

Efficient Network Partitioning: Application for Decentralized State Estimation in Power Distribution Grids

Preprint

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Efficient Network Partitioning: Application for Decentralized State Estimation in Power Distribution Grids

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Abstract—Increase in the proliferation of distributed energy resources require real-time situational awareness for efficient grid operations. State estimation plays an important role for the realtime control and management of the power grid. As the sensing infrastructure grows, aggregating and handling high volumes of data at a centralized location is extremely difficult. To address this challenge, this paper first proposes a novel and efficient hierarchical spectral clustering-based network partitioning algorithm followed by a decentralized compressive sensing (DCS)-based state estimation. The applicability of the proposed network partitioning algorithm is tested on an IEEE 123-bus network, an IEEE 8,500-node system, and a 6,000+ node distribution network. The results shows that the proposed approach efficiently divides the network into multiple sub-networks with the minimum number of edge connections among the neighbors. Then, we perform DCSbased state estimation on the 6,000+ node distribution network after dividing the network into 18 optimal partitions. Simulation results show that the DCS-based state estimation recovers the system states with high accuracy and low complexity.

Index Terms—Network partition, Spectral clustering, Decentralized state estimation (DSE), Compressive sensing, Power distribution network, alternating direction method of multipliers (ADMM).

I. INTRODUCTION

The increasing penetrations of renewable energy resources and responsive electric loads demand better situational awareness for safe and reliable grid operation. State estimation is key for maintaining advanced situational awareness. Typically, the state estimation task is designed to infer the system's state from physical measurements. State estimation in transmission networks relies on the available accurate measurements and network model information. In such cases, state estimation methods are typically formulated as least-squares variants [1]. These assumptions, however, do not work in the power

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distribution network because of limited real-time measurements compared to the unknown state variables. The model information might also be unknown or incorrect because of aging infrastructure or undocumented topology changes. One way to solve this problem is to use pseudo measurements in the form of load or energy forecasts [2]. Recently, sparsity-based approaches have found applications in distribution network state estimation [3], [4]. Sparsity-based techniques exploit the network and the data structures to achieve reliable estimation under limited observability. Further, when historical data are available, Bayesian estimation approaches are proposed to learn the underlying mapping from measurements to states [5], [6]; however, all these methods consider the whole power distribution network as a single entity and estimate the system states in a centralized location. Some major disadvantages of centralized state estimations are (i) low reliability, (ii)high computational complexity, and (iii) large communication bandwidth. On the contrary, decentralized state estimation (DSE) concentrates on estimating the system states by dividing the large network into smaller sub-networks and solves them in parallel [7], [8]. Although the control center is capable of parallel computing, the large volume of measurements transmitted from remote data sources causes a significant communication burden. This creates long delays, which further increases the response time of DSE [9]. Network partitioning plays an important role in DSE; however, limited research has been reported in the literature [10], [11]. A network reconfiguration problem for loss reduction and load balancing is presented in [10], [12]-[14]. Optimal phasor measurement units and communication links placement method for DSE in distribution networks is presented in [15]. A network partitioning approach based on a community detection algorithm for zonal voltage control is discussed in [11].

In addition to distributed state estimation, distributed optimal power flow [16], distributed Volt/VAR control [17], distributed frequency control [18], distributed optimization [19], and distributed wide-area control [20] are also gaining popularity. In all these approaches, there is a specific goal for network partitioning, such as loss reduction or local voltage control; however, none of them generalizes the network partitioning algorithm, which can work for all distributed/decentralized control or optimization problem.

In this article, we propose a generic and efficient network partitioning algorithm. The proposed approach uses hierarchical spectral clustering to obtain the optimal network partitioning. The partitioning of the network ensures minimum boundary variables among the neighboring areas. Once the network partitioning is obtained, the resultant distributed network can be used for all distributed/decentralized studies, i.e., distributed state estimation, distributed Volt/VAR control etc.

