mathbf{B}_t\left(\sum_{p=1}^{k}\mathbf{E}_t^p\right)\mathbf{B}_{t-1} = \mathbf{B}_t\mathbf{E}_t^1\mathbf{B}_{t-1} + \mathbf{B}_t\mathbf{E}_t^2\mathbf{B}_{t-1} + \ldots + \mathbf{B}_t\mathbf{E}_t^k\mathbf{B}_{t-1} \tag{20}$$

$$\mathbf{D}_t = \mathbf{B}_{t-1}\left(\sum_{p=1}^{k}\mathbf{E}_t^p\right)\mathbf{B}_t = \mathbf{B}_{t-1}\mathbf{E}_t^1\mathbf{B}_t + \mathbf{B}_{t-1}\mathbf{E}_t^2\mathbf{B}_t + \ldots + \mathbf{B}_{t-1}\mathbf{E}_t^k\mathbf{B}_t \tag{21}$$

[8] Let **A** and **C** be invertible matrix and **U** and **V** matrices of conformable dimensions. Then, $(\mathbf{A} + \mathbf{UCV})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}(\mathbf{C}^{-1} + \mathbf{V}\mathbf{A}^{-1}\mathbf{U})^{-1}\mathbf{V}\mathbf{A}^{-1}$. In case **C** is a unit matrix of dimension 1 × 1, the Woodbury identity becomes the Sherman-Morrison formula.

[9] The hypothetical extraction method (HEM) measures the contribution of a sector to the economy by zeroing both the sector's input requirements (row of the technical coefficient matrix) as well as its supply links to other industries (column). Total linkages are calculated for this modified coefficient matrix and the difference between the original output and the new output reflect the sector's importance to the economy. Different HEMs have been applied in a similar context of partitioned matrices (see Dietzenbacher *et al*., 1993).

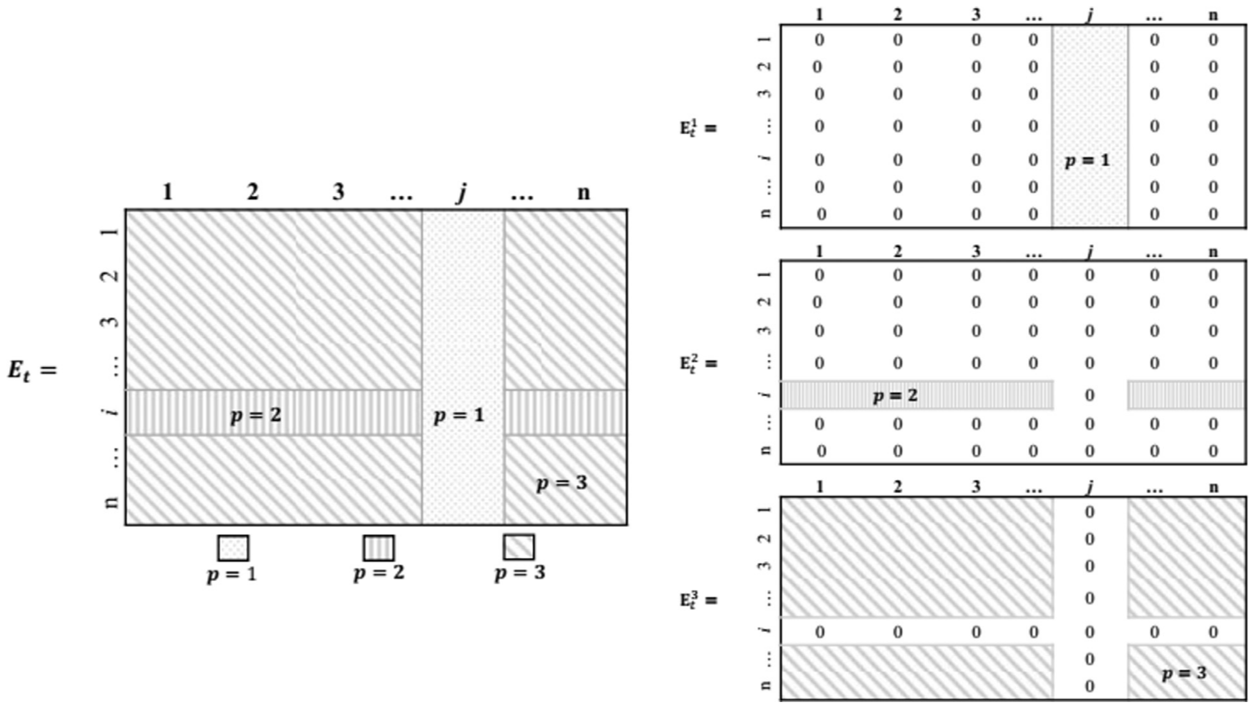

Figure 1. Example of a single-region $\mathbf{E}_t$ matrix with three p partitions ($k = 3$) and its corresponding $\mathbf{E}_t^p$ matrices.

