State Estimation for Distribution Networks with Asynchronous Sensors using Stochastic Descent

Preprint

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National Renewable Energy Laboratory

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State Estimation for Distribution Networks with Asynchronous Sensors using Stochastic Descent

Bala Kameshwar Poolla, Guido Cavraro, Andrey Bernstein

Abstract—**This paper investigates the problem of state estimation for distribution networks with asynchronous sensors comprising of a mix of smart meters and phasor measurement units (PMUs) with multiple sampling and reporting rates. We consider two independent scenarios of state estimation and tracking, with either voltages or currents as states. With these two sets, we investigate estimation under (a) full data, assuming all measurements are available and (b) limited data, where an online algorithmic approach is adopted to estimate the possibly time-varying states by processing measurements as and when available. The proposed algorithm, inspired by the classical Stochastic Gradient Descent (SGD) approach updates the states based on the previous estimate and the newly available measurements. Finally, we demonstrate the estimation and tracking efficacy through numerical simulations on the IEEE-37 test network, while also highlighting how estimation with currents as states leads to faster convergence.**

Index Terms—**state estimation, asynchronous sensors, stochastic gradient descent, distribution networks, sensor networks.**

I. Introduction

In power systems, state estimation (SE) enables us to reconstruct electrical quantities from sparse data to maintain efficient and safe system operation [1], and its application in distribution grids has received a renewed interest recently. In [2], the authors propose a new technique for robust SE in the presence of topological errors. The article [3] investigates theoretical lower bounds on the distance between the true solution and the nearest spurious local minimum and shows that adding redundant information reduces the number of spurious minima. Classic state estimators for transmission systems are designed for the case in which the system operator has an overabundance of measurements [4]. Although distribution networks were historically under-metered [5], in recent years utilities have been installing massive numbers of metering devices, such as smart meters and phasor measurement units (PMUs). However, such devices are intrinsically heterogeneous and have different sampling rates. For instance, residential smart meters take measurements typically every 15 - 60 minutes, whereas PMUs can take 30 - 60 measurements every second. Moreover, sensors do not take and report measurements all at the same time to keep communication networks free from congestion [6], [7]. As a result, data available to utilities are *asynchronized* and not enough measurements are typically available to obtain a well-conditioned state estimation problem *at any given time*. Historical load data that utilities collect for billing purposes are used to obtain pseudo-measurements, that can be fed to classic least-squares estimators, or to build load probabilistic models. When probability distributions of load demand are available, the Bayesian linear state estimator in [8] or approaches based on Kalman filtering can be used [9], [10]. Other methods rely only on measurement data, e.g., the matrix completion approach proposed in [11]. By leveraging the communication, actuation, and sensing capabilities of smart inverters, authors in [12] probe the grid by varying the power injections at selected buses, record the incurred voltage responses, and infer the complex loads at non-actuated buses.

Here, we target SE for power distribution networks with incoming streams of asynchronous measurements as in [13] but considering the exact, non-linear power-flow model. We consider two possible definitions of state: node voltages and nodal currents. While classical analysis relies on voltage-based estimation, considering the nodal currents as the network state tremendously speeds up convergence. Further, we propose an algorithm built upon the classic SGD method to process the measurements available at each time and iteratively track the network state. The main feature of the algorithm is that it provides meaningful estimates even when only a few measurements are available at each time. The proposed approach is well-suited for new-age distribution systems because of the following reasons: (a) the measurements from PMUs, DERs, and smart meters [14], [15] are usually asynchronous with considerable differences in measurement time [16], [6]; (b) the available measurements at each time can be modeled as time-varying functions of the state; (c) the integration of renewables and electric vehicles results in a highly variable loading condition [17] and makes it harder to obtain pseudo-measurements [18].

The paper is structured as follows. The power network modeling and non-linear mathematical expressions for measurements in terms of currents and voltages as states are presented in Section II. The non-linear state estimation problem formulation is introduced in Section III along with the approach adopted when there is limited availability of measurement data. The nonlinear state estimation problem and numerical validation for the IEEE-37 distribution test network are presented in Section IV, while Section V concludes the paper with some key insights and future directions of research.

