



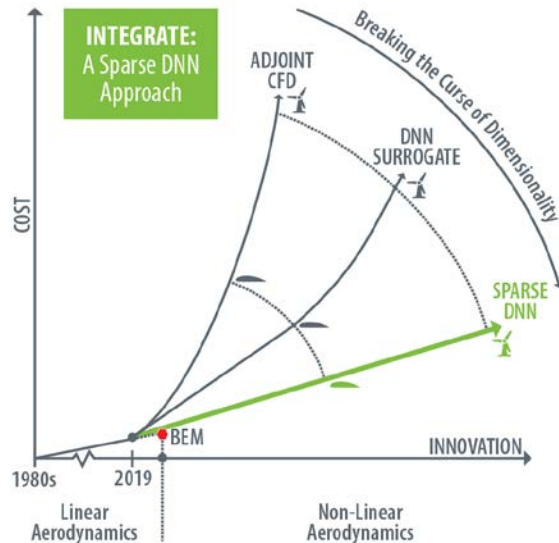
# Invertible neural networks for aerodynamic design of wind turbine blades

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# Overview

**Goal:** enable full 3D inverse design of wind turbine blades using machine learning techniques



- Design workflows generally rely on engineering tools e.g., blade element momentum (BEM) theory
  - For larger offshore rotors, nonlinear aerodynamic effects dominate, and BEM assumptions break down
- Use ML to bring computational fluid dynamics (CFD) fidelity in the design process
  - Grassmann-based shape representations
  - Invertible neural network (INN) framework
  - Trained on high-fidelity CFD data

# Inverse problems and INN

Consider a forward mapping  $f: (\mathcal{X} \subset \mathbb{R}^m) \rightarrow (\mathcal{F} \subset \mathbb{R}^d)$  with the input space weighted by  $\rho: (\mathcal{X} \subset \mathbb{R}^m) \rightarrow \mathbb{R}^+$

We seek to characterize the inverse image

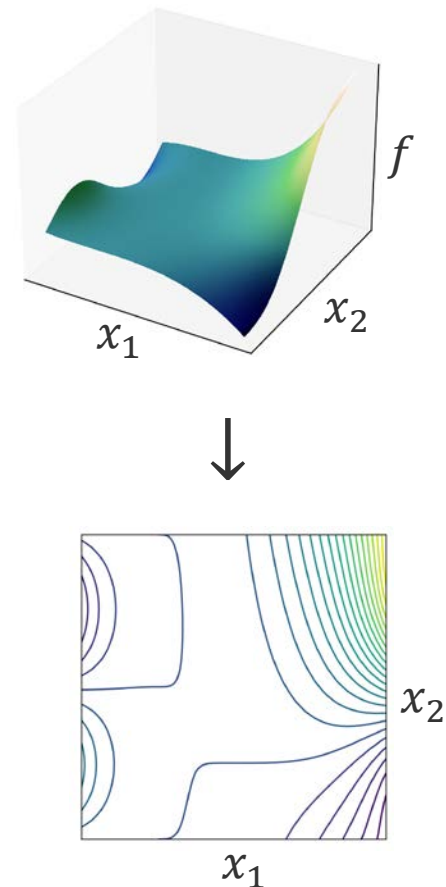
$$f^{-1}(\bar{\mathbf{f}}) = \{ \mathbf{x} \in \mathcal{X} : f(\mathbf{x}) = \bar{\mathbf{f}} \}, \quad \rho(\mathbf{x}|\bar{\mathbf{f}})$$