Notice that although the result in Eqs. 18 and 19 is equal $[\mathbf{B}_t(\sum_{p=1}^{k}\mathbf{E}_t^p)\mathbf{B}_{t-1} = \mathbf{B}_{t-1}(\sum_{p=1}^{k}\mathbf{E}_t^p)\mathbf{B}_t]$ when assessing all partitions at the system-wide level, this does not hold anymore for specific p partitions of $\mathbf{E}_t$ ($\mathbf{B}_t\mathbf{E}_t^p\mathbf{B}_{t-1} \neq \mathbf{B}_{t-1}\mathbf{E}_t^p\mathbf{B}_t$). Therefore, similar to the *average* solution commonly applied to overcome the non-uniqueness problem in a SDA[10], we allocate similar weights to the partitions' impacts matrices, such that:

$$\mathbf{D}_t^p = 0.5 \times \left(\mathbf{B}_t\mathbf{E}_t^p\mathbf{B}_{t-1} + \mathbf{B}_{t-1}\mathbf{E}_t^p\mathbf{B}_t\right) \tag{22}$$

and the temporal expansion can be decomposed into a sum of structural change partitions:

$$\mathbf{D}_t = \mathbf{D}_t^1 + \mathbf{D}_t^2 + \ldots + \mathbf{D}_t^k \tag{23}$$

Hence, one can decompose the change in current multipliers by taking a unitary vector of final demand $\mathbf{f}_t$:

$$\begin{aligned}\mathbf{x}_t = \mathbf{f}_t + (\mathbf{B}_0 - \mathbf{I})\mathbf{f}_t + \mathbf{D}_1^1\mathbf{f}_t + \mathbf{D}_2^1\mathbf{f}_t + \ldots + \mathbf{D}_t^1\mathbf{f}_t + \mathbf{D}_1^2\mathbf{f}_t + \mathbf{D}_2^2\mathbf{f}_t + \ldots \\ + \mathbf{D}_t^2\mathbf{f}_t + \ldots + \mathbf{D}_1^k\mathbf{f}_t + \mathbf{D}_2^k\mathbf{f}_t + \ldots + \mathbf{D}_t^k\mathbf{f}_t \end{aligned} \tag{24}$$

The series $\mathbf{D}_1^p\mathbf{f}_t + \mathbf{D}_2^p\mathbf{f}_t + \ldots + \mathbf{D}_t^p\mathbf{f}_t$ isolates the evolution of partition p's impacts on the structural change, where each element $\mathbf{D}_t^p\mathbf{f}_t$ is the effect of partition p at between periods t and $t-1$.

Since we have not imposed any restrictions on $\mathbf{E}_t^p$, we can measure the evolution of different drivers under *ceteris paribus* conditions by grouping partitions in diverse ways: vectors, matrices or even represented by single elements. Nevertheless, as highlighted by Dietzenbacher and Los (2000), dependencies may exist among distinct partitions due to the fact that parts of the matrix may not be able to change independently of others.[11]

[10] Dietzenbacher and Los (1998) demonstrated that the simple average of the two polar decompositions is as a good approximation of the average of the n! exact decomposition forms and it can be thus considered as a pragmatic solution to solve the non-uniqueness problem in SDA.

[11] In such cases, combining partitions would reflect changes that should occur simultaneously. The researcher should design partitions in a way that best reflect the phenomenon being analyzed, for the region under analysis.

In the next section, we propose three basic partitions that can be applied to single and multiregional IOTs, as well as a discussion on how to interpret the ETLI decomposition.

3. The Framework: Partitions and Interpretation

While the basic TLI portrays the multiplier's evolutionary path of the whole economy simultaneously changing in every period, the ETLI allows the decomposition of these changes, conveying a detailed picture of the drivers influencing the latter. As shown previously, the isolation of particular drivers is accomplished by partitioning the intertemporal change in the direct input requirement table ($\mathbf{E}_t$) in different ways. In this section, we introduce three possible partitions (Table 1 and Figures 1-2) to study the sources of hollowing-out in the economy of the Chicago region between 1980-1997 (see application in section 4).

A first natural partition is to isolate the column of sector h, i.e., the industry's "own effect". This is the impact on the multiplier if only the sector's technology, or direct input requirements, was changing *ceteris paribus*. It highlights the only part of the structural change that is mostly under the control of the sector[12]. The "own effect" also conveys some information on outsourcing, although a more comprehensive analysis requires an interregional table. In the latter case, by splitting this partition into "local own effects" (changes in local input requirements) and "external own effects" (changes in imports' structure), international outsourcing will be reflected in negative local own effect trends and positive external own effect trends.

Following the extended notion for outsourcing introduced by Romero *et al.* (2009), these first partitions could already reveal different fragmentation processes. The first case would occur when outsourcing increases the density of transactions and linkages

[12] Some of the observed changes in the column of sector h might be due to changes in supply conditions that are outside of the sector's control. For example, the closure of several steel mills in the Midwest starting in the 1980s limited the local supply of steel, forcing sectors to purchase outside the region. These effects could be more fully captured in a multiregional model.

Table 1
Basic partitions for sector *h*.

Sector *h* effect	Partitioned E^p_{ij} matrix
Own Effect "O" (equivalent to partition $p = 1$ in Fig. 1)	$\mathbf{E}^{O}_{ij} = \begin{cases} \mathbf{E}_{ij}, & if\ j = h \\ 0, & otherwise \end{cases}$
Substitution Effect "S" (partition $p = 2$ in Fig. 1)	$\mathbf{E}^{S}_{ij} = \begin{cases} \mathbf{E}_{ij}, & if\ i = h\ and\ j \neq h \\ 0, & otherwise \end{cases}$
Interrelational Effect "I" (partition $p = 3$ in Fig. 1)	$\mathbf{E}^{I}_{ij} = \begin{cases} \mathbf{E}_{ij}, & if\ i \neq h\ and\ j \neq h \\ 0, & otherwise \end{cases}$