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Notation: The lowercase (respectively, uppercase) boldface letters denote column vectors (respectively, matrices). The use of calligraphic symbols are reserved for sets. The symbol $\top$ is used to denote transposition. We use $\mathbb{0}_n$ and $\mathbb{1}_n$ to represent the vector of zeros and ones, while $\mathbf{e_n}$ is the n-th canonical vector. The symbols $\|\mathbf{x}\|$ and $\|\mathbf{X}\|$ denote the 2-norm of the vector $\mathbf{x}$ and of the matrix $\mathbf{X}$, respectively; $\|\mathbf{x}\|_{\mathbf{Q}} = \mathbf{x}^\top \mathbf{Q}\, \mathbf{x}$ for a positive definite matrix $\mathbf{Q}$. The diagonal matrix having the elements of the finite set $\{\mathbf{x}_\ell\} = \{x_1, x_2, \ldots\}$ on its diagonal is denoted by $\mathrm{diag}(\mathbf{x}_\ell)$. The expectation operator is defined by $\mathbb{E}[\cdot]$, and ∇ is used to denote the gradient.

II. Distribution Grid Modeling

An electric power network can be modeled by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the set $\mathcal{V} = \{0, \ldots, N\}$ collects the electrical buses, and the set of edges $\mathcal{E}$, with $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, captures the electrical lines. The substation, modeled as a *slack bus*, is denoted as node 0. Let the vectors collecting nodal currents and voltages be $\mathbf{i} = \{\mathrm{i}_1, \ldots, \mathrm{i}_\ell\}$ and $\mathbf{v} = \{\mathrm{v}_1, \ldots, \mathrm{v}_\ell\}$. Similarly, let the vectors of apparent power injections, active power injections, and reactive power injections at each node be denoted by $\mathbf{s}$, $\mathbf{p}$, and $\mathbf{q}$ respectively. The nodal complex current injections and voltages are related as

$$\begin{bmatrix} \mathrm{i}_0 \\ \mathbf{i} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{Y}_{00} & \mathbf{y_0}^\top \\ \mathbf{y_0} & \mathrm{Y} \end{bmatrix}}_{\mathrm{Y}_{\text{net}}} \begin{bmatrix} \mathrm{v}_0 \\ \mathbf{v} \end{bmatrix}, \tag{1}$$

where Y_{net} is the net system admittance matrix which has been properly partitioned to highlight the components associated with the slack bus. In the following, without loss of generality, we assume the slack bus voltage to be $\mathrm{v}_0 = 1\angle 0$. From equation (1), voltages can then be written as a function of currents as

$$\mathbf{v} = \mathbb{1}_n + \mathbf{X}\,\mathbf{i}, \tag{2}$$

where $\mathbf{X} := \mathbf{Y}^{-1}$. Furthermore, powers, currents, and voltages are related as

$$\mathbf{s} = \mathbf{p} + \mathrm{j}\mathbf{q} = \mathrm{diag}\,(\mathbf{v})\,\mathbf{i}^\star, \tag{3}$$

where $\star$ is the complex conjugate operator. We denote the real and imaginary parts of matrices $\mathbf{X}$ and $\mathbf{Y}$ as $\mathbf{X_R} := \Re(\mathbf{X})$, $\mathbf{X_I} := \Im(\mathbf{X})$, $\mathbf{Y_R} := \Re(\mathbf{Y})$, $\mathbf{Y_I} := \Im(\mathbf{Y})$, $\mathbf{y_{0R}} := \Re(\mathbf{y_0})$, and $\mathbf{y_{0I}} := \Im(\mathbf{y_0})$, respectively. Further, let $\mathcal{S}$ define the set of buses endowed with a measurement device. We consider two kinds of sensors: smart meters– able to measure power injections and voltage magnitudes; and phasor measurement units (PMUs)– able to measure additionally, complex voltages and complex currents [14], [15]. As we will show in the following, all the aforesaid measurable quantities can be expressed as quadratic functions of either the nodal voltages or the nodal currents. In particular, denote the m-th measurand of the sensor placed at the metered bus n[1], $n \in \mathcal{S}$, as $\tilde{y}_{m,n}$; it can be expressed in the quadratic form

$$\widetilde{y}_{m,n}(t) = \mathbf{x}(t)^\top \mathbf{A_{m,n}}(t)\, \mathbf{x}(t) + \mathbf{x}(t)^\top \mathbf{b_{m,n}}(t) + c_{m,n}(t), \tag{4}$$

where the vector $\mathbf{x}(t)$ is the state of the system (either voltages or currents as explained below); $\mathbf{A_{m,n}}(t)$, $\mathbf{b_{m,n}}(t)$, $c_{m,n}(t)$ are possibly time-varying system matrices, vectors, scalars, respectively. Clearly, the expression for the entities appearing in (4) depends on what is assumed to be the state of the network. Next, we show the form that (4) takes when the state consists of the nodal currents and the nodal voltages.