A. Contributions

Distributed/decentralized state estimation/control significantly reduces the computational time; however, a suboptimal network partitioning leads to a larger volume of data sharing among the partitions and can slow down the convergence of the decentralized/distributed algorithm. This article proposes a novel and efficient hierarchical spectral clustering-based distribution network partitioning algorithm. The major contributions of the research article are as follows:

- A novel and efficient hierarchical spectral clusteringbased network partitioning algorithm that partitions the network with minimized overlap is proposed. The proposed approach can be applied to all types of distributed/decentralized tasks in a power distribution network.
- The efficacy of the proposed approach is tested on a standard IEEE 123-bus network, an IEEE 8,500-node system, and a real 6000+ node distribution network.
- The network partitioning is applied to decentralized compressive sensing (DCS)-based state estimation and the results demonstrate a gain in the computational complexity without a loss in performance.

II. HIERARCHICAL NETWORK PARTITIONING

We model the power distribution network as a connected and undirected graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and then apply hierarchical spectral clustering to the graph to divide it into multiple subgraphs.

Consider a power distribution network that consists of n nodes and e edges. The network can be represented as a graph, $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=1,2,\cdots,n$ is the set of nodes, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges. The graph adjacency matrix \mathbf{A} , is used to represent the connection among the nodes. The entries of \mathbf{A} are defined as,

$$a_{i,j} = \begin{cases} 1, & \text{if there is a connection from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$$

For efficient optimal network partitioning, we need a similarity matrix of the network graph. For our study, we used the graph Laplacian matrix, \mathbf{L} , as the similarity matrix. The entries of matrix \mathbf{L} are defined as:

$$\mathbf{L}_{i,j} = \begin{cases} d_i, & \text{if } i = j \\ -1, & \text{if } i \text{ and } j \text{ are adjacent} \\ 0, & \text{Otherwise} \end{cases}$$
 (2)

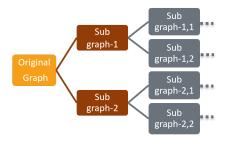


Fig. 1: Flow diagram of hierarchical network partitioning

where d_i is the degree of node i and is calculated as $d_i = \sum_{j=1}^n A_{i,j}$. Because the Laplacian matrix of a planner graph is positive semidefinite, the vector of all ones is always an eigenvector corresponding to the 0 eigenvalues, and all other eigenvalues are nonnegative. We focus on the second smallest eigenvalue, λ_2 , or the algebric connectivity of a graph, and the corresponding eigenvector, v, called the Fiedler vector. For any vector, \mathbf{u} , the Fiedler value, λ_2 , of a graph is given by:

$$\lambda_2 = \min_{\mathbf{u} \perp \mathbf{1}} \frac{\mathbf{u}^T \mathbf{L} \mathbf{u}}{\mathbf{u}^T \mathbf{u}},\tag{3}$$

The minimum value of (3) is obtained when $\mathbf{u} = v$, i.e, the unknown vector is the Fiedler vector. In this work, we exploit a key theorem validated in [21].

Theorem 1. Let $\mathcal{G}=(\mathcal{V},\mathcal{E})$ be a graph on n nodes of maximum degree, d_{max} . Let L be the Laplacian matrix and ϕ be the isoperimetric number. For any vector $\mathbf{u} \in \mathbb{R}^n$ such that $\mathbf{u}^T\mathbf{1}=0$:

$$\frac{\mathbf{u}^T \mathbf{L} \mathbf{u}}{\mathbf{u}^T \mathbf{u}} \ge \frac{\phi^2}{2d_{max}}.$$
 (4)

Moreover, there exists a splitting value, s, for which $(\{i: u_i \leq s\}, i: x_i > s)$ has a ratio at most $\frac{\phi^2}{2d_{max}}$. More details on this theorem can be found in [21].

Using Theorem 1, with v the Fiedler vector of the graph Laplacian, the spectral partitioning is to find a splitting value, s, that partitions the vertices of $\mathcal G$ into sets such that $v_i \leq s$ and $v_i > s$. Although there are several values to choose for s, we choose the $sign\ cut$ for which s=0. We successively apply the discussed method to obtain the desired number of subgraphs for an equivalent large graph. We limit the subgraph creation based on the computational capability of the particular region. Fig. 1 shows a pictorial representation of the hierarchical spectral graph partitioning, and the step-by-step procedure is given in Algorithm 1. Next, we discuss the DSE proposed in [22] using the optimally partitioning algorithm discussed.