for some conditional value  $\bar{\mathbf{f}} \in \mathcal{F}$

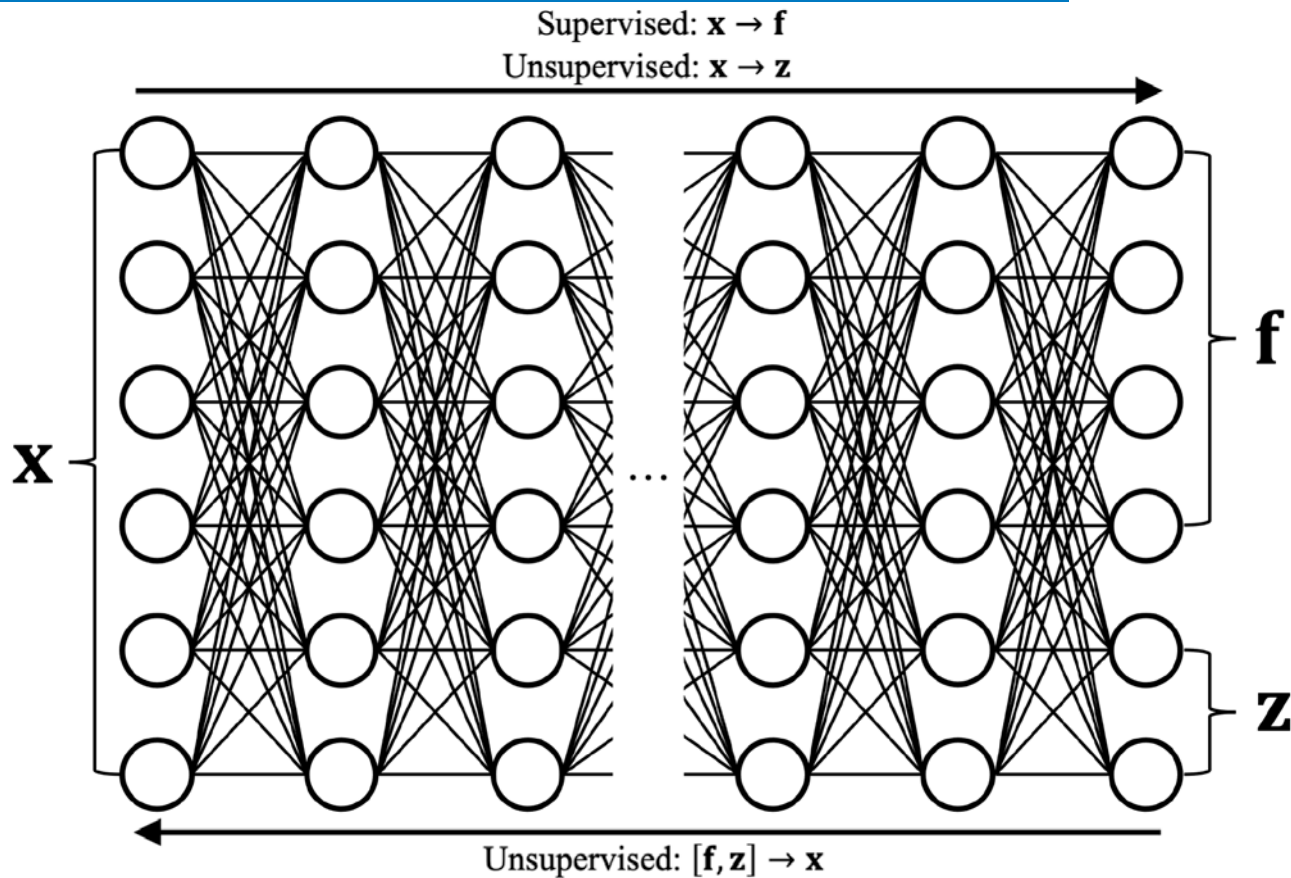
Use an invertible neural network that learns a bijection

$$f_{INN}(\mathbf{x}; \Theta) = \begin{bmatrix} \mathbf{f} \\ \mathbf{z} \end{bmatrix}$$

where the latent variables  $\mathbf{z} \rightarrow \mathbb{R}^{m-d}$  parameterize the set  $f^{-1}$



# INN architecture



# Invertible blocks

Partition the incoming vector into two equal pieces and apply

$$\phi_i(\mathbf{x}_i) = \phi_i\left(\begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix}\right) = \begin{bmatrix} \mathbf{u}_i \odot e^{s_1(\mathbf{v}_i)} + t_1(\mathbf{v}_i) \\ \mathbf{v}_i \odot e^{s_2(\mathbf{u}_i)} + t_2(\mathbf{u}_i) \end{bmatrix} = \begin{bmatrix} \mathbf{u}_{i+1} \\ \mathbf{v}_{i+1} \end{bmatrix} = \mathbf{x}_{i+1}$$

which has the closed-form invertible mapping

$$\phi_i^{-1}(\mathbf{x}_{i+1}) = \phi_i^{-1}\left(\begin{bmatrix} \mathbf{u}_{i+1} \\ \mathbf{v}_{i+1} \end{bmatrix}\right) = \begin{bmatrix} (\mathbf{v}_{i+1} - t_2(\mathbf{u}_{i+1})) \odot e^{-s_2(\mathbf{u}_{i+1})} \\ (\mathbf{u}_{i+1} - t_1(\mathbf{v}_{i+1})) \odot e^{-s_1(\mathbf{v}_{i+1})} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \mathbf{x}_i$$

regardless of the form of  $s_1, t_1, s_2, t_2: \mathbb{R}^{m/2} \rightarrow \mathbb{R}^{m/2}$

[Ardizzone, et al., 2019]

# Training losses

- Supervised quantities are trained using MSE losses

$$\mathcal{L}_f = \|\mathbf{f} - \mathbf{f}_{\text{INN}}(\mathbf{x}; \Theta)_{[1:d]}\|_2^2$$

- Unsupervised quantities are trained using the maximum mean discrepancy (MMD)

[Gretton, et al. 2012]

