A Transactive Approach for Service Restoration Utilizing Customer Load Flexibility and Grid-Edge Resources

Preprint

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Presented at the 2024 IEEE Power and Energy Society General Meeting
Seattle, Washington
July 21–25, 2024

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Contract No. DE-AC36-08GO28308

Conference Paper
NREL/CP-5D00-87975
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Suggested Citation
Dugan, Jesse, Kumar Utkarsh, and Fei Ding. 2024. *A Transactive Approach for Service Restoration Utilizing Customer Load Flexibility and Grid-Edge Resources: Preprint*. Golden, CO: National Renewable Energy Laboratory. NREL/CP-5D00-87975. https://www.nrel.gov/docs/fy24osti/87975.pdf.

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A Transactive Approach for Service Restoration Utilizing Customer Load Flexibility and Grid-Edge Resources

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***Abstract*—This paper develops a transactive energy system model to restore electricity to customers in an isolated distribution system after an outage. The model engages a variety of customer types – prosumers, flexible loads, critical/noncritical customers, and distributed generators – as active participants in the restoration process. Unlike many existing transactive approaches, the proposed model is developed for service restoration and accounts for various customer types and their *autonomy* and *privacy* through an iterative approach to determine the optimal market price, while maintaining system-level power flow and voltage constraints. The advantages of the proposed approach are numerically validated on a modified IEEE 123-bus test system.**

***Index Terms*—distributed energy resources, distribution grid, service restoration, resilience, transactive energy system.**

I. Introduction

Power distribution system restoration has traditionally been carried out by utility field crews who manually repair the affected grid area over several days or weeks. As an alternative, utilities have begun to adopt autonomous approaches – such as fault-location, isolation, and service restoration – that enable the distribution system to detect and recover from faults or disturbances without human intervention; however, most utilities currently do not account for the flexibility and generation capability provided by customers with flexible loads or behind-the-meter (BTM) generation, such as rooftop photovoltaic systems and/or batteries. Such distributed energy resources (DERs) offer an opportunity to enhance distribution system flexibility and improve service restoration in the outage-affected areas, especially isolated circuits.

Direct load control, which can be used to take advantage of customer flexibility and DERs, poses privacy and autonomy concerns; therefore, transactive energy systems have been proposed as alternatives to incentivize DER and prosumer participation in the distribution system through price signals and market mechanisms. Although transactive energy systems have been well studied under normal operations [1], their use during outage conditions is a topic of emerging research [2].

Market mechanisms to engage BTM resources for service restoration that maintain customer autonomy and privacy while considering system-level power flow constraints are limited in the literature. Double-auction market mechanisms have been used to incentivize DER reactive power support for additional switching operations [3] and voltage support [5]- [6] during restoration. Ref. [7] proposes a transactive energy rationing mechanism in conjunction with a double-auction market for an islanded power system. Ref. [4] introduces a two-stage resource commitment and service recovery transactive market mechanism to engage black-start resources as well as critical, flexible, and nonflexible loads for active power restoration during a contingency. These prior works, however, have overlooked prosumers as a market resource to enhance distribution system resilience, and they do not consider customer privacy, assuming that all supply and demand curves are known to the market operator. Further, [4] and [7] overlook power flow and voltage constraints, which are necessary for a realistic distribution system restoration solution.

To fill these gaps, this paper develops a transactive energy system model to engage a variety of customer types, including prosumers, distributed generators (DGs), and critical, noncritical, and flexible customers to aid service restoration in an isolated section of a distribution system. The proposed model maintains customer privacy through an iterative market approach where the market operator cannot access private customer behavior (modeled as customer utility functions); and the customers are assumed to act rationally but with autonomy, i.e., they choose their consumption/generation based on their own perceived benefits and price signals from the market operator, and the anticipated power at the customer node is the only information exchanged in the model.

The rest of this paper is organized as follows. Section II describes the methodology, including the customer utility functions and surplus models, the solution algorithm, the resource-level optimization formulation, and the power flow and voltage constraints. Section III presents and discusses the simulation results for a case study on a modified 123-bus test system. Finally, Section IV presents concluding remarks.

