# A Medium-/Low-Voltage Joint State Estimator Through Linear Uncertainty Propagation

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## Introduction

As grid-edge technologies advance and become more affordable, distributed energy resources could better manage their power output or energy usage portfolios and proactively participate in system operation. Compared with passive distribution systems, where utility-owned control devices operate under timed set points to ensure system reliability and resilience, active distribution systems face an increasing level of variability and uncertainty in their voltage profiles, especially near the end users; therefore, situational awareness at the grid edge becomes critical. Fortunately, the existence of sensor devices installed in the low-voltage (LV) networks, e.g., cable television (CATV) voltage sensors [1], provides a promising solution to address this need. CATV voltage sensors transmit voltage magnitude measurements in real time (at a 5-min resolution) through the secure, high-bandwidth, low-latency CATV communications.

## Proposed Algorithm

Despite the extensive discussion on DSSE, MV and LV state estimations are investigated in the literature as separate topics. The interdependence between the primary and secondary distribution networks is generally disregarded. To address the growing need for visibility at the grid edge and to embrace the opportunities introduced by untapped CATV measurements, this study proposes an MV/LV joint state estimator.

### Framework of the joint MV/LV state estimation

The framework of the joint MV/LV state estimation arises naturally from the hierarchy of distribution networks. As shown in Fig.1, we divide the distribution system into a primary MV subnetwork and multiple secondary LV subnetworks following the border-bus overlapping partition approach. The MV subnetwork overlaps with the LV subnetworks with shared secondary feeder heads (i.e., primary side of the secondary transformer). Given the two-level hierarchical nature of the problem, we propose solving the joint state estimation in an iterative manner where primary and secondary estimators alternatively solve their state estimation problems (1.a) and (1.b) during the iterations while exchanging the estimated boundary conditions, $b_i^{p-s}$ and $b_i^{s-p}$, calculated based on (2.a) and (2.b), respectively:

$$\boldsymbol{z^p} = h^p(\boldsymbol{x^p}) + \boldsymbol{e^p} \quad (1.a)$$

$$\boldsymbol{z_i^s} = h_i^s(\boldsymbol{x_i^s}) + \boldsymbol{e_i^s} \quad (1.b)$$

$$\boldsymbol{b_i^{p-s}} = h_i^{p-s}(\boldsymbol{x^p}) \quad (2.a)$$

$$\boldsymbol{b_i^{s-p}} = h_i^{s-p}(\boldsymbol{x_i^s}) \quad (2.b)$$

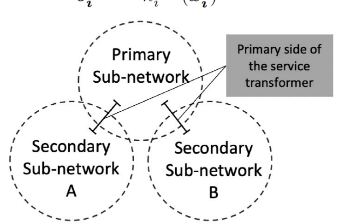


**Fig. 1**: Partition of the distribution network.

### Linear measurement function

To ensure the computational efficiency of the joint state estimator, linearized primary and secondary measurement matrices are derived based on reference [2], as given in (3.a):

$$\begin{bmatrix} P'^* \\ Q'^* \\ |V|'^* \end{bmatrix} = H^* \begin{bmatrix} \Re\{V^*\} \\ \Im\{V^*\} \end{bmatrix} = \begin{bmatrix} H^*_{inj} \\ H^*_{vmag} \end{bmatrix} \begin{bmatrix} \Re\{V^*\} \\ \Im\{V^*\} \end{bmatrix} \quad (3.a)$$

Where:

$$H^*_{inj} = \begin{bmatrix} \Re\{M^*\} & \Im\{M^*\} \\ \Im\{M^*\} & -\Re\{M^*\} \end{bmatrix}^{-1} \quad (3.b)$$

$$H^*_{vmag} = \begin{bmatrix} K_1^* & K_2^* \end{bmatrix} H^*_{inj} \quad (3.c)$$

$$|V|'^* = |V|^* + \begin{bmatrix} K_1^* & K_2^* \end{bmatrix} H^*_{inj} \begin{bmatrix} \Re\{m^*\} \\ \Im\{m^*\} \end{bmatrix} - k^* \quad (3.d)$$

$$\begin{bmatrix} P'^* \\ Q'^* \end{bmatrix} = \begin{bmatrix} P^* \\ Q^* \end{bmatrix} + H^*_{inj} \begin{bmatrix} \Re\{m^*\} \\ \Im\{m^*\} \end{bmatrix} \quad (3.e)$$

$$w^* = -(Y^*_{LL})^{-1} Y^*_{L0} V^*_0 \quad (3.f)$$

$$W^* = diag(w^*) \quad (3.g)$$

$$M^* = (Y^*_{LL})^{-1} diag(\overline{w}^*)^{-1} \quad (3.h)$$

$$m^* = w^* \quad (3.i)$$

$$G^* = V_0^* \overline{Y_{0L}}^* \overline{M}^* \quad (3.j)$$

$$g^* = V_0^* (\overline{Y_{00}}^* \overline{V_0}^* + \overline{Y_{0L}}^* \overline{w}^*) \quad (3.k)$$

$$K_1^* = |W^*| \Re\{(W^*)^{-1} M^*\} \quad (3.l)$$

$$K_2^* = |W^*| \Re\{(W^*)^{-1}(-jM^*)\} \quad (3.m)$$

$$k^* = |w^*| \quad (3.n)$$

### Uncertainty propagation

Linearized boundary condition functions (3) and (4) are also derived using the linear power flow in reference [2], which results in the uncertainty propagation rule given in (8) for estimating the error covariance matrix associated with $b_i^{s-p}$.