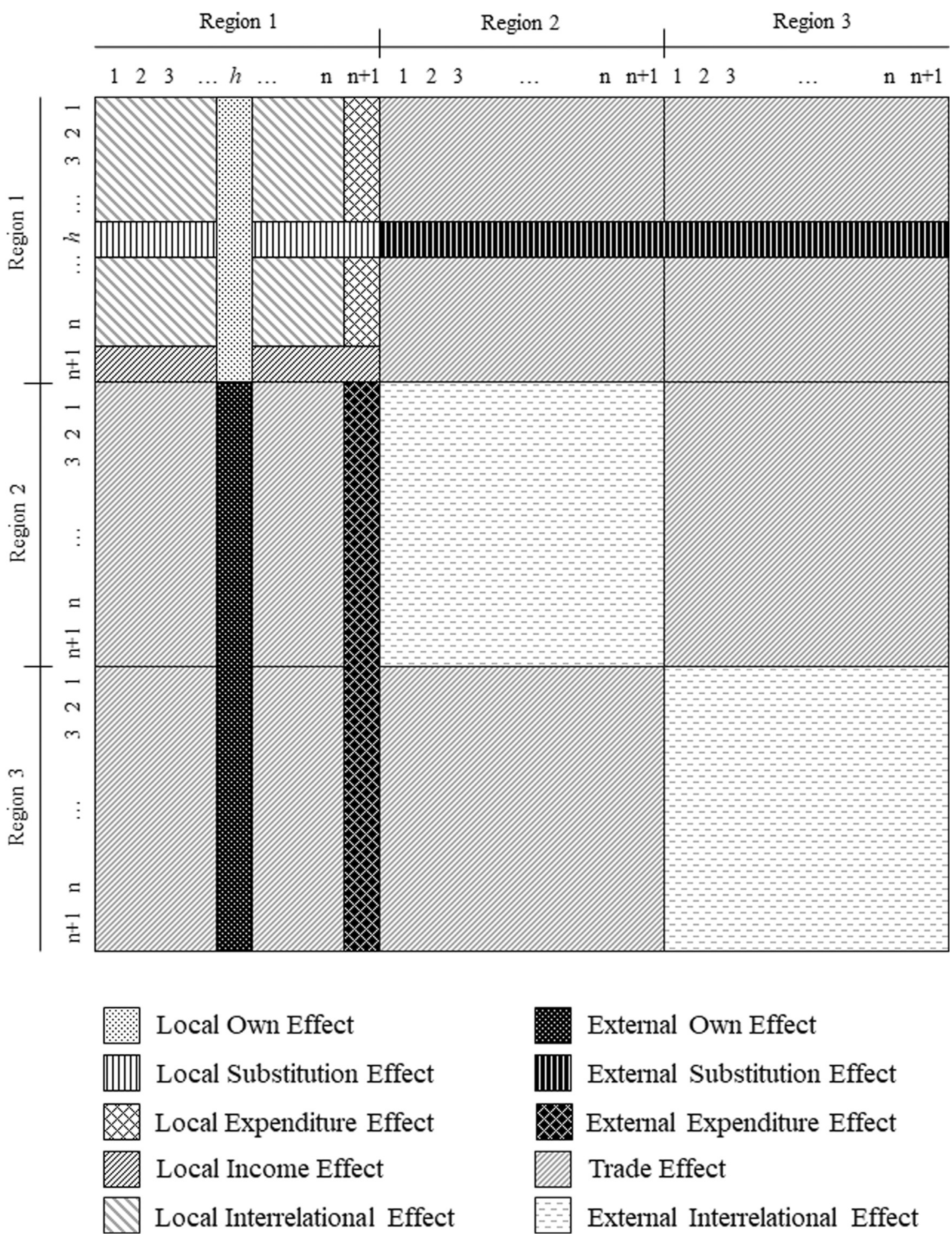

Figure 2. Basic Partitions in a Multi-Regional IOT.