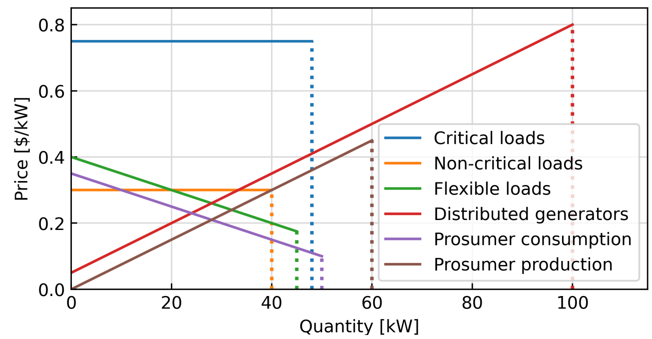

Fig. 1: Example utility functions for different customer types.

II. Methodology

The market operator's objective is to maximize social welfare subject to power flow and voltage constraints; however, this cannot be directly evaluated due to private customer information. The objective of each customer is to maximize their benefit, but those decisions might conflict with the overall system needs; therefore, the following approach is introduced to determine the optimal market price that maximizes social welfare while preserving customer privacy while adhering to network power flow and voltage constraints. We assume that each participating customer is equipped with a home energy management system (HEMS), which has been pre-programmed by the customer for their utility function and can communicate with the market operator. The proposed approach is executed once during each market interval (e.g., every 5 minutes), and the customers' HEMS will not execute their power set points until the market converges for that interval.

A. Customer utility functions

Linear utility functions are used to represent the marginal benefit that a customer can gain from buying (selling) energy from (to) the market. The line intercepts represent the start-up cost of production or the maximum value of the restored load for consumption, and the slopes of the lines represent the marginal cost (benefit) of energy production (consumption). Per-phase utility functions for each customer type are defined as follows, and example utility functions are depicted in Fig 1.

1) Critical and noncritical customers: Critical and noncritical loads are considered to have no flexibility. Eq. (1) describes the utility function for critical loads, where $A^c_{i\varphi}$ is the maximum value of the restored load, $\overline{P^c_{i\varphi}}$ is the maximum demand, and $p^c_{i\varphi}$ is the power set point for load i and phase φ. Similarly, Eq. (2) describes the utility function for noncritical loads. We assume that $A^c_{i\varphi} \gg A^n_{i\varphi}$ because critical loads have a much higher value of restored load than noncritical loads.

$$u(p^c_{i\varphi}) = A^c_{i\varphi} \text{ if } p^c_{i\varphi} \leqslant \overline{P}^c_{i\varphi};\ 0 \text{ otherwise} \tag{1}$$

$$u(p^n_{i\varphi}) = A^n_{i\varphi} \text{ if } p^n_{i\varphi} \leqslant \overline{P}^n_{i\varphi};\ 0 \text{ otherwise} \tag{2}$$

2) Flexible consumers: Eq. (3) describes the utility function for flexible consumers, where $A^{f1}_{i\varphi}$ is the marginal (decreasing) benefit of the energy consumption, $A^{f2}_{i\varphi}$ is the maximum value of the restored load, $\overline{P^f_{i\varphi}}$ is the maximum demand, and $p^f_{i\varphi}$ is the power set point for load i and phase φ.

$$u(p^f_{i\varphi}) = -A^{f1}_{i\varphi}p^f_{i\varphi} + A^{f2}_{i\varphi} \text{ if } p^f_{i\varphi} \leqslant \overline{P}^f_{i\varphi};\ 0 \text{ otherwise} \tag{3}$$

3) Distributed generators: Eq. (4) describes the utility function for DGs, where $A^{g1}_{i\varphi}$ is the marginal (increasing) cost of energy production, $A^{g2}_{i\varphi}$ is the minimum start-up cost, $\overline{P^g_{i\varphi}}$ is the maximum generation, and $p^g_{i\varphi}$ is the power set point for load i and phase φ.