A. Currents as states

Let the vector $\mathbf{x} = \begin{bmatrix} \mathbf{i}_r^\top & \mathbf{i}_i^\top \end{bmatrix}^\top$ collect the real ($\mathbf{i}_r = \Re(\mathbf{i})$) and imaginary components ($\mathbf{i}_i = \Im(\mathbf{i})$) of the nodal current injections $\mathbf{i}$. Then, it can be shown that the sensor measurements can be expressed with the quadratic form (4). In particular, after defining $\mathbf{x_{Rn}} := \mathbf{X_R}\,\mathbf{e_n}$, $\mathbf{x_{In}} := \mathbf{X_I}\,\mathbf{e_n}$, we derive the following expressions for the:

(1) squared voltage magnitude $|\mathrm{v}_n|^2$

$$\mathbf{A_{1,n}} = \begin{bmatrix} \mathbf{x_{Rn}}\,\mathbf{x_{Rn}}^\top + \mathbf{x_{In}}\,\mathbf{x_{In}}^\top & \mathbf{x_{In}}\,\mathbf{x_{Rn}}^\top - \mathbf{x_{Rn}}\,\mathbf{x_{In}}^\top \\ \mathbf{x_{Rn}}\,\mathbf{x_{In}}^\top - \mathbf{x_{In}}\,\mathbf{x_{Rn}}^\top & \mathbf{x_{Rn}}\,\mathbf{x_{Rn}}^\top + \mathbf{x_{In}}\,\mathbf{x_{In}}^\top \end{bmatrix},$$

$\mathbf{b_{1,n}}^\top = \begin{bmatrix} 2\,\mathbf{x_{Rn}}^\top & -2\,\mathbf{x_{In}}^\top \end{bmatrix}$, and $c_{1,n} = 1$;

(2) active power injection p_n

$$\mathbf{A_{2,n}} = \frac{1}{2} \begin{bmatrix} 2\,\mathbf{e_n}\,\mathbf{x_{Rn}}^\top & \mathbf{x_{In}}\,\mathbf{e_n}^\top - \mathbf{e_n}\,\mathbf{x_{In}}^\top \\ \mathbf{e_n}\,\mathbf{x_{In}}^\top - \mathbf{x_{In}}^\top\,\mathbf{e_n}^\top & 2\,\mathbf{e_n}\,\mathbf{x_{Rn}}^\top \end{bmatrix},$$

$\mathbf{b_{2,n}}^\top = \begin{bmatrix} \mathbf{e_n}^\top & \mathbb{0}_\mathbf{n}^\top \end{bmatrix}$, and $c_{2,n} = 0$;

(3) reactive power injection q_n

$$\mathbf{A_{3,n}} = \frac{1}{2} \begin{bmatrix} 2\,\mathbf{e_n}\,\mathbf{x_{In}}^\top & \mathbf{x_{Rn}}\,\mathbf{e_n}^\top - \mathbf{e_n}\,\mathbf{x_{Rn}}^\top \\ \mathbf{e_n}\,\mathbf{x_{Rn}}^\top - \mathbf{x_{Rn}}^\top\,\mathbf{e_n}^\top & 2\,\mathbf{e_n}\,\mathbf{x_{In}}^\top \end{bmatrix},$$

$\mathbf{b_{3,n}}^\top = \begin{bmatrix} \mathbb{0}_\mathbf{n}^\top & \mathbf{e_n}^\top \end{bmatrix}$, and $c_{3,n} = 0$;

(4) complex current real part $(\mathbf{i_r})_n$

$$\mathbf{A_{4,n}} = \mathbb{0},\ \mathbf{b_{4,n}}^\top = \begin{bmatrix} \mathbf{e_n}^\top & \mathbb{0}_\mathbf{n}^\top \end{bmatrix},\ \text{and } c_{4,n} = 0;$$

(5) complex current imaginary part $(\mathbf{i_i})_n$

$$\mathbf{A_{5,n}} = \mathbb{0},\ \mathbf{b_{5,n}}^\top = \begin{bmatrix} \mathbb{0}_\mathbf{n}^\top & \mathbf{e_n}^\top \end{bmatrix},\ \text{and } c_{5,n} = 0;$$