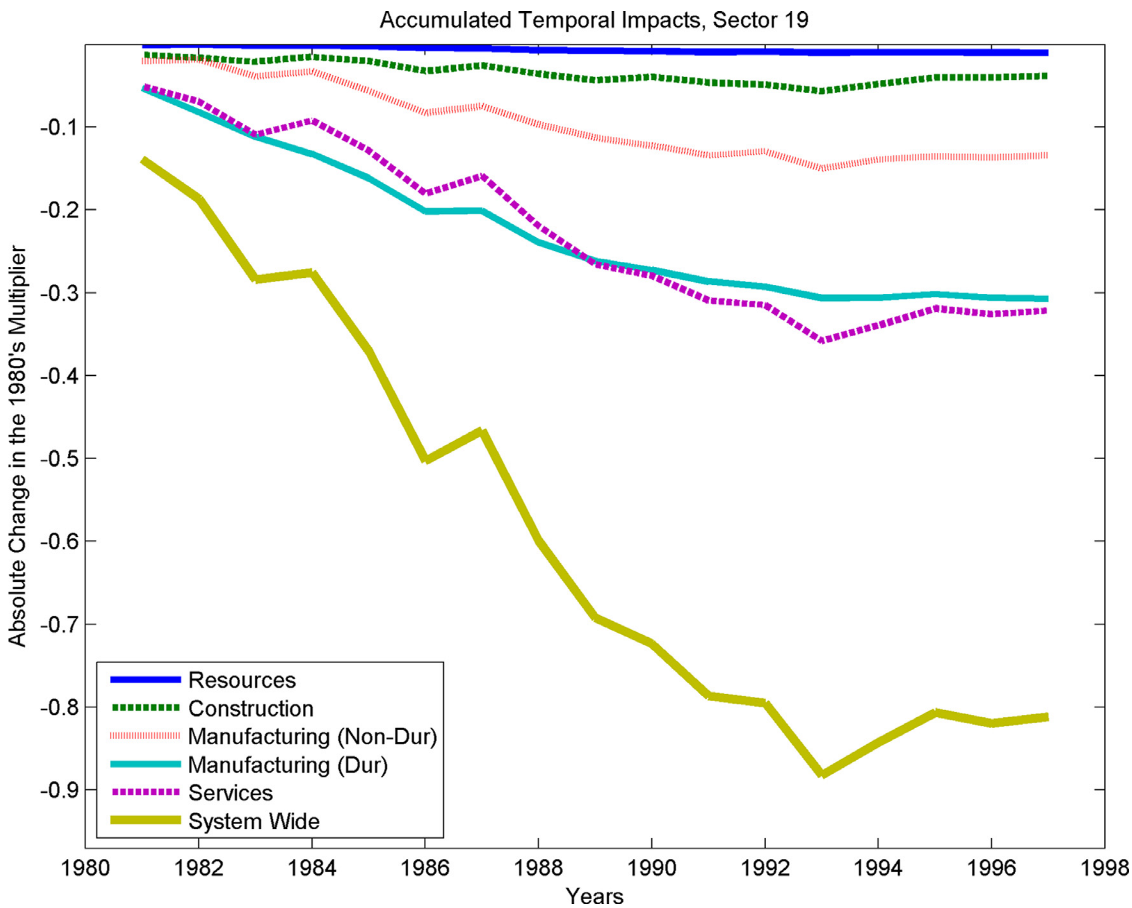

Figure 3. Trends in accumulated temporal impacts of the demand increase in Sector 19

within an economy, i.e. an increase in the “local own effects” due to a *functional fragmentation* of the production processes. This type of fragmentation is related to industries focusing on those functions considered as their main competitive advantages and subcontracting other parts of their value chains (in this case, outsourcing would be the cause of increasing local multipliers). The second type of fragmentation described in Romero *et al.* (2009) is called *spatial fragmentation* and happens due to industrial relocations. In this case, regional or national economies might be losing some internal linkages showing a trend of declining “local own effects” and growing “external own effects” (outsourcing would be the cause of shrinking local multipliers) [13].

By isolating the row of sector h in a partition, we measure the “substitution effect” of the structural change.[14] The partition shows, *ceteris paribus*, how the change in the sector's sales structure affects its multiplier via forward linkages. Negative substitution effects combined with negative own effects highlight a hollowing-out phenomenon when output in the sector is not declining. Moreover, in interregional tables, one can separate local substitution effects from external substitution effects (changes in the sector's export pattern) to assess shifts in global value added chains to local and foreign spillovers.

A third basic partition is the one that excludes all direct linkages with sector h, i.e., changes in row and column h are ignored. This partition measures “interrelational effects”: the effects in sector h from changes all other industries' linkages. This procedure reveals the evolution of the indirect economic leakages and the impact they impose on the multiplier of sector h (e.g., negative trends indicate increasing leakages). Furthermore, several subpartitions can be performed to isolate the influence of specific elements of the interrelational effect, as a particular industry, cluster, region (in interregional tables), etc.

Notice that these are only a few examples of possible partitions (see Figure 2 for a multi-regional perspective). Different decompositions can isolate distinct drivers of interest for the researcher, so partitions need to be tailored to specific research questions/theories.

4. Case Study: Results of a second (expanded) look at Okuyama *et al.* 2006

Okuyama *et al.* (2006) applied the basic TLI to analyze the phenomenon of hollowing-out in Chicago during the 1980s and 1990s. Hollowing-out refers to the process of reducing local dependency due to declining levels of purchases and sales, and it is evidenced by shrinking local multipliers with constant or increasing output. By tracing the evolution of linkages from 1980 to 1997, the TLI highlighted this pattern in both Sector 19 (Industrial Machin-

[13] In this paper, we follow the logic behind the concept *spatial fragmentation* that was originally proposed by Jones and Kierzkowski (1990, 2005), and in which outsourcing is interpreted as a form of division of production processes into subsequent phases that are undertaken in different locations separated by distance. It should be noted however that this process of fragmentation does not necessarily happen only in a global framework. As discussed in Yuskavage *et al.* (2008), Rugman *et al.* (2009) or Bernhardt *et al.* (2015), there is also a parallel phenomenon of domestic fragmentation within large economies.

[14] This partition does not include purchases from the sector itself ($\mathbf{E}_{hh}$) since they are accounted for in the own effect. As this element influences both backward and forward linkages, it could potentially be removed from the own effect and studied separately in its own partition (similar to the idea of self-generated changes used by Sonis *et al.* (1996)).