Two probability distributions are identical if and only if

$$MMD(p, q) := \sup_{\phi \in \mathcal{H}} |\mathbb{E}_{x \sim p}[\phi(x)] - \mathbb{E}_{y \sim q}[\phi(y)]| = 0$$

In practice, we compute

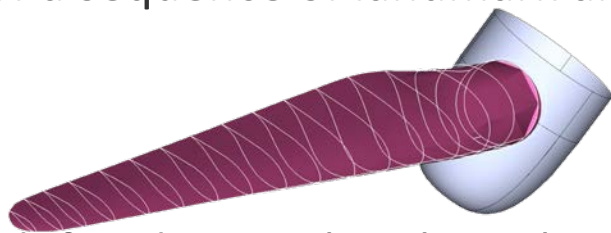
$$MMD^2(p, q) = \mathbb{E}_{x, x' \sim p}[k(x, x')] - 2\mathbb{E}_{x \sim p, y \sim q}[k(x, y)] + \mathbb{E}_{y, y' \sim q}[k(y, y')]$$

where  $k(\cdot, \cdot)$  is some kernel function

$$\rightarrow \text{for this work, we use } k(x, y) = \frac{1}{1 + \|x - y\|_2^2}$$

# Blade Shape Representation

- We need a framework for blade shape representations that to enable design with the INN model
  - Blade is comprised of a sequence of landmark airfoils

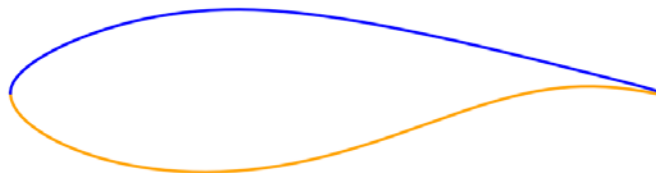


- Each airfoil shape is defined using the Class-Shape Transformation (CST)

$$\zeta(\psi) = C_{N2}^{N1}(\psi)S(\psi) + \psi\zeta_T$$

$$C_{N2}^{N1} = \psi^{N1}(1 - \psi)^{N2}$$

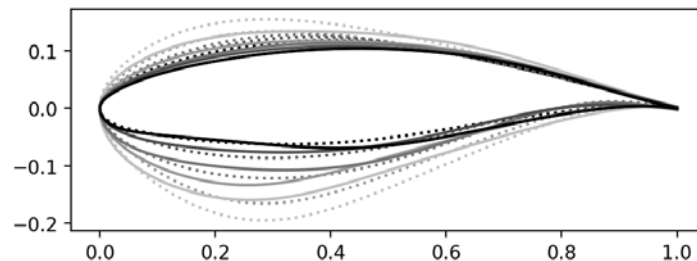
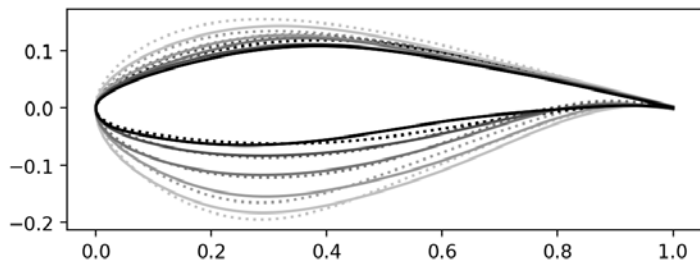
$$S(\psi) = \sum_{i=0}^n a_i S_i$$



- 5 landmark shapes  $\times$  20 shape parameters = 100 dimensions
  - Doesn't account for chord or twist

# Blade Shape Representation

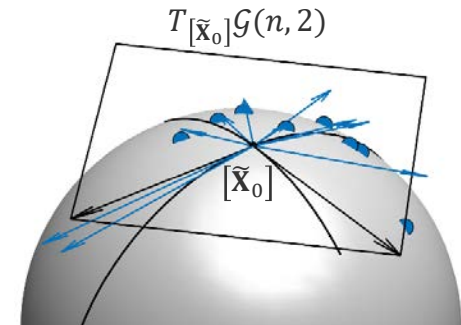
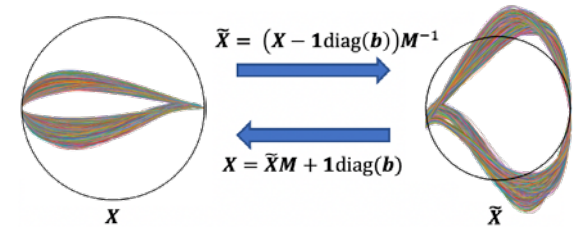
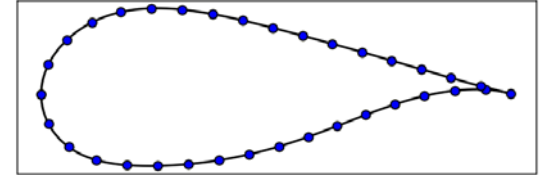
- Treating airfoil shape perturbations independently can result in bad blade shapes with undesirable features (e.g., kinks or dimples)
  - Define cohesive perturbations applied to each airfoil
  - Pathway for dimension reduction
- Consistent perturbations applied to CST coefficients does not map to consistent shape deformations