(6) complex voltage real part $(\mathbf{v_r})_n$

$$\mathbf{A_{6,n}} = \mathbb{0},\ \mathbf{b_{6,n}}^\top = \begin{bmatrix} \mathbf{x_{Rn}}^\top & -\mathbf{x_{In}}^\top \end{bmatrix},\ \text{and } c_{6,n} = 1;$$

(7) complex voltage imaginary part $(\mathbf{v_i})_n$

$$\mathbf{A_{7,n}} = \mathbb{0},\ \mathbf{b_{7,n}}^\top = \begin{bmatrix} \mathbf{x_{In}}^\top & \mathbf{x_{Rn}}^\top \end{bmatrix},\ \text{and } c_{7,n} = 0.$$

B. Voltages as states

Let the vector $\mathbf{x} = \begin{bmatrix} \mathbf{v}_r^\top & \mathbf{v}_i^\top \end{bmatrix}^\top$ collect the real ($\mathbf{v}_r = \Re(\mathbf{v})$) and imaginary components ($\mathbf{v}_i = \Im(\mathbf{v})$) of the nodal voltage injections $\mathbf{v}$. Similar to the previous subsection, the sensor measurements can be expressed with the quadratic form (4). More precisely, we have the following expression for the:

(1) squared voltage magnitude $|\mathrm{v}_n|^2$

$$\mathbf{A_{1,n}} = \begin{bmatrix} \mathbf{e_n}\,\mathbf{e_n}^\top & \mathbf{0} \\ \mathbf{0} & \mathbf{e_n}\,\mathbf{e_n}^\top \end{bmatrix},$$

$\mathbf{b_{1,n}}^\top = \begin{bmatrix} \mathbb{0}_\mathbf{n}^\top & \mathbb{0}_\mathbf{n}^\top \end{bmatrix}$, and $c_{1,n} = 0$;

(2) active power injection p_n

$$\mathbf{A_{2,n}} = \begin{bmatrix} \mathbf{Y_R}\,\mathbf{e_n}\,\mathbf{e_n}^\top & \mathbf{Y_I}\,\mathbf{e_n}\,\mathbf{e_n}^\top \\ -\mathbf{Y_I}\,\mathbf{e_n}\,\mathbf{e_n}^\top & \mathbf{Y_R}\,\mathbf{e_n}\,\mathbf{e_n}^\top \end{bmatrix},$$

$\mathbf{b_{2,n}}^\top = \begin{bmatrix} \mathbf{e_n}\,\mathbf{e_n}^\top\,\mathbf{y_{0R}} & \mathbf{e_n}\,\mathbf{e_n}^\top\,\mathbf{y_{0I}} \end{bmatrix}$, and $c_{2,n} = 0$;

[1]We assume each reporting node has either a PMU or a smart meter.

(3) reactive power injection q_n

$$\mathbf{A_{3,n}} = \begin{bmatrix} -\mathbf{Y_I}\,\mathbf{e_n}\,\mathbf{e_n}^\top & \mathbf{Y_R}\,\mathbf{e_n}\,\mathbf{e_n}^\top \\ -\mathbf{Y_R}\,\mathbf{e_n}\,\mathbf{e_n}^\top & -\mathbf{Y_I}\,\mathbf{e_n}\,\mathbf{e_n}^\top \end{bmatrix},$$

$\mathbf{b_{3,n}}^\top = \begin{bmatrix} -\mathbf{e_n}\,\mathbf{e_n}^\top\,\mathbf{y_{0I}} & \mathbf{e_n}\,\mathbf{e_n}^\top\,\mathbf{y_{0R}} \end{bmatrix}$, and $c_{3,n} = 0$;

(4) complex current real part $(\mathbf{i_r})_n$

$\mathbf{A_{4,n}} = \mathbb{0}$, $\mathbf{b_{4,n}}^\top = \begin{bmatrix} \mathbf{e_n}^\top\mathbf{Y_R} & -\mathbf{e_n}^\top\mathbf{Y_I} \end{bmatrix}$, and $c_{4,n} = 0$;