III. DISTRIBUTED STATE ESTIMATION

State estimation in the distribution grid involves the characterization of the voltage magnitude and the angle at each node based on the available measurements. With accurate model information, the voltage phasors at all nodes are enough to find other electrical quantities, such as nodal power consumption/injection and line current flows; however, accurate

Algorithm 1: Hierarchical spectral clustering-based network partitioning

Input Admittance matrix: \mathbf{Y} , Minimum number of nodes in a single partition kInitialize: K=0while k>K do

Obtain: Adjacency matrix \mathbf{A} Obtain: Degree matrix of graph \mathbf{D} Obtain: graph Laplacian $\mathbf{L}=\mathbf{D}-\mathbf{A}$ Obtain: The Fiedler vector \mathbf{v} of \mathbf{L} Obtain: $Subgraph_1 \leftarrow \mathbf{v} > 0$ and $Subgraph_2 \leftarrow \mathbf{v} \leq 0$ Update $K=\max(Volume(Subgraph_1), Volume(Subgraph_2))$ end

network information might not be always available to the distribution network operator. This is due to the change in line parameters and/or undocumented changes in network topology. Therefore, we propose a model-free compressive sensing-based distributed state estimation to recover all the nodal voltage phasors using limited measurements in power distribution grids. Toward this end, we first briefly review the compressive sensing technique, followed by the distributed compressive sensing-based state estimation in power distribu-

Return: Number of partitions with their node index

A. Compressive Sensing

tion grids.

Compressive sensing finds the sparse solution to an undetermined linear system by exploiting the sparsity of the signal on a linear transformation basis [23]. Let $\mathbf{x} \in \mathbb{R}^N$ be the original signal of interest compressible in a linear transformation basis Ψ , i.e., $\mathbf{x} = \Psi \mathbf{b}$, where, \mathbf{b} has at most K << N significant coefficients. If the sensing mechanism is $\mathbf{y} = \Phi \mathbf{x}$, where $\mathbf{y} \in \mathbb{R}^M$ is the measurement vector, and $\mathbf{\Phi} \in \mathbb{R}^{M \times N}$ is projection matrix. The entries of $\mathbf{\Phi}$ could be i.i.d random Gaussian variables with zero mean and 1/M variance. The goal of compressive sensing is to recover \mathbf{x} from \mathbf{y} . The most common approach to recover \mathbf{x} is to solve the l_1 minimization problem as:

$$\hat{\mathbf{a}} = \min_{\mathbf{q} \in \mathbb{R}^N} \|\mathbf{q}\|_1$$
 subject to $\mathbf{y} = \mathbf{\Phi} \mathbf{\Psi} \mathbf{q}$ (5)

The optimization problem of (5) reconstructs $\hat{\mathbf{x}} = \mathbf{\Psi}\hat{\mathbf{a}}$. The state recovery using (5) becomes computationally expensive as the system size increases.

B. Decentralized Compressive Sensing

To reduce the computational complexity of compressive sensing-based state estimation in large distribution networks, we propose a DCS-based state estimation approach in power distribution grids. First, we use the hierarchical network partitioning algorithm, as discussed in Section II, to divide the power distribution network into \mathcal{D} subareas. Next, we use DCS to recover the states in each subarea.

Let us define the sates of the power network for a particular area, $\mathbf{x_a}$ $\left[\Re(\mathbf{s}_a)^T,\Im(\mathbf{s}_a)^T,\Re(\mathbf{v}_a)^T,\Im(\mathbf{v}_a)^T,|\mathbf{v}_a|^T\right]^T$ the concatenation of active power, reactive power, real part of the voltage, imaginary part of the voltage, and voltage magnitude. The state variables, x_a , include internal variables, $\mathbf{x}_{a,int}$, and boundary variables, $\mathbf{x}_{a,adj}$. Boundary variables are the variables that overlap with other subareas. Although the optimization problem (5) is used to recover the system states in each subarea, we must ensure that the boundary variables in each area are equal. To make the boundary variables equal, we introduce a global variable, $g_{a,b}$, for two neighboring areas, a and b, as $g_{a,b} = \mathbf{x}_{a,adj}^{ab} = \mathbf{x}_{b,adj}^{ab}$. Note that the global variable for any two adjacent areas, a and b, should be equal, i.e., $g_{a,b} = g_{b,a}$. The decentralized model of the compressive sensing-based state estimation problem corresponds to:

where $f_a(\mathbf{x}_a) = ||\Psi_a\mathbf{x}_a||_1 + \frac{\lambda_a}{2}||\mathbf{y}_a - \Phi_a\Psi_a\mathbf{x}_a||_2^2$, and \mathbf{y}_a is the available measurement in area a. The alternating direction method of multipliers (ADMM) is the most commonly used method for solving (6). ADMM takes the form of a decomposition-coordination procedure, in which solutions to small local problems are coordinated to find a solution to a large global problem [24]. In the context of the formulation in (6), the dual variables $\lambda_{a,b}$ correspond to consensus boundary constraints of areas a and b; the cardinality of the neighbor sets $|\mathcal{J}^a|$ is the number of neighbors of area a. Define the local variables $\lambda_a = \{\lambda_{a,b}|b \in \mathcal{J}^a\}$ and $\mathbf{g}_a = \{\mathbf{g}_{a,b}|b \in \mathcal{J}^a\}$ as the concatenation of dual and global variables, respectively; and λ and \mathbf{g} as the concatenation of the corresponding local and global variables. The augmented Lagrangian formulation for (6) is given as:

$$\mathcal{L}(\mathbf{x}, \mathbf{g}, \boldsymbol{\lambda}) = \sum_{a=1}^{D} \mathcal{L}_a \left(\{ \mathbf{x}_a \}, \{ \mathbf{g}_a \}, \{ \boldsymbol{\lambda}_a \} \right)$$
(7)

where:

$$\mathcal{L}_{a}(\{\mathbf{x_{a}}\}, \{\mathbf{g_{a}}\}, \{\lambda_{a}\}) = f_{a}(\mathbf{x}_{a}) + \sum_{b \in \mathcal{J}^{a}} \left[\lambda_{a}^{T}(\mathbf{x}_{a,adj} - \mathbf{g}_{a}) + \frac{\rho}{2} ||\mathbf{x}_{a,adj} - \mathbf{g}_{a}||_{2}^{2} \right]$$
(8)

where ρ is a predefined constant. The ADMM iterates through:

$$\mathbf{x}_{a}^{t+1} = \underset{\mathbf{x}_{a}}{\operatorname{argmin}} \ \mathcal{L}_{a}(\mathbf{x}_{a}, \mathbf{g}_{a}^{t}, \boldsymbol{\lambda}_{a}^{t}) \tag{9}$$

$$\mathbf{g}_{a}^{t+1} = \underset{\mathbf{g}_{a}}{\operatorname{argmin}} \ \mathcal{L}_{a}(\mathbf{x}_{a}^{t+1}, \mathbf{g}_{a}, \boldsymbol{\lambda}_{a}^{t})$$
 (10)

$$\boldsymbol{\lambda}_{a}^{t+1} = \boldsymbol{\lambda}_{a}^{t} + \rho(\mathbf{x}_{a,adj}^{t+1} - \mathbf{g}_{a}^{t+1}) \ \forall b \in \mathcal{J}^{a}$$
 (11)

The iterates (9)–(11) converge to a solution of (6) [24]. The decentralized algorithm described in (9)–(11) can be solved in

a parallel manner. The subareas estimate their respective states based on their respective available measurements [22].

C. Computational Complexity

The compressive sensing-based state estimation method involves an l_1 minimization that is solved by linear programming. Let M be the number of variables and N the number of constraints. The computational complexity of solving one Newton step is O(MN.min(M,N)). The same argument is valid for DCS; however, the number of variables in a particular area reduces, reducing execution time. The execution time in the DSE method is always less than the centralized state estimation method. The total time for the DSE method depends on the time taken for the estimation process in the area having the highest number of variables and the intercommunication among the neighboring areas. The proposed hierarchical spectral clustering approach minimizes the overlap among partitions, reducing the communication burden across subareas/partitions.