$$u(p^g_{i\varphi}) = A^{g1}_{i\varphi}p^g_{i\varphi} + A^{g2}_{i\varphi} \text{ if } p^g_{i\varphi} \leqslant \overline{P}^g_{i\varphi};\ 0 \text{ otherwise} \tag{4}$$

4) Prosumers: Prosumers can operate as producers and/or consumers in a single market interval. Eq. (5) describes the utility function for the prosumer energy production, where $A^{p1}_{i\varphi}$ is the marginal (increasing) cost of energy production, $A^{p2}_{i\varphi}$ is the minimum start-up cost, $\overline{P^{pg}_{i\varphi}}$ is the maximum generation, and $p^p_{i\varphi}$ is the power set point for load i and phase φ. Eq. (6) describes the utility function for the prosumer energy consumption, where $A^{p3}_{i\varphi}$ is the marginal (decreasing) benefit of energy production, $A^{p4}_{i\varphi}$ is the maximum value of the restored load, $\overline{P^{pd}_{i\varphi}}$ is the maximum demand, and $p^m_{i\varphi}$ is the power set point for load i and phase φ.

$$u(p^p_{i\varphi}) = A^{p1}_{i\varphi}p^p_{i\varphi} + A^{p2}_{i\varphi} \text{ if } p^p_{i\varphi} \leqslant \overline{P}^{pg}_{i\varphi};\ 0 \text{ otherwise} \tag{5}$$

$$u(p^m_{i\varphi}) = -A^{p3}_{i\varphi}p^m_{i\varphi} + A^{p4}_{i\varphi} \text{ if } p^m_{i\varphi} \leqslant \overline{P}^{pd}_{i\varphi};\ 0 \text{ otherwise} \tag{6}$$

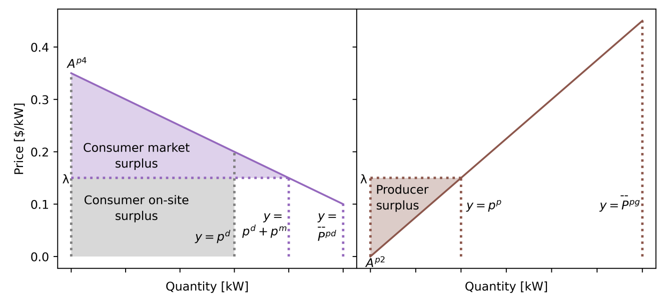

Fig. 2: Three types of prosumer surplus, including the consumer market, consumer on-site, and producer surplus.

B. Customer surpluses

The *customer surplus*, or the total benefit from buying and/or selling energy, is defined as the definite integral of the customer's utility function from 0 to the power set point [8]. Consumer surplus comes from buying energy at a lower marginal price than what they were willing to pay for, while the producer surplus comes from selling energy at a higher marginal price than their production cost. A prosumer can operate as both a producer and a consumer, receiving surplus from one or both modes of operation, as shown in Fig. 2.

C. Solution algorithm

The primal-dual algorithm [1] is used to compute the competitive equilibrium, as described in Algorithm 1. It is a subgradient algorithm that has been shown to converge to an optimal solution of the power set points and price pair; however, it is not strategy-proof, and customers might benefit from providing false information [1]. After an initial guess of the market price, the market operator broadcasts the price to the customers, who then determine their optimal set points at that price and send the decisions back to the operator. The operator then checks for power flow and voltage violations and updates the market price as necessary. When the price change between two consecutive iterations gets small enough, the market has converged to the optimal price.