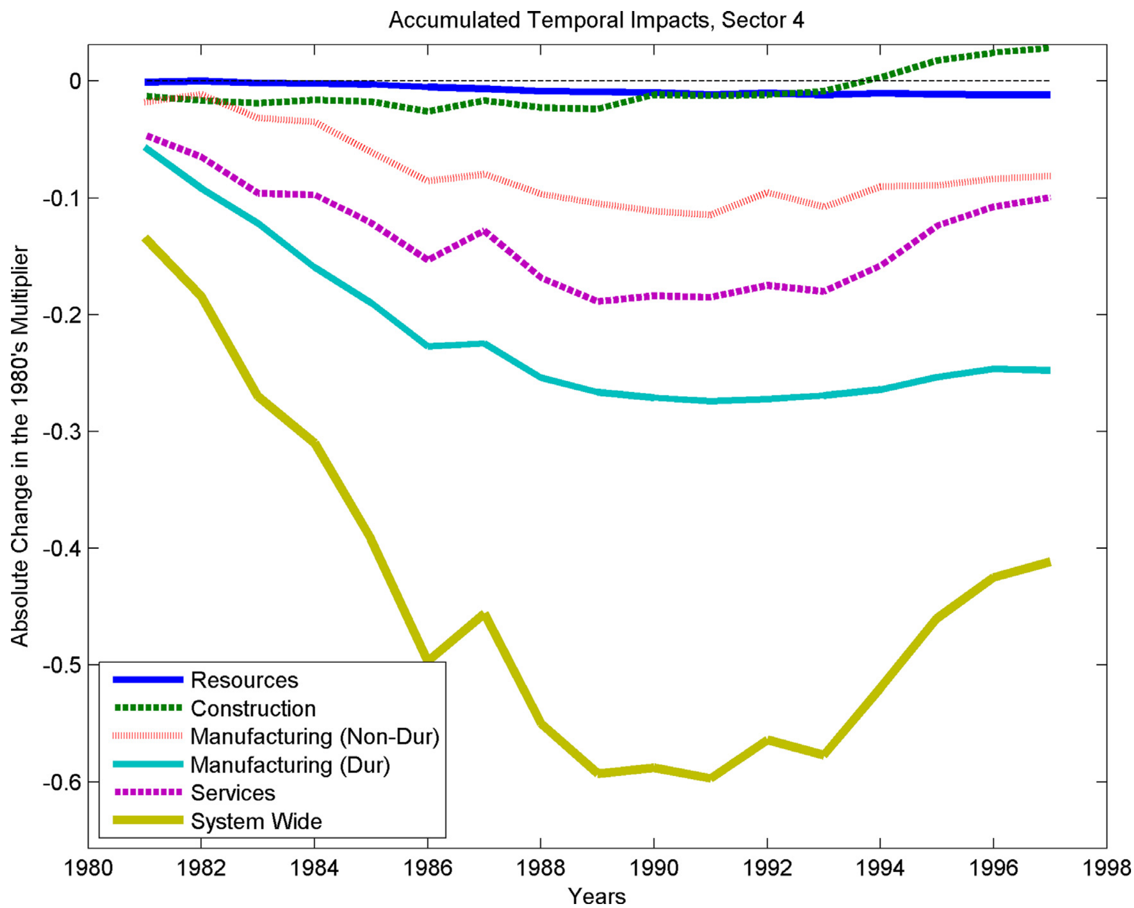

Figure 4. Trends in accumulated temporal impacts of the demand increase in Sector 4

ery and Equipment) and Sector 4 (Construction). Using the same dataset as the authors, we first reproduce the results from the paper and then apply our linear decomposition to uncover the underlying drivers of such phenomenon.[15]

The system-wide multiplier of sector 19 plunged 81% from the 1980s, in a steady decline until 1993 (-82%) and then negligible recovery until 1997 (Figure 3). The clusters of sectors that contributed the most for this pattern were Manufacturing of Durable Goods (to which sector 19 belongs) and Services; together, they were responsible for more than half of the decline. This result shows that linkages between sector 19 and these other sectors consistently weakened during the period. Since output of this sector grew (see Figure 1 in Okuyama *et al.* (2006)), the authors conclude that there is evidence of hollowing-out.

As shown in Figure 4, sector 4 also reduced its local dependency (-41.2% from 1980). A constant decline is observed throughout the 1980s (when it reaches -59.7%) with a stabilization during the early 1990s and later rebound (mainly due to Services and Construction). In fact, dependency on the own sector increased 2.9% from 1980. Considering this overall trend, the authors also assert the existence of a hollowing-out process in the sector (Okuyama *et al.*, 2006).

Okuyama *et al.* (2006) devise a typology of sectors depending on the shape of their temporal trends. A full explanation is provided in their paper and is outside of the scope of this analysis. Here, we focus on sector 28 (Finance and Insurance) that had the most extreme case of a tilted J-shape curve among all sectors (Figure 5). Despite a decreasing trend in the 1980s, the sector exhibits a strong upward trend in the 1990s with multipliers growing 39% from 1980. All sectoral clusters, with the exception of Resources and Manufacturing of Durables Goods, positively contributed to such surge, although since the early 1990s there is a generalized upward trend in linkages (except resources).

However, this previous work does not address the main forces driving such trends (own effects, substitution effects, interrelational effects, etc.) and whether they actually reflect a hollowing-out process or just indirect changes in the rest of the economy. Is the own sector changing input requirements and becoming less locally connected, or is the rest of the economy depending less from this sector or is it a combination of both? Are the aggregate trends masking interrelational effects? The original TLI does not produce any insights on these issues as it only captures the effect of the whole economy changing at once. Hence, we apply our linear decomposition to understand the sources of these temporal changes.

Following the partitions suggested in section 3, we separate (1) changes in local input requirement of the sector under analysis, so we can isolate "own effects" (O); (2) changes in local demand for the sector under analysis, isolating "substitution effects" (S); (3) changes in the rest of the matrix, isolating "interrelational effects" (I). The left panel of the following Figures 6-8 is the same as Figures 3-5, while the other three panels isolate the aforementioned partitions.

For sector 19, the aggregate declining trend observed from the basic TLI is consistent throughout the three-fold decomposition (Figure 6). Most of the plunge in the multiplier of sector 19 is due to own sector changes, i.e., the Industrial Machinery and Equipment sector is sourcing less inputs locally and this input requirement change by itself is responsible for around 70% of the over-

[15] The code and data for this decomposition are available upon request.