# Grassmannian Shape Representations

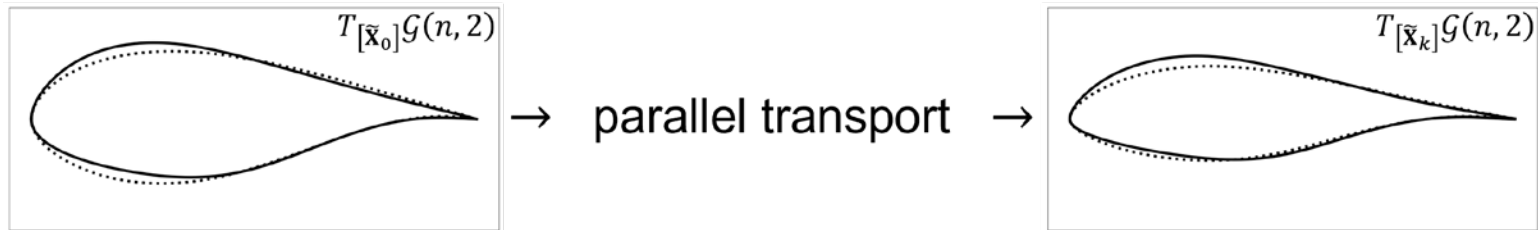
- Represent shapes as  $n$   $(x, y)$ -landmarks along the curve
- Perform landmark-affine (LA) standardization to shapes
  - Treats each airfoil shape as an element of the Grassmann manifold  $\mathcal{G}(n, 2)$
  - Remaining differences in shapes are driven by higher order variations
- Perform principal geodesic analysis on Grassmann shapes
  - A generalization of principal component analysis (PCA) to Riemannian manifolds
  - Defines principal components within the central tangent space  $T_{[\tilde{\mathbf{X}}_0]} \mathcal{G}(n, 2)$  at some point  $[\tilde{\mathbf{X}}_0]$



# PGA-based Blade Design

Goal: Seek to apply cohesive perturbations to the landmark airfoils in the blade blade

- PGA coordinates are defined relative to a central tangent space of the Grassmannian defined at a specific point  $T_{[\tilde{\mathbf{x}}_0]} \mathcal{G}(n, 2)$
- Parallel transport is a process by which we can smoothly translate the PGA coordinates to a new tangent space  $T_{[\tilde{\mathbf{x}}_k]} \mathcal{G}(n, 2)$



- Blade shape is defined by four PGA coordinates and a thickness value

# Outer blade section design

Building towards full blade design, we two perform INN-based designs on the outboard section of the blade



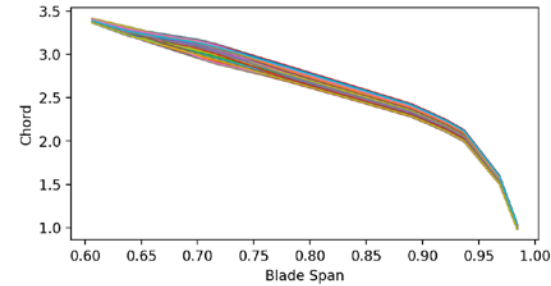
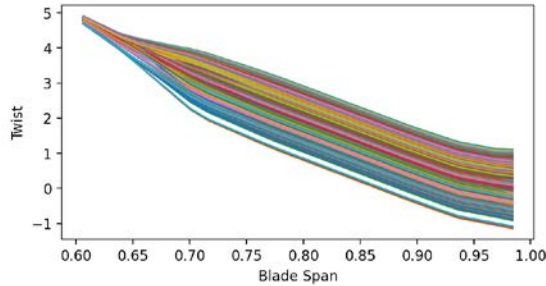
**Goal:** Trade-off some power to mitigate loads

Consider two design problems:

1. Design of chord & twist profiles based on 3D CFD
2. Design of blade tip shape and chord & twist profiles based on BEM

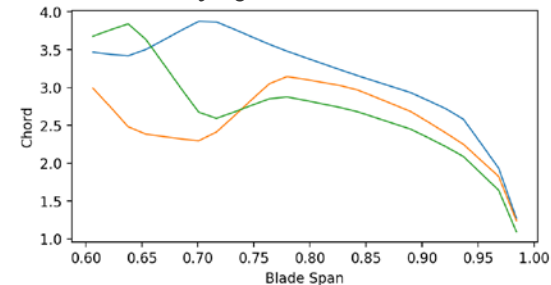
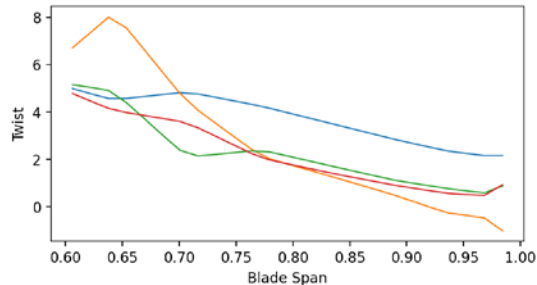
# Blade Twist & Chord

- Need to encode twist and chord profiles along the blade span in a manner that is compatible with our INN framework



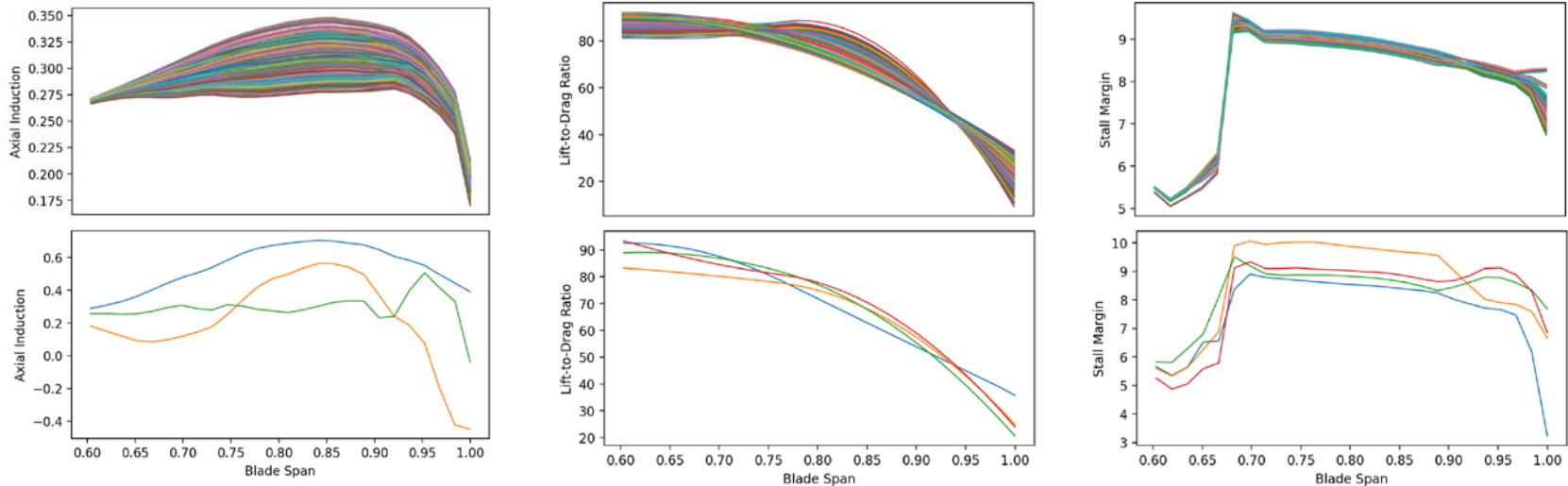
- Parameterize profiles using a Karhunen–Loève basis expansion

$$\int k(f(\tilde{r}), f(r)) \phi(\tilde{r}) d\tilde{r} = \lambda \phi(r) \rightarrow f(\alpha) \approx \sum_{i=0}^N c_i \phi_i(r)$$



