Solving the Unit Commitment Problem:

Polyhedral Theory, Symmetry, and Power Flow

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Unit Commitment Problem (UC)

- Schedule grid participants (generators, load, etc.) to balance supply and demand for electricity for the next week, next day, or next few hours
- Typically modeled as a Mixed-Integer Programing (MIP) problem (Garver 1962)
- Lots of components:
  - Binary variables & combinatorial constraints (thermal units, storage, variable load)
  - Uncertainty (fixed load, variable generation output)
  - Nonlinearity (AC power flow)
  - Reliability (Transmission & Generation contingencies)
- All ISOs in the U.S. use commercial MIP solvers to solve UC
  - Transition from heuristics based on Lagrangian relaxation were estimated to save ~$5B USD annually in 2017 (O'Neill 2017)

UC in Practice

- Binary variables & combinatorial constraints (thermal units, storage, variable load)
  - Model directly using MIP; relax / approximate combined cycle units
  - Variety of practices for storage models, most do not manage energy balance
- Uncertainty (fixed load, variable generation output)
  - Ramping reserve products, other ad-hoc rules & out-of-market corrections
- Nonlinearity (AC power flow)
  - Linearize around AC base point & use sensitivity factors for a small subset of lines (likely to be) binding
- Reliability (Transmission & Generation contingencies)
  - Generation contingencies: regulation, spinning, non-spinning reserve
  - Transmission contingencies: sensitivity factors; offline studies

Contents

Mixed Integer Programming Formulation

$$\min \sum_{g \in \mathcal{G}} c^g \qquad (1)$$

subject to

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) = D \qquad (2)$$

$$(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G}. \qquad (3)$$

- Objective function (1) minimizes generation/system operation cost
- Constraints (2) are the system operating constraints (load satisfaction, transmission thermal limits, reserve requirements, etc.)
- Constraints (3) are the technical limits/constraints and cost of operation c^g for schedule u^g, p^g, for each generator. Variables u^g are the "commitment" decisions for the generator (generally understood to be discrete) and variables p^g are the "dispatch" decisions for the generator.

Mixed Integer Programming Formulation

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$$(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G}. \qquad (3)$$

Formulation of constraints (2):

- Power flow (Van den Bergh et al. 2014, Mozlahn & Hiskens 2019)
- Reserve products (Wang & Chen 2020)
- Integration with Π^g (Lu 2016, Bendotti et al. 2018)

Mixed Integer Programming Formulation

$$\min \sum_{g \in \mathcal{G}} c^g \qquad (1)$$

subject to

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) = D \qquad (2)$$

$$(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G}. \qquad (3)$$

Formulation of Constraints (3):

- Thermal Generator (K., Ostrowski, & Watson 2020)
- Combine Cycle Units (Hua et al. 2019)
- Storage (Baldick et al. 2021)

Strategies for Formulating Π^g

In practice, Π^g is represented as a mixed-integer linear set; i.e.,
$\Pi^g = \{(p, u) \in \mathbb{R}^n \times \mathbb{Z}^m \mid Ap + Bu = d\}$.

A traditional research question is: what's the "best" formulation for Π^g given set of requirements?

"Best" could mean:

- Fewest number of constraints / variables (compact)
- Convex hull representation of Π^g (tight)
- Computational performance with a MIP solver

Modeling Thermal Generators

Deep academic literature on model thermal generators as defined by Carrion and Arroyo (2006).

Features:

- Binary on/off state of the generator
- Minimum/Maximum power production when committed
- Minimum uptime & downtime constraints
- Ramping constraints (both between consecutive on-periods and transitions)
- Downtime-dependent startup costs

Modeling Thermal Generators

Deep academic literature on model thermal generators as defined by Carrion and Arroyo (2006).

Theoretical Results:

- Optimize over a single generator in $O(T^3)$ time (Frangioni & Gentile 2006)
- Convex hull description of single generator with $O(T^3)$ variables and constraints (K. et al. 2018, Bacci et al. 2019)

Convex hull description is too large for use within UC MIP formulation with hundreds or thousands of generators!

(A smaller formulation, e.g., with $O(T^2)$ variables and constraints isn't mathematically ruled out)

Modeling Thermal Generators

What to do? Mathematics informed engineering!