IV. SIMULATION RESULTS

In this section, we present the simulation results of the efficient hierarchical graph partitioning algorithms followed by the DSE. We test the proposed hierarchical spectral clusteringbased graph partitioning algorithm on the standard IEEE 123bus system and the IEEE 8,500-node system. We also test the algorithm on a real power distribution network having 6,000+ single-phase nodes. The distribution network data is taken from a distribution network operator in Kansas, USA. The 5-area partitioning of the IEEE 123-bus system and 12-area partitioning of the IEEE 8,500-node system are shown in Fig. 2 and Fig. 3, respectively. As shown, the hierarchical spectral clustering divides the network such that the neighboring area shares minimum edges. Similar performance is also observed for the 6,000+ node distribution network. An 18-area partition of the network is shown in Fig.4. A comparative analysis of the proposed hierarchical spectral network partitioning with the kmeans partitioning is given in TABLE. I. It can be seen, that the total number of shared links among the neighbors using the proposed algorithm is much less than k-means network clustering.

Next, we evaluate the performance of the DCS-based state estimation on the 18-area partitioned distribution network.

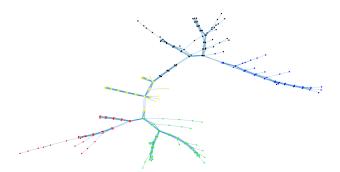


Fig. 2: Five-area optimal partitioning of the IEEE 123-bus network

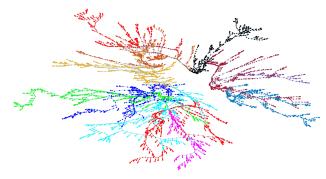


Fig. 3: Twelve-area optimal partitioning of the IEEE 8,500-node system

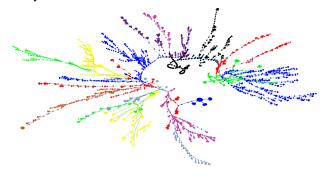


Fig. 4: Eighteen-area optimal partitioning of the real 6,000+ node system

We evaluate the performance of the DSE methods using mean absolute percentage error (MAPE) for the voltage magnitude and the mean integrated absolute error (MIAE) for the voltage angle, respectively [4]. In the results, we present the MAPE and MIAE for the voltage magnitude and angle estimations, respectively.

Fig. 5 (a) and (b) show the voltage magnitude and voltage angle recovery of the 6,000+ node distribution network. It is observed that the estimation error both in voltage magnitude

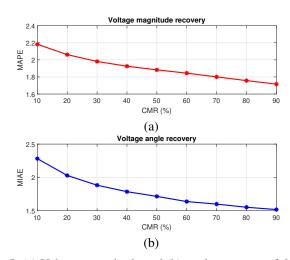


Fig. 5: (a) Voltage magnitude and (b) angle recovery of 6,000+ node system

TABLE I: Performance of proposed partitioning algorithm

Test system	No of	No of shared links	
iest system	partitions	k-means	Hierarchical
		partition	network partition
IEEE 8500	12	123	86
6000+	18	500+	140

and angle is maximum when the compressed measurement ratio (CMR) is 10%. The MAPE and MIAE in the voltage magnitude and the angle decrease as the CMR (%) increases. This is because as the CMR value increases, the available measurement increases, which increases the estimation accuracy and reduces the estimation error.

Next, we present a comparative analysis of computation time in centralized and decentralized state estimation method in Table II. It can be seen that, with effective partitioning, the simulation time reduced drastically from 631.72 seconds in the centralized approach to 20.19 seconds in the proposed decentralized approach.

TABLE II: Simulation time for 1 run and 1 CMR

Approach	Time(sec)
Centralized CS based state estimation	631.72
Decentralized CS based state estimation	20.19

V. CONCLUSION AND FUTURE WORK

This paper proposes an efficient hierarchical spectral clustering-based network partitioning algorithm. The proposed algorithm divides the network with minimum edge sharing among the neighboring partitions/areas. Because the information shared among the neighboring areas is limited, this can be effective in reducing the communication overhead in distributed/DSE and control. We tested the algorithm performance on standard IEEE systems and a real 6,000+ distribution network. Next, we performed decentralized compressive sensingbased state estimation on the 6,000+ distribution network by optimally dividing the network into 18 smaller subnetworks and reported the simulation results. The simulation results show that the estimation error in the voltage magnitude and the voltage angle is 2.2% and 2.4%, respectively, even when the CMR is only 10%, and the estimation accuracy further increases with an increase in CMR%. For future work, we will test the accuracy of DSE under cyber attacks [25].

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