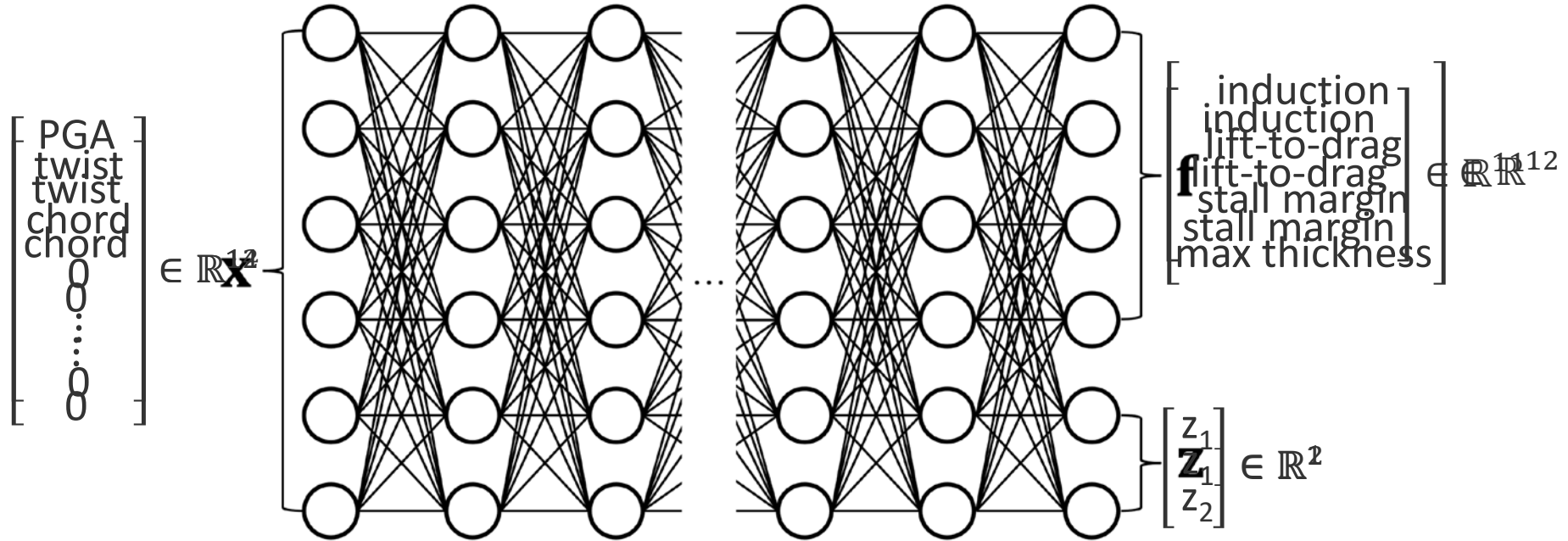
# Design Metrics

- Aerodynamic design metrics are (i) axial induction factor, (ii) lift-to-drag ratio, and (iii) stall margin



- Max thickness-to-chord ratio is included as a structural design for the BEM design problem

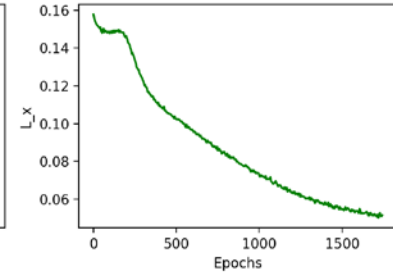
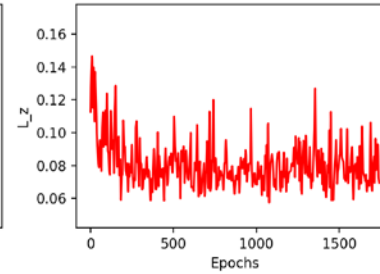
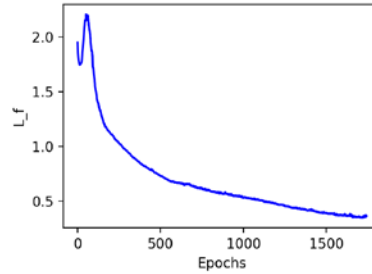
# INN architecture – Scenario 2 – BEM



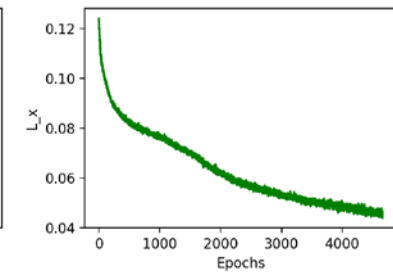
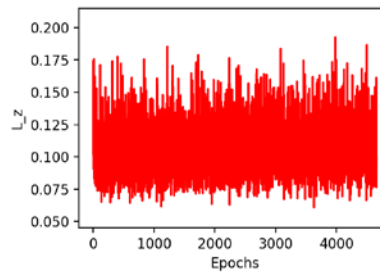
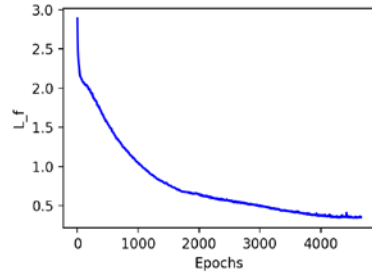
# Training

- MSE losses of aerodynamic quantities improve by an order of magnitude before leveling off
- Unsupervised losses
  - Gaussian distribution over latent variables is nearly recovered at initialization
  - Target input distribution converges relatively quickly