$$\begin{bmatrix} \Re\{V^p_{0,i}\} \\ \Im\{V^p_{0,i}\} \end{bmatrix} = I_i^s \begin{bmatrix} \Re\{V^p\} \\ \Im\{V^p\} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} P^s_{0,i} \\ Q^s_{0,i} \end{bmatrix} = H_i^{s-p} \begin{bmatrix} \Re\{V_i^s\} \\ \Im\{V_i^s\} \end{bmatrix} - H_i^{s-p} \begin{bmatrix} \Re\{m_i^s\} \\ \Im\{m_i^s\} \end{bmatrix} + \begin{bmatrix} \Re\{g_i^s\} \\ \Im\{g_i^s\} \end{bmatrix} \quad (5)$$

$$H_i^{s-p} = \begin{bmatrix} \Re\{G_i^s\} & -\Im\{G_i^s\} \\ \Im\{G_i^s\} & \Re\{G_i^s\} \end{bmatrix} H^s_{inj,i} \quad (6)$$

$$I_i^s = \begin{bmatrix} e_j{}^T & 0^T \\ 0^T & e_j{}^T \end{bmatrix} \quad (7)$$

$$\Sigma_{b_i^{s-p}} = H_i^{s-p} (H_i^s)^T (\Sigma_{z_i^s})^{-1} H_i^s (H_i^{s-p})^T \quad (8)$$

## Case Study

We examine the performance of the joint state estimator on a modified IEEE 13-bus system model. Detailed secondary circuits are modeled following the starlike topology and attached to the primary subnetwork via step-down service transformers. Results demonstrate the advantage of the joint state estimator over its disjointed counterparts. It is also shown that reasonable estimations of the uncertainty propagation from measurements to boundary conditions play an important role in improving the accuracy of the joint state estimation during iterations.

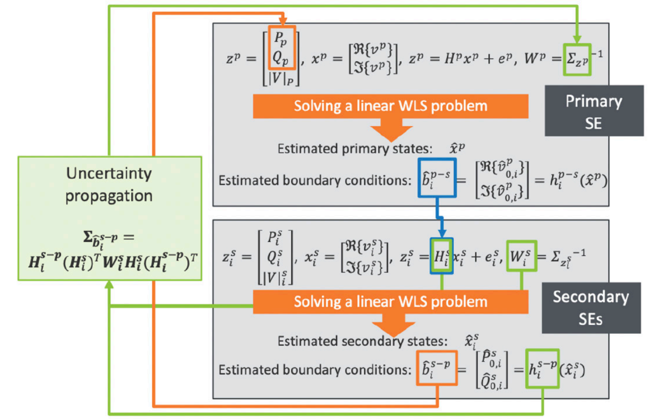


**Fig. 2**: Data flow within the joint state estimator.

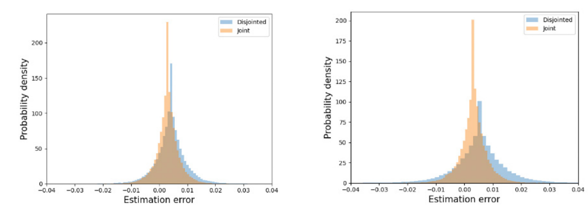


**Fig. 3**: Comparison between the primary (left figure) and secondary (right figure) state estimation errors generated by the joint versus the disjointed state estimators.

**Table 1**: Statistics of the error distributions generated by the joint and disjointed state estimations (in p.u.)

| | Primary states | | Secondary states | |
|---|---|---|---|---|
| | Mean | Standard deviation | Mean | Standard deviation |
| No uncertainty propagation | 0.0046 | 0.0062 | 0.0037 | 0.0056 |
| Uncertainty propagation | 0.0032 | 0.0043 | 0.0033 | 0.0051 |

**Table 2**: Statistics of the error distributions with different settings of uncertainty propagation (in p.u.)

| | Primary states | | Secondary states | |
|---|---|---|---|---|
| | Mean | Standard deviation | Mean | Standard deviation |
| No uncertainty propagation | 0.0046 | 0.0062 | 0.0037 | 0.0056 |
| Uncertainty propagation | 0.0032 | 0.0043 | 0.0033 | 0.0051 |
| Uncertain propagation and nonzero error covariance | 0.0030 | 0.0041 | 0.0032 | 0.0049 |

**Table 3**: Statistics of the error distributions under different secondary voltage magnitude measurement coverage levels (in p.u.)

| Method | Measurement coverage | Mean | Standard deviation |
|---|---|---|---|
| Disjointed state estimator | 0% | 0.0051 | 0.0075 |
| Joint state estimator | 20% | 0.0046 | 0.0071 |
| | 40% | 0.0038 | 0.0051 |
| | 60% | 0.0037 | 0.0047 |
| | 80% | 0.0037 | 0.0047 |
| | 100% | 0.0037 | 0.0047 |

## References

1. R. F. Cruickshank, III, B.-M. S. Hodge, and A. R. Florita, "Heterogeneous network topology management and control." [Online]. Available: https://www.osti.gov/biblio/1805392

2. A. Bernstein and E. Dall'Anese, "Linear power-flow models in multiphase distribution networks," in 2017 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2017, pp. 1–6.

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