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On Mixed-Integer Programming Formulations for the Unit Commitment Problem

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Modeling Thermal Generators

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
CA	600.0	43.333%	24	0	0
MLR	340.5	0.0555%	3	4	1
ALS	394.7	0.0933%	7	2	2
KOW	390.2	0.0117%	2	3	5
T	**268.6**	**0.0104%**	**1**	**8**	4
Co	309.5	0.0596%	3	4	5
R1	308.9	0.0480%	3	3	**6**
R2	373.1	0.0665%	4	0	1

CA formulation uses the least number of variables and constraints

T formulation has:

1. convex hull description without ramping and downtime-dependent startup costs
   $O(T)$ variables and constraints
2. heuristically adds strengthening variables and constraints for ramping and downtime-dependent startup costs (but only a linear number in T).

Modeling Other Devices

- Arriving at performant models for Carrion & Arroyo (2006)'s thermal generator was the culmination of more than a decade of research
  - Over 100,000 unique mathematically equivalent formulations!
- Greater challenges ahead for devices with energy constraints which cross time, e.g., storage, flexible load
  - Adding a single total energy consumed constraint makes single generator problem NP-hard (Pan et al. 2022)
    - Still NP-hard even if minimum up/down time and ramping constraints are removed.
- Implies individual storage devices do not have a compact convex hull description!
  - Good-enough approximations for the convex hull? (Baldick et al. 2021)
  - Cut generation?
    - Will also be NP-hard, but on a much smaller problem than the whole UC

Contents

Handling Identical Devices

- Symmetry caused by identical devices is often present in practical UC problems
- Even the sophisticated symmetry detection and avoidance strategies available in modern commercial MIP codes cannot fully avoid certain solutions with identical objective value (K., Ostrowski, & Watson 2018).
- Identical units tend to be collocated and owned and/or operated by the same entity

- Ex: consider two identical generators g^1 and g^2 and two solutions:

$$u^{g_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad u^{g_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad u^{g_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad u^{g_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

- These two solutions are *not* symmetric, but *do* have identical objective value

Handling Identical Devices

- K., Ostrowski, & Watson (2018) propose handling this through reformulation.
- Aggregate identical devices:
  - can be done with optimality and feasibility guarantees if the device formulation satisfies certain (restrictive) properties
- These alternative optimal solutions have a single representation in the aggregate reformulation

$$u^{g_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad u^{g_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad u^{g_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad u^{g_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

Handling Identical Devices

- When does aggregation preserve optimality and feasibility?
  - Minimally need convex hull descriptions for the devices
  - Not sufficient in general; see Baum & Trotter (1978)
- The convex hull descriptions for thermal generators *do* enable this aggregation
  - Doesn't always make a huge computational difference, but can be significant on harder UC instances
  - Can be difficult to implement correctly (need to recover non-aggregated solution)
- Identical devices seem to be a fact of life:
  - Can we exploit similar ideas for storage models?

Contents

Transmission Constraints

- Transmission Constraints serve to limit the flow of electricity through a transmission line or transformer
  - More flow -> more heat -> line expansion and sagging
  - Transformers have their own power ratings for reliable operation
- In typical unit commitment and economic dispatch problems the line flow calculation used is a *linear approximation* of the AC power flow equations:

$$\sum_{l \in F^+(i)} f_l - \sum_{l \in F^-(i)} f_l = n_i \quad \forall i \in B$$

$$f_l = B_l\left(\theta_{l(i)} - \theta_{l(j)}\right) \quad \forall l \in L$$

$$-F_l \leq f_l \leq F_l \qquad \forall l \in L$$

$$\theta_{ref} = 0$$

$$n_i = \sum_{g \in G(i)} p_g - L_i \quad \forall i \in B$$

Variables: $2|B| + |L| + |G|$
Equalities: $2|B| + |L| + 1$

Decision variables: p_g

Transmission Constraints

$$\sum_{l \in F^+(i)} f_l - \sum_{l \in F^-(i)} f_l = n_i \quad \forall i \in B$$
$$f_l = B_l\left(\theta_{l(i)} - \theta_{l(j)}\right) \quad \forall l \in L$$
$$-F_l \le f_l \le F_l \quad \forall l \in L$$
$$\theta_{ref} = 0$$

$$\boldsymbol{n_B} = \boldsymbol{A}^T \cdot \boldsymbol{f_L}$$
$$\boldsymbol{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$\boldsymbol{f_L} = \boldsymbol{B_d} \cdot \boldsymbol{A} \cdot \boldsymbol{\theta_B}$$
$$-\boldsymbol{F_L} \le \boldsymbol{f_L} \le \boldsymbol{F_L}$$