Algorithm 1 Algorithm for transactive energy market

Input: Customer utility functions and resource limits ($A^{c,n,f,g,p,d,m}_{i\varphi}, \overline{P}^{c,n,f,g,p,d}_{i\varphi}, \overline{S}^{g,p}_{i\varphi}$)
Output: Optimal market price, $\overline{\lambda}$

```
Initialize market price, λ_k = λ_0, done = False
while not done do
    Solve resource-level optimizations to maximize customer surpluses at price λ_k (Eq. (7)–(21)).
    Check for power flow, voltage violations (Eq. (22)–(32)).
    Calculate the updated price (λ_{k+1}) based on Eq. (33).
    if (|λ_{k+1} − λ_k|/λ_k ⩽ 0.00001) then
        λ̄ = λ_k, done = True
    else
        update market price, λ_k = λ_{k+1}
    end if
end while
return λ̄
```

D. Resource-level optimization

The objective function for the resource-level optimization is to maximize the customer surplus at the given price, subject to BTM resource limits and electricity demand limits. The formulation for each customer type is as follows.

1) Critical and noncritical customers: If the market price is low enough (i.e., $\lambda \leqslant A^c_{i\varphi}, A^n_{i\varphi}$), these customers will consume their entire demand ($\overline{P}^c_{i\varphi}, \overline{P}^n_{i\varphi}$).

2) Flexible consumers: These customers will alter their consumption by evaluating their optimal power set point ($p^{f*}_{i\varphi}$) based on the market price (λ) and their utility function, up to a maximum demand value ($\overline{P}^f_{i\varphi}$).

$$p^{f*}_{i\varphi}(\lambda) = \arg\max \frac{1}{2} p^f_{i\varphi}(A^{f2}_{i\varphi} - \lambda) \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{7}$$

$$\text{s.t.} \quad p^f_{i\varphi} \leqslant \beta^f_{i\varphi}(\lambda - A^{f2}_{i\varphi})/A^{f1}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{8}$$

$$0 \leqslant p^f_{i\varphi} \leqslant \overline{P}^f_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{9}$$

$$\beta^f_{i\varphi} = 1 \text{ if } \lambda \leqslant A^{f2}_{i\varphi};\ 0 \text{ otherwise} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{10}$$

3) Distributed generators: The DGs determine their optimal power output ($p^{g*}_{i\varphi}$) based on the market price and their utility functions to maximize their producer surplus subject to generation limits ($\overline{P}^g_{i\varphi}$).

$$p^{g*}_{i\varphi}(\lambda) = \arg\max \frac{1}{2} p^g_{i\varphi}(\lambda - A^{g2}_{i\varphi}) \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{11}$$

$$\text{s.t.} \quad p^g_{i\varphi} \leqslant \beta^g_{i\varphi}(\lambda - A^{g2}_{i\varphi})/A^{g1}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{12}$$

$$0 \leqslant p^g_{i\varphi} \leqslant \overline{P}^g_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{13}$$

$$\beta^g_{i\varphi} = 1 \text{ if } \lambda > A^{g2}_{i\varphi};\ 0 \text{ otherwise} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{14}$$

4) Prosumers: Because prosumers might have BTM DERs as well as flexible loads, they will determine the optimal values of energy bought from the market ($p^{m*}_{i\varphi}$), BTM generation consumed on-site ($p^{d*}_{i\varphi}$), and BTM generation sold to the market ($p^{p*}_{i\varphi}$) to maximize the sum of their consumer surplus and their producer surplus. Total consumption ($p^d_{i\varphi} + p^m_{i\varphi}$) will not exceed total demand ($\overline{P}^{pd}_{i\varphi}$), and the total production ($p^d_{i\varphi} + p^p_{i\varphi}$) will not exceed the maximum generation limits ($\overline{P}^{pg}_{i\varphi}$).