(5) complex current imaginary part $(\mathbf{i_i})_n$

$\mathbf{A_{5,n}} = \mathbb{0}$, $\mathbf{b_{5,n}}^\top = \begin{bmatrix} \mathbf{e_n}^\top\mathbf{Y_I} & \mathbf{e_n}^\top\mathbf{Y_R} \end{bmatrix}$, and $c_{5,n} = 0$;

(6) complex voltage real part $(\mathbf{v_r})_n$

$\mathbf{A_{6,n}} = \mathbb{0}$, $\mathbf{b_{6,n}}^\top = \begin{bmatrix} \mathbf{e_n}^\top & \mathbb{0}_\mathbf{n}^\top \end{bmatrix}$, and $c_{6,n} = 0$;

(7) complex voltage imaginary part $(\mathbf{v_i})_n$

$\mathbf{A_{7,n}} = \mathbb{0}$, $\mathbf{b_{7,n}}^\top = \begin{bmatrix} \mathbb{0}_\mathbf{n}^\top & \mathbf{e_n}^\top \end{bmatrix}$, and $c_{7,n} = 0$;

Although the use of nodal voltages as network states is a fairly standard practice, we advocate the use of nodal currents due to the enhanced speed of convergence and illustrate this through simulations.

III. The State Estimator

In this section, we introduce the state estimator for distribution networks with asynchronous sensors. We recall that the key objective of the estimation problem is to effectively track the system states, namely, currents or voltages, using smart meter data (voltage magnitude, active and reactive power) and PMU measurements (real and imaginary currents, voltages). As we observed in the previous section, the sensor data can be expressed as quadratic functions of either currents or voltages as state variables. Let the m-th measurement taken at node $n \in \mathcal{S}$ be denoted as

$$y_{m,n}(t) = \widetilde{y}_{m,n}(t) + \eta_{m,n}(t),$$

with $\eta_{m,n}(t)$ as the measurement noise. Let the vector $\mathbf{y} = [y_{1,1}^\top \ldots y_{m,n}^\top]^\top$ collect the set of measurements from the smart meters and the PMUs for all nodes in the network. Then the standard weighted least squares approach to estimate the system states $\mathbf{x}$ is given by the optimization problem

$$\min_{\mathbf{x}} f(\mathbf{x}) = \min_{\mathbf{x}} \sum_n \sum_m w_m^2\, \|f_{m,n}(\mathbf{x})\|^2, \tag{5}$$

where $f_{m,n}(\mathbf{x}) = y_{m,n} - (\mathbf{x}^\top\mathbf{A_{m,n}}\mathbf{x} + \mathbf{x}^\top\mathbf{b_{m,n}} + c_{m,n})$, the matrices/vectors $\mathbf{A_{m,n}}$, $\mathbf{b_{m,n}}$, and $c_{m,n}$ are the terms from the quadratic expressions of the measurements presented in the previous section, and w_m is the weight associated with each measurement. To solve problem (5) at time t, several solutions have been developed in the literature, e.g., gradient-based or Newton-Raphson techniques [19]. Typically, the system operator needs to compute the gradient

$$\nabla f(\mathbf{x}(t)) = \sum_{n\in\mathcal{S}(t)} \sum_m -2w_m^2\, f_{m,n}(\mathbf{x}(t))(2\mathbf{A_{m,n}}\mathbf{x}(t) + \mathbf{b_{m,n}}).$$

However, in a setup where the sensors report at different rates and the network state changes rapidly, there is not enough information to obtain $\nabla f(\mathbf{x}(t))$. Thus, we are unable to compute the closed-form gradient and apply off-the-shelf gradient-based algorithms.

Let $\mathcal{S}(t)$ be the set of sensors reporting at time t and φ_n be the reporting frequency of sensor n, i.e., the number of times it sends measurements to the system operator each hour. Without loss of generality, we assume that sensor n can report at most once every minute, or mathematically, that $\varphi_n \le 60$. Moreover, let $\pi_n = \varphi_n/60$, where we interpret π_n as the probability that sensor n reports measurements to the system operator. We introduce the vector

$$\nabla \hat{f}(\mathbf{x}(t)) = \sum_{n\in\mathcal{S}(t)} \frac{60}{\varphi_n}\Big(\sum_m -2w_m^2\, f_{m,n}(\mathbf{x}(t))(2\mathbf{A_{m,n}}\,\mathbf{x}(t) + \mathbf{b_{m,n}})\Big),$$

which is an approximation of the gradient $\nabla f(\mathbf{x}(t))$, built using only data (and weighted to account for the probability of availability of measurements) coming from reporting sensors.