Rewrite in matrix notation:

- $\boldsymbol{A}$ is the $|L| \times (|B| - 1)$ incidence matrix
  - $a_{l,i} = 1$ if line l starts at bus i
  - $a_{l,i} = -1$ if line l ends at bus i
  - Remove the column corresponding to $\theta_{ref} = 0$
- $\boldsymbol{B_d}$ is a $|L| \times |L|$ diagonal matrix with B_l on the diagonals
- $\boldsymbol{\theta_B}$ is the vector of θ_i variables, $i \neq ref$
- $\boldsymbol{f_L}$ is the vector of f_l variables
- $\boldsymbol{n_B}$ is the vector of n_i variables, $i \neq ref$

$$\boldsymbol{n_B} = \boldsymbol{A}^T \cdot \boldsymbol{B_d} \cdot \boldsymbol{A} \cdot \boldsymbol{\theta_B}$$
$$\boldsymbol{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$\boldsymbol{f_L} = \boldsymbol{B_d} \cdot \boldsymbol{A} \cdot \boldsymbol{\theta_B}$$
$$-\boldsymbol{F_L} \le \boldsymbol{f_L} \le \boldsymbol{F_L}$$

Calculating Flows from $\boldsymbol{n_B}$

$$\boldsymbol{n_B} = \boldsymbol{A^T} \cdot \boldsymbol{B_d} \cdot \boldsymbol{A} \cdot \boldsymbol{\theta_B}$$
$$\mathbf{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$\boldsymbol{f_L} = \boldsymbol{B_d} \cdot \boldsymbol{A} \cdot \boldsymbol{\theta_B}$$
$$-\boldsymbol{F_L} \leq \boldsymbol{f_L} \leq \boldsymbol{F_L}$$

$$\mathbf{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$-\boldsymbol{F_L} \leq \boldsymbol{PTDF^{L\times B}} \cdot \boldsymbol{n_B} \leq \boldsymbol{F_L}$$

With:

$$n_i = \sum_{g \in G(i)} p_g - L_i \quad \forall i \in B$$

Variables: $|B| + |G|$

Equalities: $|B| + 1$

Given $\boldsymbol{\hat{n}_B}$ with $\mathbf{1}^T \boldsymbol{n_B} + n_{ref} = 0$:

- Solve $\boldsymbol{\hat{n}_B} = (\boldsymbol{A^T B_d A}) \cdot \boldsymbol{\theta_B}$ -> $\boldsymbol{\hat{\theta}_B}$
- $\boldsymbol{\hat{f}_L}$ <- $(\boldsymbol{B_d A}) \cdot \boldsymbol{\hat{\theta}_B}$

How to put in algebraic model?

- $\boldsymbol{\theta_B} = \left(\boldsymbol{A^T B_d A}\right)^{-1} \cdot \boldsymbol{n_B}$
- $\boldsymbol{f_L} = \underbrace{(\boldsymbol{B_d A}) \cdot \left(\boldsymbol{A^T B_d A}\right)^{-1}}_{\boldsymbol{PTDF^{L\times B}}} \cdot \boldsymbol{n_B}$

See Van den Bergh et al. (2014) for details

Comparing Transmission Models

$$\boldsymbol{n_B} = (\boldsymbol{A}^T \boldsymbol{B_d} \boldsymbol{A}) \cdot \boldsymbol{\theta_B}$$
$$\boldsymbol{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$-\boldsymbol{F_L} \leq (\boldsymbol{B_d} \boldsymbol{A}) \cdot \boldsymbol{\theta_B} \leq \boldsymbol{F_L}$$

- Sparse if $\boldsymbol{A}$ is sparse
- $2|B| - 1$ variables
- $|B|$ equalities
- $|L|$ range constraints