$$p^{p*}_{i\varphi}(\lambda), p^{d*}_{i\varphi}(\lambda), p^{m*}_{i\varphi}(\lambda) = \arg\max \frac{1}{2} p^p_{i\varphi}(\lambda - A^{p2}_{i\varphi}) + \frac{1}{2}(p^d_{i\varphi} + p^m_{i\varphi})(A^{p4}_{i\varphi} - \lambda) + \lambda p^d_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{15}$$

$$\text{s.t.} \quad 0 \leqslant p^p_{i\varphi} \leqslant \beta^p_{i\varphi}(\lambda - A^{p2}_{i\varphi})/A^{p1}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{16}$$

$$0 \leqslant p^p_{i\varphi} + p^d_{i\varphi} \leqslant \overline{P}^{pg}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{17}$$

$$0 \leqslant p^d_{i\varphi} + p^m_{i\varphi} \leqslant \beta^d_{i\varphi}(\lambda - A^{p4}_{i\varphi})/A^{p3}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{18}$$

$$0 \leqslant p^d_{i\varphi} + p^m_{i\varphi} \leqslant \overline{P}^{pd}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{19}$$

$$\beta^{pg}_{i\varphi} = 1 \text{ if } \lambda > A^{p2}_{i\varphi};\ 0 \text{ otherwise} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{20}$$

$$\beta^{pd}_{i\varphi} = 1 \text{ if } \lambda \leqslant A^{p4}_{i\varphi};\ 0 \text{ otherwise} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{21}$$

E. Power flow and voltage constraints

The optimal power set points are sent to the operator, who checks for power flow and voltage violations. The power flow constraints (23)-(30) are based on a linearized, three-phase, unbalanced branch flow model [9]. Slack variables ($p^{s+}_{i\varphi}, p^{s-}_{i\varphi}, q^{s+}_{i\varphi}, q^{s-}_{i\varphi}, v^{s+}_{i\varphi}$ and $v^{s-}_{i\varphi}$) are introduced into the power flow constraints at the slack bus, and the voltage constraints across the system are minimized in the objective function.

$$\min \sum_{i \in \mathcal{N}^r} \sum_{\varphi \in \Phi} p^{s+}_{i\varphi} + p^{s-}_{i\varphi} + q^{s+}_{i\varphi} + q^{s-}_{i\varphi} + \sum_{i \in \mathcal{N}} \sum_{\varphi \in \Phi} v^{s+}_{i\varphi} + v^{s-}_{i\varphi} \tag{22}$$

$$\text{s.t.} \quad p_{ij\varphi} + p^{g*}_{j\varphi} + p^{p*}_{j\varphi} - p^{c*}_{j\varphi} - p^{n*}_{j\varphi} - p^{f*}_{j\varphi} - p^{m*}_{j\varphi} = \sum_{k:(j,k) \in \mathcal{L}} p_{jk\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{23}$$

$$p^{g*}_{i\varphi} + p^{p*}_{i\varphi} - p^{c*}_{i\varphi} - p^{n*}_{i\varphi} - p^{f*}_{i\varphi} - p^{m*}_{i\varphi} + p^{s+}_{i\varphi} - p^{s-}_{i\varphi} = \sum_{j:(i,j) \in \mathcal{L}} p_{ij\varphi} \quad \forall i \in \mathcal{N}^r, \varphi \in \Phi \tag{24}$$

$$q_{ij\varphi} + q^g_{j\varphi} + q^p_{j\varphi} - q^c_{j\varphi} - q^n_{j\varphi} - q^f_{j\varphi} - q^m_{j\varphi} = \sum_{k:(j,k) \in \mathcal{L}} q_{jk\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{25}$$

$$q^g_{i\varphi} + q^p_{i\varphi} - q^c_{i\varphi} - q^n_{i\varphi} - q^f_{i\varphi} - q^m_{i\varphi} + q^{s+}_{i\varphi} - q^{s-}_{i\varphi} = \sum_{j:(i,j) \in \mathcal{L}} p_{ij\varphi} \quad \forall i \in \mathcal{N}^r, \varphi \in \Phi \tag{26}$$

$$q^c_{i\varphi} = \alpha_{i\varphi} p^{c*}_{i\varphi},\ q^n_{i\varphi} = \alpha_{i\varphi} p^{n*}_{i\varphi},\ q^f_{i\varphi} = \alpha_{i\varphi} p^{f*}_{i\varphi}, \quad q^m_{i\varphi} = \alpha_{i\varphi} p^{m*}_{i\varphi}, q^m_{i\varphi} = \alpha_{i\varphi} p^{d*}_{i\varphi} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{27}$$