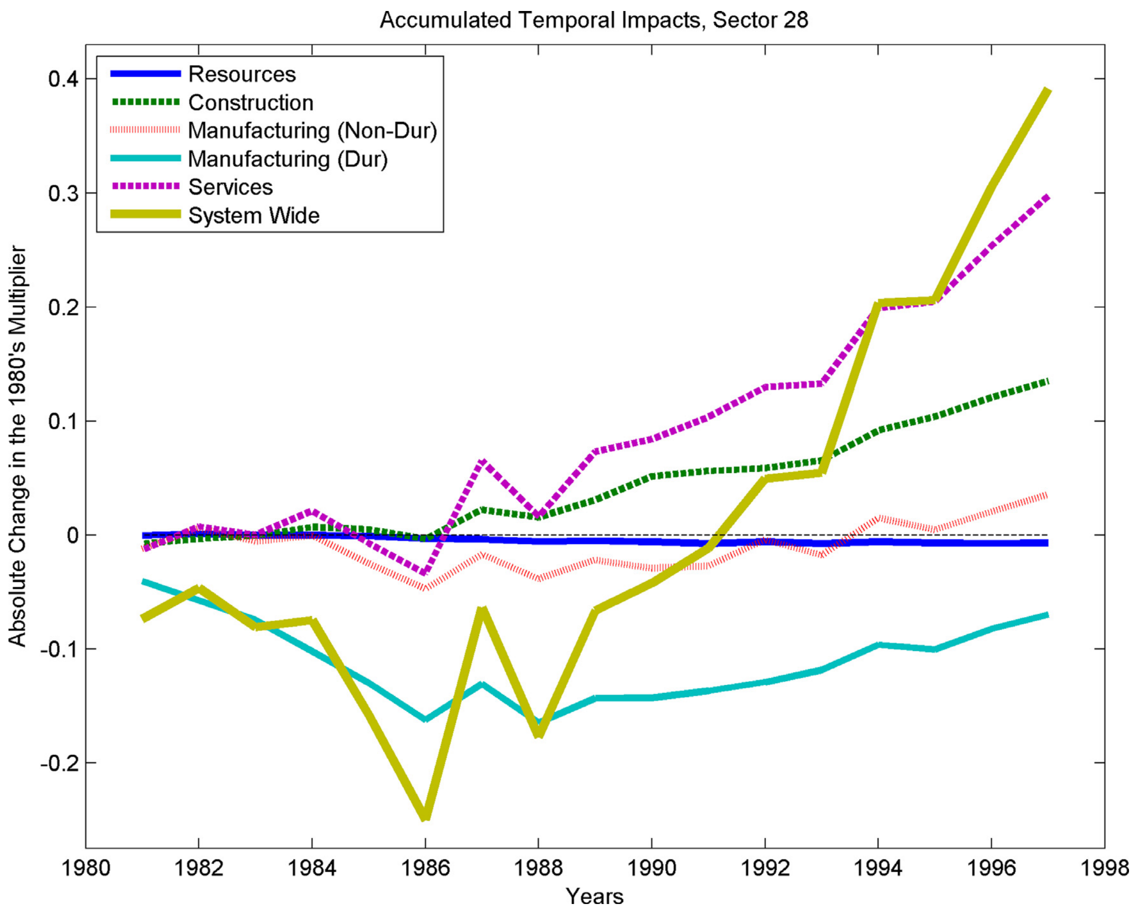

Figure 5. Trends in accumulated temporal impacts of the demand increase in Sector 28

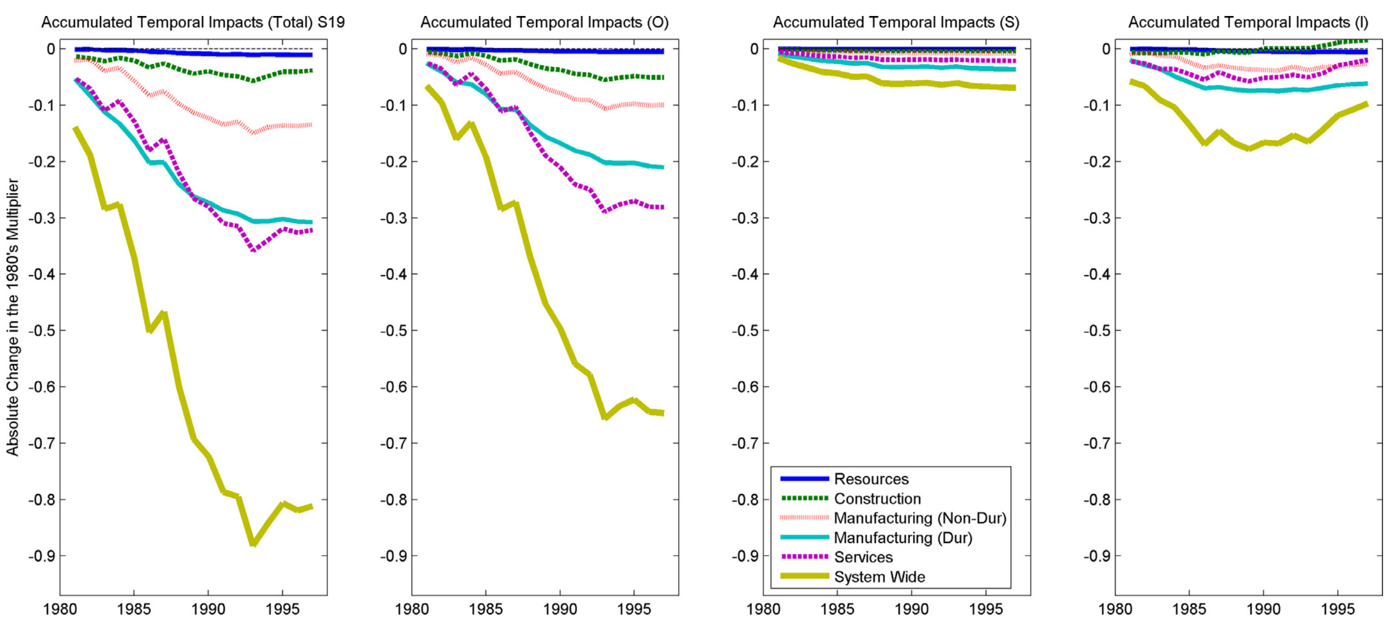

Figure 6. Decomposed evolutionary path for Sector 19

all decline. The other sectors in the economy are also purchasing less from sector 19, although this substitution effect alone accounts for only 8%. Finally, the last component of the multiplier reduction is the indirect impact of changes in the rest of the economy. It means that even if sector 19 had not changed technology and all other sectors had not change the proportion of inputs they purchase from this sector, the multiplier in 1997 would be 16.6% less than in 1980. Hence, the indirect change in the economic structure of Chicago is weakening the fields of influence in sector 19. Notice that the indirect effects rebound in the late 1990s. In sum, the negative behavior of own and substitution effects suggests a signif-