Let the state estimate at time t be denoted as $\hat{\mathbf{x}}(t)$. We propose to update the state estimate every time a new set of measurements become available with the following rule

$$\hat{\mathbf{x}}(t+1) = \hat{\mathbf{x}}(t) - \alpha\nabla\hat{f}(\mathbf{x}(t)). \tag{6}$$

where α is a suitable positive constant. The update rule (6) can be interpreted as a *stochastic gradient descent* of the cost in (5). Indeed, resorting to the probabilistic interpretation of the parameters π_ns, it is easy to note that

$$\mathbb{E}[\nabla\hat{f}(\mathbf{x}(t))] = \sum_{n\in\mathcal{S}(t)} \frac{60\,\pi_n}{\varphi_n}\Big(\sum_m -2w_m^2\, f_{m,n}(\mathbf{x}(t))(2\mathbf{A_{m,n}}\,\mathbf{x}(t) + \mathbf{b_{m,n}})\Big), = \nabla f(\mathbf{x}(t)).$$

In case the state of the network is fixed, i.e., $\mathbf{x}(t) \equiv \bar{\mathbf{x}}$ for a certain $\bar{\mathbf{x}}$, algorithm (6) inherits the convergence properties of the stochastic gradient descent [20]: it does not necessarily converge to the true optimal solution and, under certain conditions on the parameters, only convergence to a norm-ball around the true optimizer can be guaranteed. However, for time-varying system states, a theoretical stability characterization of the algorithm is still a challenging problem and has been numerically shown in the next section.

IV. Simulation Results

We consider the IEEE-37 bus distribution test feeder shown in Figure 1 for the dynamic SE problem discussed in the previous sections. The buses in the network host two types of measurement devices: (a) smart meters measuring active, reactive power, voltage magnitude, and (b) Phasor Measurement Units (PMUs) measuring currents, voltage magnitude, and voltage angle.

A. A gradient approach for traditional state estimation.

Here, we considered a classic SE problem in which the information/data available is sufficient to build an estimate of the system state, or in other words, the distribution grid is observable. More precisely, we assume that measurements from all the sensors are available and hence, problem (5) admits a local solution. After initializing the optimization from a flat voltage profile, a local minimum is found through the standard gradient descent scheme. Notice that here we are considering a traditional SE framework in which the goal is to numerically solve the problem defined in (5) rather

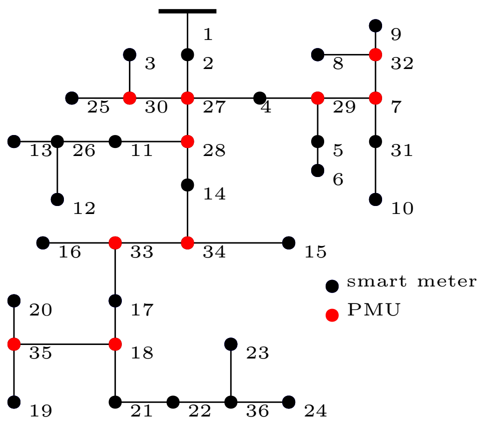

Fig. 1. IEEE-37 network indicating the location of smart meters and PMUs.

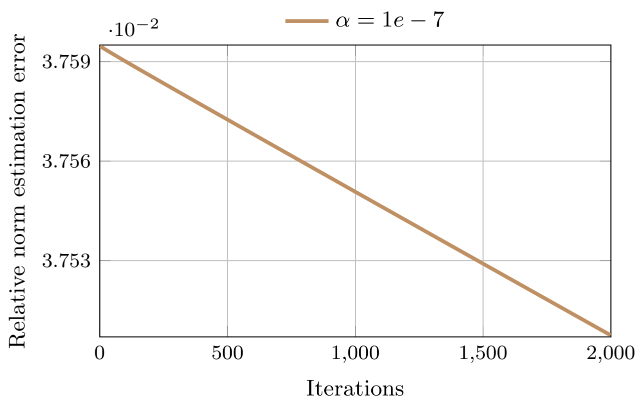

Fig. 2. Relative norm of the estimation error for voltage tracking when initialized around the flat voltage profile.