$$-\sqrt{\overline{S}^{p^2}_{i\varphi} - p^{p*2}_{i\varphi}} \leqslant q^p_{i\varphi} + q^d_{i\varphi} \leqslant \sqrt{\overline{S}^{p^2}_{i\varphi} - p^{p*2}_{i\varphi}} \forall i \in \mathcal{N}, \varphi \in \Phi \tag{28}$$

$$-\sqrt{\overline{S}_{i\varphi}^{g^2} - p_{i\varphi}^{g*^2}} \leqslant q_{i\varphi}^{g} \leqslant \sqrt{\overline{S}_{i\varphi}^{g^2} - p_{i\varphi}^{g*^2}} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{29}$$

$$\begin{aligned} v_{it\varphi} &= v_{jt\varphi} + 2(R_{ij\varphi\varphi}p_{ijt\varphi} + X_{ij\varphi\varphi}q_{ijt\varphi}) - \\ &(R_{ij\varphi\varphi'}p_{ijt\varphi'} + R_{ij\varphi\varphi''}p_{ijt\varphi''} + X_{ij\varphi\varphi'}q_{ijt\varphi'} + \\ &X_{ij\varphi\varphi''}q_{ijt\varphi''}) + \sqrt{3}(X_{ij\varphi\varphi'}p_{ijt\varphi'} - X_{ij\varphi\varphi''}p_{ijt\varphi''} - \\ &R_{ij\varphi\varphi'}q_{ijt\varphi'} + R_{ij\varphi\varphi''}q_{ijt\varphi''}) \ \forall (i,j) \in \mathcal{L}, \varphi \in \Phi \end{aligned} \tag{30}$$

$$\check{V} - v_{i\varphi}^{s-} \leqslant v_{i\varphi} \leqslant \hat{V} + v_{i\varphi}^{s+} \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{31}$$

$$p_{i\varphi}^{s+}, p_{i\varphi}^{s-}, q_{i\varphi}^{s+}, q_{i\varphi}^{s-}, v_{i\varphi}^{s+}, v_{i\varphi}^{s-} \geqslant 0 \quad \forall i \in \mathcal{N}, \varphi \in \Phi \tag{32}$$

Constraints (23) and (25) balance the per-phase active ($p_{ij\varphi}$) and reactive ($q_{ij\varphi}$) power flows along line (i,j) into bus j, the power injections at bus j, and the power flows out of bus j along line (j,k). These constraints are relaxed at the slack bus ($\mathcal{N}^r \in \mathcal{N}$) through the slack variables $p_{i\varphi}^{s+}, p_{i\varphi}^{s-}, q_{i\varphi}^{s+}$, and $q_{i\varphi}^{s-}$, which accommodate the power imbalance that might occur before the algorithm has converged. The variables $p_{i\varphi}^{s+}$ and $q_{i\varphi}^{s+}$ represent excess demand, and $p_{i\varphi}^{s-}$ and $q_{i\varphi}^{s-}$ represent excess supply. We assume that consumers will maintain a constant per-phase power factor $\theta_{i\varphi}$ and that the reactive power set points are therefore related to the active power set points through $\alpha_{i\varphi} = \tan(\theta_{i\varphi})$ in Constraint (27). We also assume that the reactive power injections of the DGs and prosumers can be remotely controlled by the operator and are allowed to vary between the limits defined by Constraints (28) and (29). Constraint (30) defines the squared magnitude of the voltage ($v_{it\varphi}$) at bus i for phase φ based on the voltage at an adjacent, downstream bus ($v_{j\varphi}$), the per-phase active and reactive power flows from bus i to bus j, and the resistance ($R_{ij\varphi\varphi'}$) and reactance ($X_{ij\varphi\varphi'}$) of line (i,j) between phase φ and φ'. For phase φ, φ' and φ'', denote the phase that lags and leads φ by 120 degrees, respectively (e.g., for $\varphi = a, \varphi' = b$, and $\varphi'' = c$). Finally, Constraint (31) define the voltage limits and corresponding voltage slack variables ($v_{i\varphi}^{s+}$ and $v_{i\varphi}^{s-}$), and Constraint (32) enforces non-negativity.