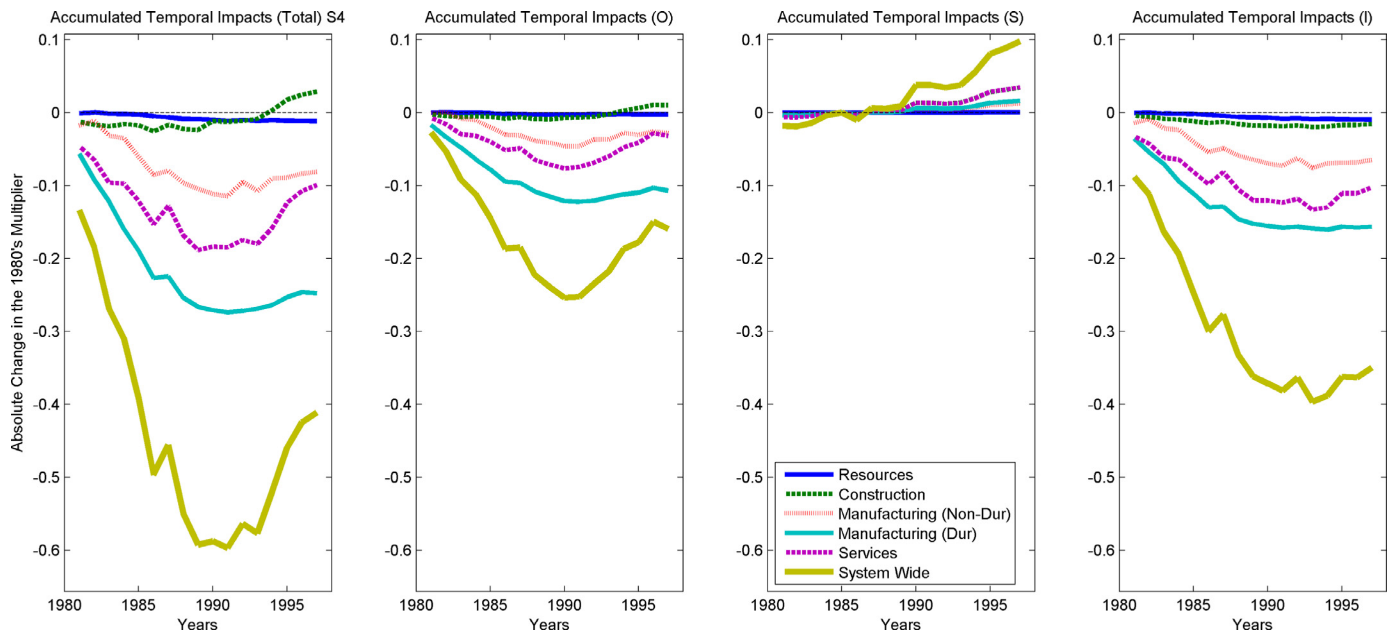

Figure 7. Decomposed evolutionary path for Sector 4

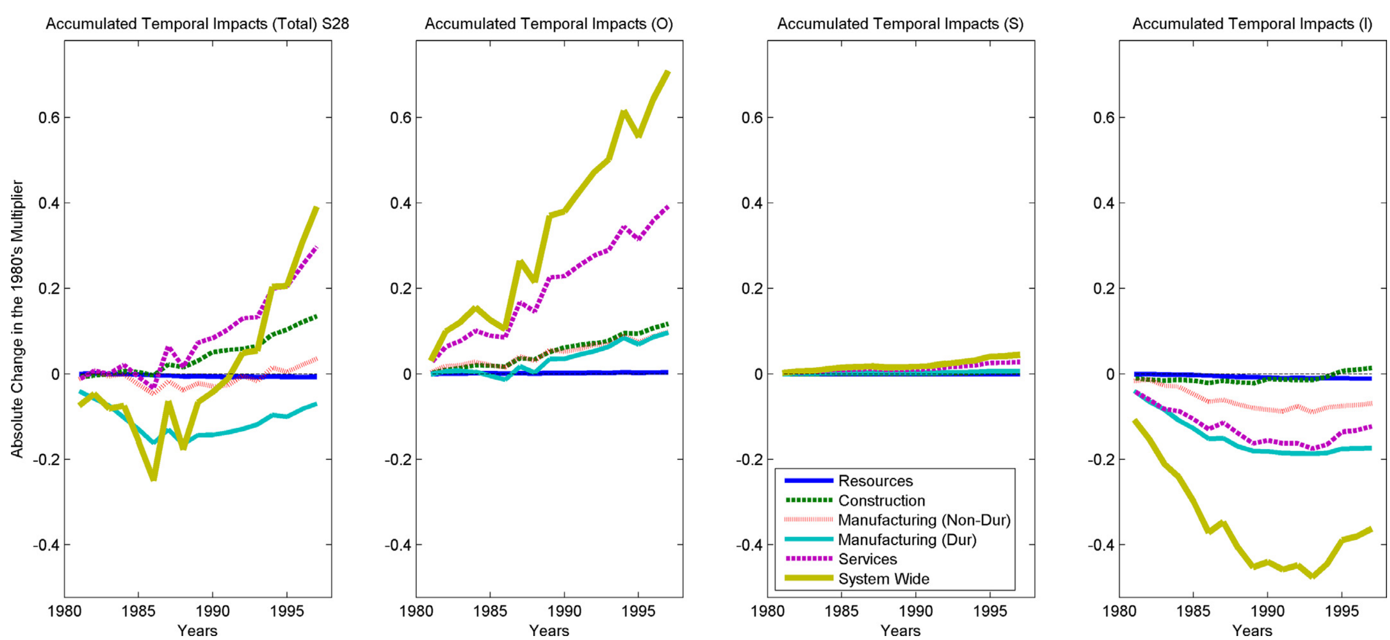

Figure 8. Decomposed evolutionary path for Sector 28

icant hollowing-out process in the sector, confirming the previous conclusion.

The U-shaped curves in Figure 4 (and first panel of Figure 7) are also observed for own effects (second panel) and interrelational effects (last panel) for sector 4. In contrast to the prior analysis of aggregated changes, there is a positive substitution effect, i.e., changes in the sales structure of the sector are increasing local linkages. If the sales structure was the only change observed each year, sector 4's multiplier in 1997 would have been 9.8% higher than in 1980. The negative impacts of own and indirect effects, however, were stronger than these gains, leading to the overall plunge in the multiplier. Interrelational effects contributed more to this result than own effects. Alone, the former would have decreased the multiplier in 35%, while the latter by only 16%. In this case, the weakening of ripple effects is greater than the sector's local input requirement changes. Therefore, evidence of hollowing-out is dubious for sector 4, a sharp contrast to conclusions of Okuyama *et al.* (2006). For this sector, not decomposing the basic TLI masks the main driver of the negative trend observed in Figure 4, interrelational effects, that absorb the positive substitution effect.

Finally, in the case of sector 28, the aggregated tilted J-curve is actually a composition of positive own effects and negative interrelational effects (Figure 8). This alludes to a hollowing-in process during the entire period. The sector itself is increasing local linkages[16] and, *ceteris paribus*, should have increased the 1980's mul-

[16] As explained by Romero *et al.* (2009), this increase in the local own linkages can be also explained by a functional fragmentation. This could imply that, during the period studied, the Finance and Insurance sector were progressively shifting

tiplier in 70.9% by 1997. Nonetheless, weakening ripple effects, especially in the 1980s decreased the overall multiplier until being compensated by own sector effects in the 1990s, creating the J-shaped pattern. Forward linkages positively influenced the multiplier throughout the period.

5. Conclusions

The ETLI framework proposed in this paper is aimed at analyzing the structural change in a time-series of IOTs when the focus in only on industrial linkages (i.e., abstracting from the contribution of final demand). It allows exploring how industrial networks evolve through time by isolating the contribution of changes in subsets of direct connections year-by-year to the most recent state of the economy. Besides identifying the intertemporal contribution of changes in the economy, this extended framework permits decomposing this contribution by driver in each year of the time-series.