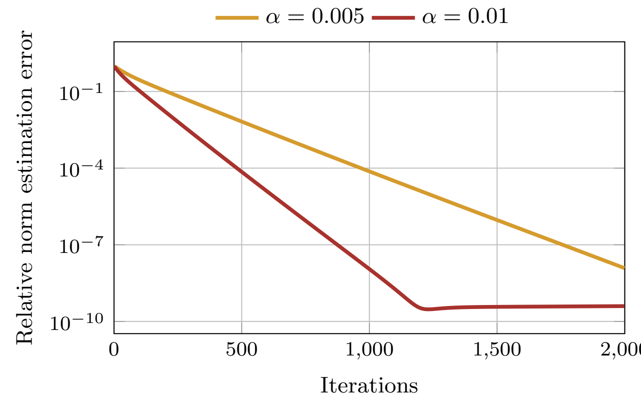

Fig. 3. Relative norm of the estimation error (log scale) for current tracking when initialized around the flat voltage profile.

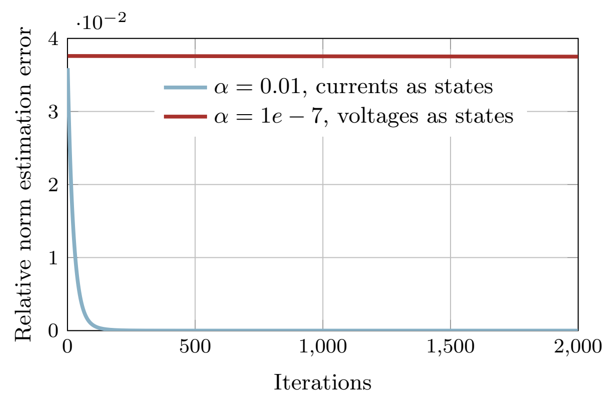

Fig. 4. Relative norm of the estimation error for voltages, when using voltages/currents as states and initialized around the flat voltage profile.

than tracking the state over time. Even though traditional approaches for solving problem (5) rely on more efficient algorithms, e.g., Newton-Raphson techniques, we believe it is interesting to report the convergence rate of the gradient scheme. When voltages are considered as the network states, Figure 2 illustrates the relative norm estimation error $\|\mathbf{v}-\mathbf{v}^{\star}\|/\|\mathbf{v}^{\star}\|$, as a function of number of iterations. (Note that the step-size α is numerically optimized for superior convergence). On the other hand, Figure 3 demonstrates the case with the nodal currents as the network states and plots the relative norm estimation error $\|\mathbf{i}-\mathbf{i}^{\star}\|/\|\mathbf{i}^{\star}\|$. Notably, the convergence to the solution of (5) is in general much faster in the second case. Finally, in Figure 4, the relative norm estimation error for voltages is plotted for the two cases, i.e., Figure 2 is overlaid with voltages derived from current profiles in Figure 3. This observation highlights that currents, rather than voltages, should be considered as the network states in the dynamic SE setting described in this paper.

B. Dynamic state estimation

Due to the faster convergence for estimation of currents as discussed above, for the scenario with limited measurement information at asynchronous rates, we investigate current tracking. To this end, we consider a varying power injection with updates every minute. To compute the statistics of this load variation (represented by σ_{P} and σ_{Q}), we used the 15 minute rolling-horizon data from [21] and obtained a relative standard deviation of 0.05%. The sensors reporting at different rates are affected by measurement noise such that, (a) smart meters provide measurements once every 15 minutes and introduce noise that is modeled as a zero-mean Gaussian random variable with a relative standard deviation $\sigma_{\text{SM}} = 0.5\%$, (b) PMUs provide measurements every minute and introduce noise that is modeled as a zero-mean Gaussian random variable with a relative standard deviation $\sigma_{\text{PMU}} = 0.05\%$, i.e., we have

$$\begin{aligned}
\mathbf{p}(t+1) &= (\mathbf{p}(t) + \sigma_{\text{P}} \cdot \mathbf{p}_{\text{nom}} \cdot \zeta)\,(1+\sigma_{\text{SM}} \cdot \lambda), \\
\mathbf{q}(t+1) &= (\mathbf{q}(t) + \sigma_{\text{Q}} \cdot \mathbf{q}_{\text{nom}} \cdot \zeta)\,(1+\sigma_{\text{SM}} \cdot \lambda),
\end{aligned}$$

where ζ, λ are random scalars drawn from normal distribution and $\mathbf{p}_{\text{nom}}$, $\mathbf{q}_{\text{nom}}$ are the nominal active, reactive power vectors.