### Scenario 1 – CFD

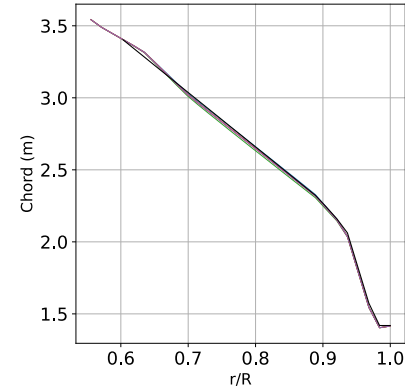
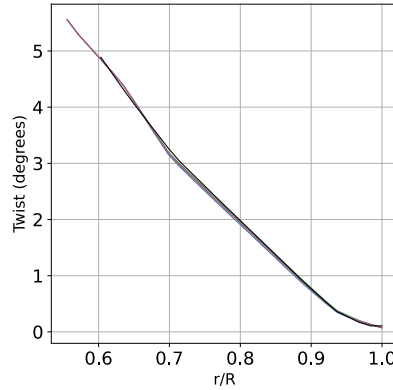


### Scenario 2 – BEM

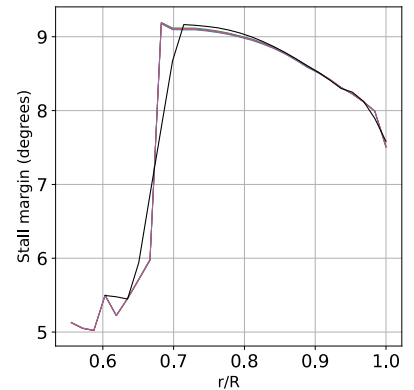
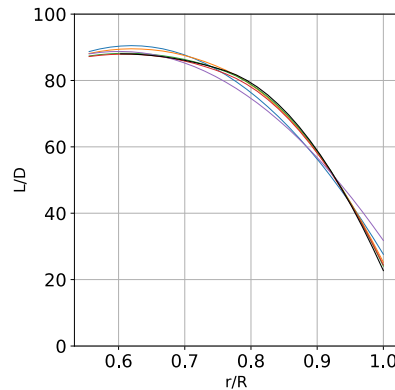
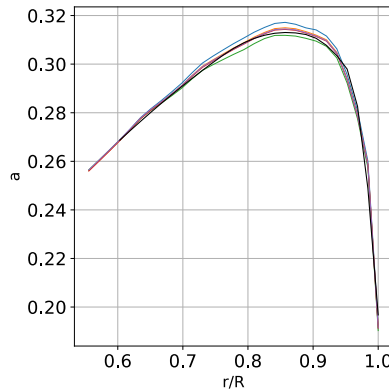


# Results – Scenario 1 – CFD

Five generated  
twist/chord profiles



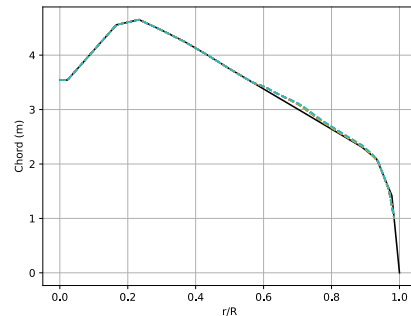
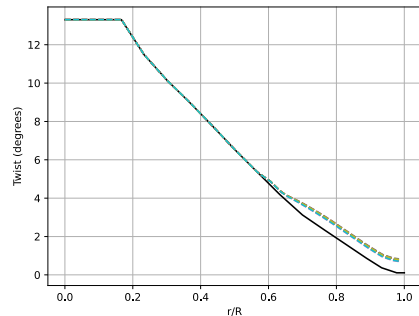
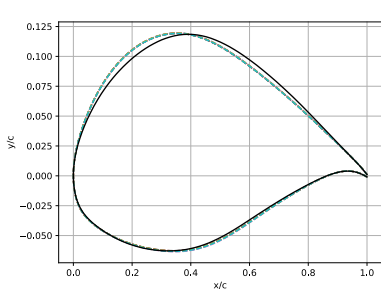
Match target  
outputs within  
error thresholds



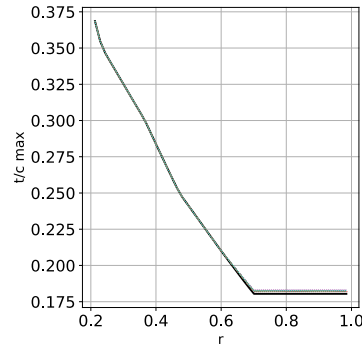
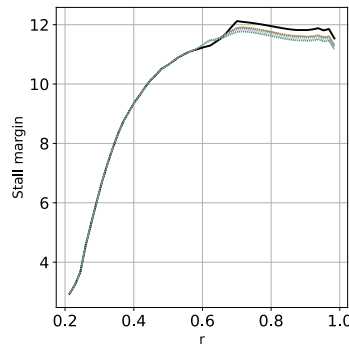
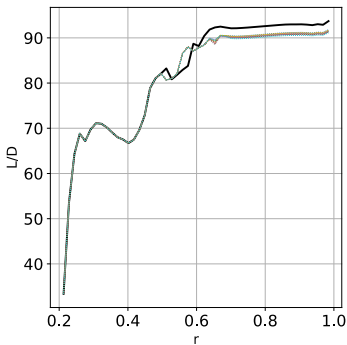
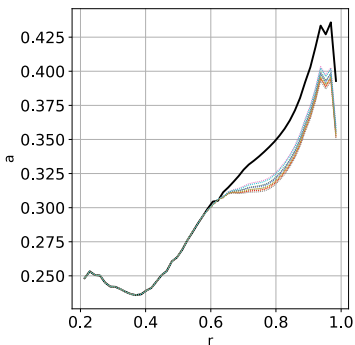