F. Price update equation

The market price (λ_{k+1}) for the next iteration ($k+1$) is based on the current market price (λ_k), the step size (γ), and the slack variables. The price increases if the anticipated active power demand is greater than the anticipated supply or if there is undervoltage in the system (attributed to an overloaded system). Likewise, the price decreases when the anticipated supply is greater than the anticipated demand or if there is overvoltage in the system.

$$\lambda_{k+1} = \lambda_k + \gamma \sum_{i \in \mathcal{N}} \sum_{\varphi \in \Phi} (p_{i\varphi}^{s+} - p_{i\varphi}^{s-} + v_{i\varphi}^{s-} - v_{i\varphi}^{s+}) \tag{33}$$

III. Simulation Results and Discussion

The proposed transactive model is implemented on a modified IEEE 123-bus test system [10], and a selection of 27 buses and their respective loads are assigned at random to each customer class of critical, noncritical, flexible loads, and prosumers. Similarly, 16 DGs, each with a capacity of 100 kW, are allotted to buses in the distribution system at random.

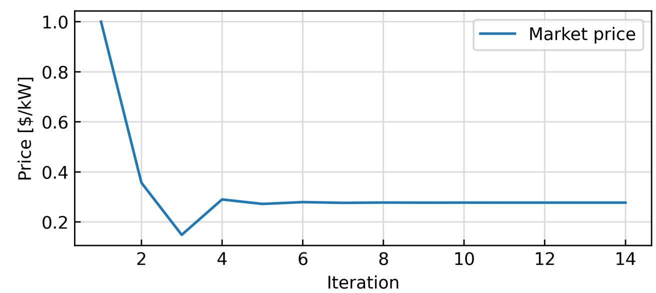

Fig. 3: Market price at each iteration.

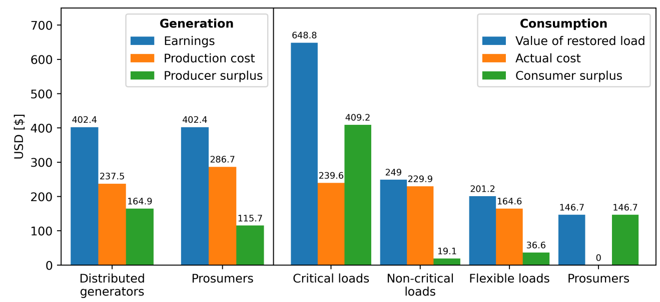

Fig. 4: Cost plots for each customer type at the optimal price.

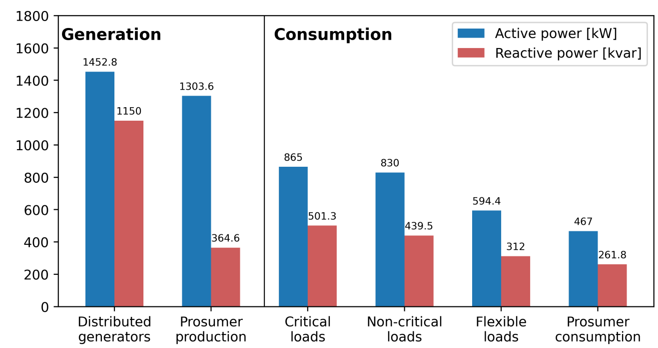

Fig. 5: Active and reactive power generation and consumption by customer type at the optimal price.

An outage scenario is simulated where the entire system is disconnected from the distribution substation, thus forming an isolated network. As such, DGs and prosumers must be incentivized to restore some or all the customer loads. The simulation results that follow are for a single 5-minute market interval. The algorithm is implemented in Python, and the optimization programs are modeled in Pyomo [11] and solved using GLPK [12] with the default solver settings.