We illustrated how this extension can reveal underlying drivers of change by revisiting some of the conclusions obtained in Okuyama *et al.* (2006) regarding hollowing-out effects in the Chicago region between 1980-1997. Using the original TLI methodology, the authors infer the existence of a hollowing-out process in sectors 19 (Industrial Machinery and Equipment) and 4 (Construction) by analyzing the basic TLI (overall linkages' evolution). We confirm the authors' conclusion of a hollowing-out process in sector 19 and show that the phenomenon is mainly driven by increasing outsourcing of inputs in that sector. When assessing sector 4, however, the hollowing-out process is less clear. The declining overall linkages observed in that paper are due primarily to interrelational effects and secondarily to outsourcing in the sector, while substitution effects *per se* would indicate a hollowing-in process. Hence, we see weaker signs of hollowing-out than argued by the authors. We also show a hollowing-in process for the entire period in sector 28 (Finance and Insurance) that was hidden underneath overall linkages that only become positive after 1990.

In sum, the ETLI framework provides an intertemporal view of the impacts of several drivers to the change in technical coefficients observed in a particular sector, extracting the most information from an IOT time-series. Besides the aforementioned applications, it can be used to explore other phenomena using different sets of partitions that reflect a particular model or *a priori* hypothesis, and potentially generate time-series variables for econometric forecasts of structural change.

In a time when production processes are very much fragmented, companies and government need a deeper understanding of the global value chain to anticipate ripple effects of disruptions in the supply chain as consequence of unexpected events, such as earthquakes, tsunamis, political upheavals, trade wars, man-made catastrophes or global pandemics. Such risks can result in material shortages and delivery delays that propagate downstream the supply chain and lead to logistics bottlenecks, rising prices and reductions in productivity. As a result of COVID-19 outbreak (Fortune, 2020), 94% of the Fortune 1000 companies have experienced disruption to their supply chains. In such a turbulent environment, new applications of the ETLI considering interregional and multinational interactions can provide insights into the evolution of value-added chains and the impact of unpredicted changes in trade patterns. This information can help countries and industries to get a better control over the entire supply chains and protect against disruptions at any stage throughout the system (e.g., accepting the temporal shortages; applying contingency plans; changing the operation policies) (Ivanov, 2020).

their focus to their core competences and subcontracting other parts of their value chain locally (increasing the degree of intermediation and their internal linkages with other sectors) to gain efficiency.

Uncited References:

(Casler and Hadlock, 1997)

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