# Results – Scenario 2 – BEM

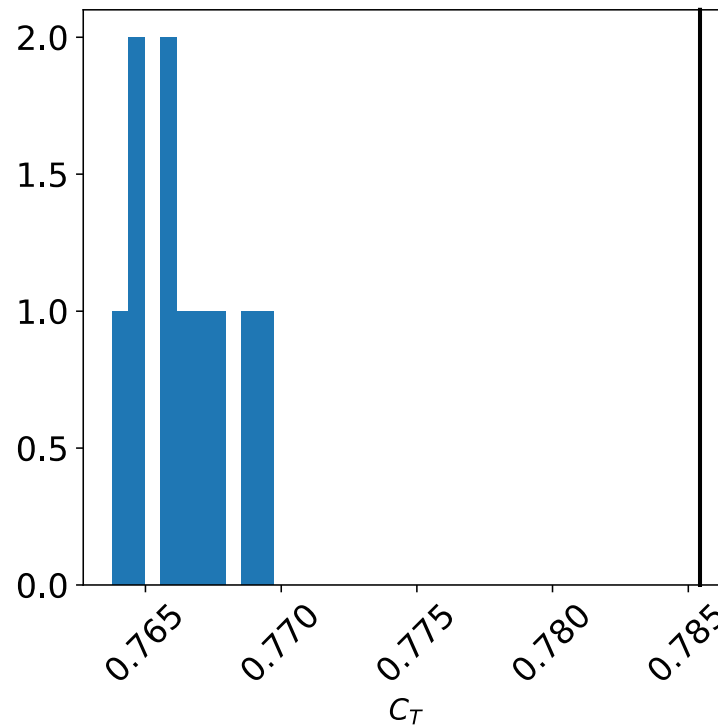
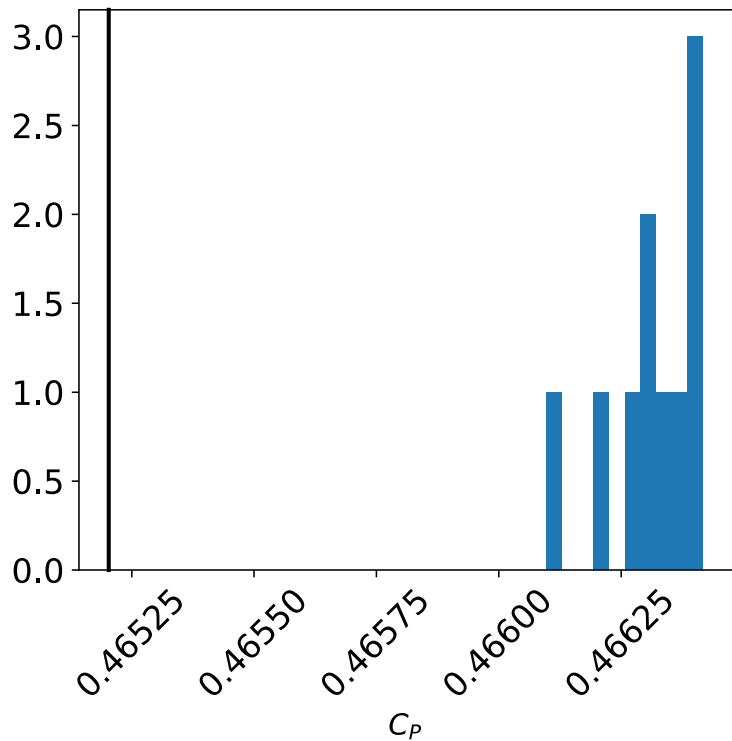
Ten generated  
blade designs



Match target  
outputs within  
error thresholds



# Results – Scenario 2 – BEM



# Conclusion & Next Steps

- Used invertible neural network (INN) architecture to perform design of the outboard section of the blade
  - Learns a bijection between inputs and outputs
  - Relies on Grassmann airfoil shape representations and KL expansions for blade span profile quantities
- Next steps: Perform full 3D blade shape design using the INN framework
  - Used PGA perturbations to sample new blade shapes
  - Used Mercury 3D CFD framework to evaluate over 5,000 blades
  - Train the INN using this data and validate the generated designs

# Thanks!

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