We used the Stochastic Gradient Descent approach previously described to for estimate the nodal currents in the network. In Figure 5, we consider a weighted estimation problem for varying α with a ratio $w_{\text{PMU}}\!:\!w_{\text{SM}}$=10 : 1[2] and with measurements corrupted by white noise as above. Furthermore, to account for the asynchronicity in reporting, the terms associated with the smart meters are scaled by a factor of 15 (as they provide scarce measurements, compared to the minute-scale resolution of PMUs). Recall from our previous discussion that the data is weighted with an inversely

[2] w_{SM} corresponds to m=1, 2, 3, and w_{PMU} to m=4, 5, 6, 7

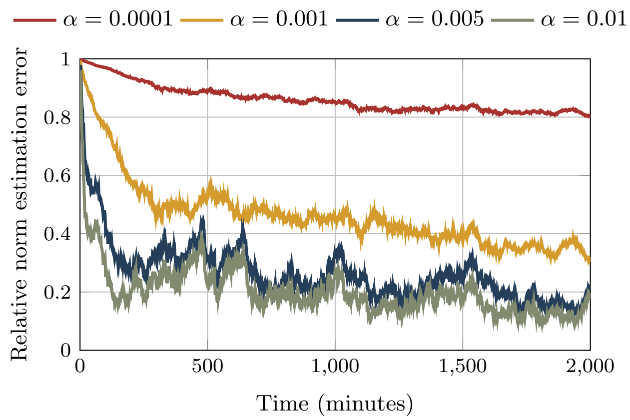

Fig. 5. Relative norm of the estimation error for current tracking with varying step size α when initialized around the flat voltage profile with a $\sigma_{\mathrm{P}} = \sigma_{\mathrm{Q}} = 0.2$ for load variation, $\sigma_{\mathrm{SM}} = 0.5\%$, $\sigma_{\mathrm{PMU}} = 0.05\%$ for measurement noise. We employ a weight of $w_{\mathrm{PMU}} : w_{\mathrm{SM}}$=$10 : 1$ for PMUs : smart meters.

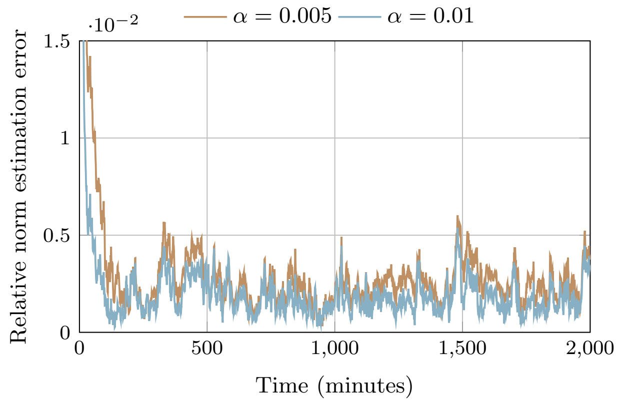

Fig. 6. Relative norm of the estimation error for voltages corresponding to the currents in Figure 5 for $\alpha = 0.005$, $\alpha = 0.01$.

proportional factor, $1/\pi_n$ to account for the probability that measurements at $n \in \mathcal{S}(t)$ are available at iteration t.

In Figure 6, we reconsider the weighed estimation problem for nodal currents as in Figure 5 and compute the nodal voltages in the network from the estimated currents using the power flow relation (2). Note that the relative error for voltages is of the order 10^{-2} even under load variation and measurement noise, thus highlighting the effective tracking performance of our proposed algorithm.

V. Conclusions

In this paper, we investigated a dynamic SE problem for systems with heterogeneous sensors and multiple sampling, reporting rates. We considered two scenarios: one in which full measurement data was available; one with partial data availability from sensors. The gradient of the SE problem's cost could be computed in the first scenario but not in the second, preventing the use of classical gradient descent algorithms. Hence, we proposed an SGD-like algorithm to estimate the system states. Furthermore, we observed that estimating currents (both real and imaginary) resulted in a faster convergence rate than with voltages as states. Finally, we demonstrated our algorithm on the IEEE-37 distribution test bed with real-world load-variation statistics data and white noise-corrupted measurements, thereby highlighting the efficacy of our non-linear estimation approach in terms of tracking performance and the relative norm estimation error. As future work, we plan to theoretically characterize the algorithmic stability for time-varying system states.

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