- No computation to implement
- Solver must solve $\boldsymbol{n_B} = (\boldsymbol{A}^T \boldsymbol{B_d} \boldsymbol{A}) \cdot \boldsymbol{\theta_B}$ to calculate and enforce **any** line's flow

$$\boldsymbol{1}^T \boldsymbol{n_B} + n_{ref} = 0$$
$$-\boldsymbol{F_L} \leq \boldsymbol{PTDF}^{\boldsymbol{L \times B}} \cdot \boldsymbol{n_B} \leq \boldsymbol{F_L}$$

- Dense even if $\boldsymbol{A}$ is sparse
- $|B|$ variables
- 1 equality
- $|L|$ range constraints

- Need to calculate rows of $\boldsymbol{PTDF}^{\boldsymbol{L \times B}}$ for **active** lines
- Need a lazy constraint generation algorithm to be effective
- Roald & Molzahn (2019) show only a small subset (~1%) of these need to be enforced for a given load profile

Sparsity-Preserving Algorithm for PTDF-model (Egret Implementation)

- Factorize $\boldsymbol{A}^T\boldsymbol{B_d}\boldsymbol{A} = \mathbf{LU}$
- Initialize $L^A = \emptyset$; $viol \leftarrow True$
- While $viol$:
  - $\widehat{\boldsymbol{n}}_{\boldsymbol{B}} \leftarrow$ Solve PTDF-DCOPF with L^A
  - Check for violations by calculating:
    - $\widehat{\boldsymbol{\theta}}_{\boldsymbol{B}} \leftarrow \mathbf{LU}.\mathrm{solve}(\widehat{\boldsymbol{n}}_{\boldsymbol{B}})$
    - $\widehat{\boldsymbol{f}}_{\boldsymbol{L}} \leftarrow (\boldsymbol{B_d}\boldsymbol{A}) \cdot \widehat{\boldsymbol{\theta}}_{\boldsymbol{B}}$
  - $viol \leftarrow any(\widehat{\boldsymbol{f}}_{\boldsymbol{L}} > \boldsymbol{F_L}, \widehat{\boldsymbol{f}}_{\boldsymbol{L}} < -\boldsymbol{F_L})$
  - Update L^A by adding at least one violated line
  - $\boldsymbol{PTDF}^{l\times B} \leftarrow \mathbf{LU}.\mathrm{solve}((\boldsymbol{B_d}\boldsymbol{A})_l, \mathrm{'T'})$

PTDF-DCOPF

$$\min \sum_{g\in G} c^g(p^g)$$

$$\underline{P}_g \leq p_g \leq \overline{P}^g \quad \forall g \in G$$

$$n_i = \sum_{g\in G(i)} p_g - L_i \quad \forall i \in B$$

$$\mathbf{1}^T\boldsymbol{n_B} + n_{ref} = 0$$

$$-F_l \leq \boldsymbol{PTDF}^{l\times B} \cdot \boldsymbol{n_B} \leq F_l \quad \forall l \in L^A$$

Egret is available under a BSD license at https://github.com/grid-parity-exchange/Egret

Network Formulation has a big impact on Production Cost Model Runtimes

- Week-long simulation of the RTS-GMLC system using Prescient:
  - 73 buses
  - 120 branches
- XpressMP solver
- Solved Unit Commitment problems (7 total) to various MIP Gaps

MIP Gap	EGRET B-theta	EGRET Lazy PTDF	% Improvement
1.00%	252 s	213 s	15.4%
0.10%	311 s	231 s	25.8%
0.01%	552 s	257 s	53.5%
0.00%	621 s	336 s	45.9%

Contents

Conclusions

- Many challenges remain: UC is a stochastic mixed-integer nonlinear optimization problem which ISOs practically approximate:
  - Better models for AC power flow
  - Incorporating storage devices
  - Controllable loads / DERs
  - Virtual bidders
  - Uncertainty management
  - Many others …
- IEEE PES Task Force Report on UC:
  - IEEE Task Force on Solving Large Scale Optimization Problems in Electricity Market and Power System Applications: Security-Constrained Unit Commitment for Electricity Market: Modeling, Solution Methods, and Future Challenges. IEEE PES-TR96.
  - Journal version: Chen et al. (2022)

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Q&A

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