The slope and intersect for the utility functions are shown in Fig. 1. We assume that any customers within the same class have the same utility function slope and intercept but that they might differ based on the total demand. Note that these values are for simulation purposes only and do not represent an accurate value of the restored load.

As shown in Fig. 3, the simulation converged from an initial price of 1.0 \$/kW to an optimal price of 0.277 \$/kW in 13 iterations and 12.7 seconds. The results of the numerical simulations at the optimal market price of 0.277 \$/kW are displayed in Figs. 4–7. Fig. 4 shows the total dollars earned by the DGs and prosumers, the total actual cost of producing

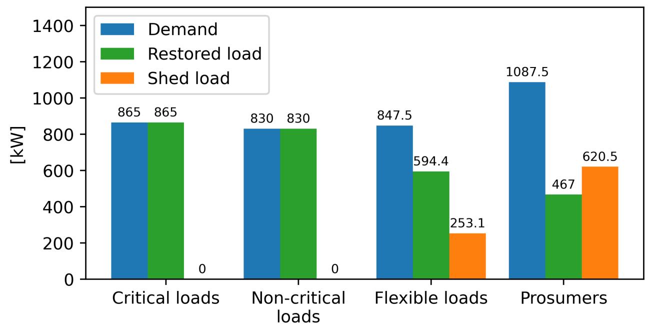

Fig. 6: Active power demand, restored load, and shed load.

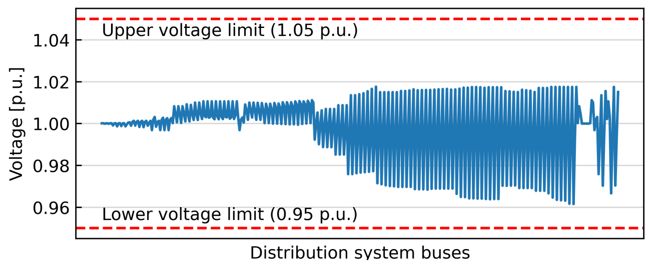

Fig. 7: System voltages at the optimal market price.

the power, and the total net profit. Likewise, it shows the total maximum value of the restored load for consumers along with how much they actually paid and the net monetary benefit achieved. The sum of the green columns represents the total customer surplus and the overall social welfare achieved, which is $892.2. Fig. 5 shows the final cleared quantities of the DG and prosumer production along with the total active (2756.4 kW) and reactive power (1514.6 kvar) load consumption for each consumer type.

Customer load restoration is shown in Fig. 6, which presents the active power demand, the restored load, and the shed load for each consumer type. Due to the market design and their lack of flexibility, critical and noncritical loads will all be fully restored or all be fully shed. In this case, the overall social welfare is highest when both load types are fully restored. Conversely, the flexible consumers and the prosumers adjust their demand to maximize their individual surpluses, resulting in significant shed load for both customer types. Finally, Fig. 7 reports the voltage profile across the system at the optimal market price, and it can be observed that there are no voltage violations beyond the constrained bounds of $\pm 5\%$.

IV. Conclusions

This paper presents a transactive model for distribution system service restoration following an outage and preserves customer privacy and autonomy while considering system power flow constraints. The performance of the proposed model is demonstrated on a modified IEEE-123 bus test system considering a variety of customer types. The simulation results provide a proof of concept for a market mechanism to leverage grid-edge flexibility and DERs that would not otherwise participate in service restoration, thereby helping to improve the overall system resilience by providing much needed electricity to critical and residential customers in isolated networks following an outage; however, this, and any economic approach for improved resilience, relies on accurate utility functions to represent the value of the restored load for different customer classes. Because outages can result in nonmonetary and indirect costs [13], it is imperative to ensure that these utility functions are accurate and that the resulting transactive energy system are equitable.

V. Acknowledgments

This work was authored in part by the National Renewable Energy Laboratory, operated by Alliance for Sustainable Energy, LLC, for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. Funding provided by U.S Department of Energy Office of Electricity under Agreement #38428. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

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