Network-Aware and Welfare-Maximizing Dynamic Pricing for Energy Sharing

Preprint

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Presented at the Conference on Decision and Control (CDC 2024)
Milan, Italy
December 16-19, 2024

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Contract No. DE-AC36-08GO28308

Conference Paper
NREL/CP-5D00-90821
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Suggested Citation
Alahmed Ahmed S., Cavraro Guido, Andrey Bernstein, and Lang Tong. 2024. *Network-Aware and Welfare-Maximizing Dynamic Pricing for Energy Sharing: Preprint.* Golden, CO: National Renewable Energy Laboratory. NREL/CP-5D00-90821. https://www.nrel.gov/docs/fy25osti/90821.pdf.

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Network-Aware and Welfare-Maximizing Dynamic Pricing for Energy Sharing

Ahmed S. Alahmed, Guido Cavraro, Andrey Bernstein, and Lang Tong

***Abstract*— The proliferation of behind-the-meter (BTM) distributed energy resources (DER) within the electrical distribution network presents significant supply and demand flexibilities, but also introduces operational challenges such as voltage spikes and reverse power flows. In response, this paper proposes a network-aware dynamic pricing framework tailored for energy-sharing coalitions that aggregate small, but ubiquitous, BTM DER downstream of a distribution system operator's (DSO) revenue meter that adopts a generic net energy metering (NEM) tariff. By formulating a Stackelberg game between the energy-sharing market leader and its prosumers, we show that the dynamic pricing policy induces the prosumers toward a network-safe operation and decentrally maximizes the energy-sharing social welfare. The dynamic pricing mechanism involves a combination of a locational *ex-ante* dynamic price and an *ex-post* allocation, both of which are functions of the energy sharing's BTM DER. The *ex-post* allocation is proportionate to the price differential between the DSO NEM price and the energy sharing locational price. Simulation results using real DER data and the IEEE 13-bus test systems illustrate the dynamic nature of network-aware pricing at each bus, and its impact on voltage.**

I. Introduction

WHILE the small, but ubiquitous, BTM DER are primarily adopted to provide prosumer services such as bill savings and backup power, they can also be leveraged, under proper consumer-centric mechanism design, to provide various grid services such as voltage control, system support during contingencies, and new capacity deferral [2]. Harnessing the flexibility of BTM DER participation in grid services is usually challenged by the DSO's lack of visibility and controllability on BTM DER alongside the absence of network-aware pricing mechanisms that can induce favorable prosumer behaviors.

The rising notion of energy sharing of a group of prosumers under the DSO's tariff presents a compelling solution to optimize DER utilization, mitigate grid constraints, and promote renewable energy integration. A major barrier facing the practical implementation of energy-sharing markets is the incorporation of distribution network constraints into the energy-sharing pricing mechanism, and aligning the objectives of the self-interested energy-sharing prosumers with the global objective of maximizing the coalition's welfare.

Despite the voluminous literature on energy-sharing systems' DER control and energy pricing, network constraints are rarely considered due to the theoretical complexity they introduce. A short list of recent works on energy communities and energy sharing that neglected network constraints can be found here [3], [4], [5], [6], [7]. Some works considered a coarse notion of network constraints by incorporating operating envelopes (OEs) at the point of common coupling between the energy sharing system and the DSO [8], [9] that limit the export and imports between the two entities. At best, some works consider OEs at the prosumer's level [10], [11]. Few papers considered network-aware pricing mechanisms in distribution networks, such as [12], [13], [14] and the line of literature on distribution locational marginal prices (dLMP), e.g., [15], [16]. Our work differs from the existing literature in two important directions. Firstly, we consider network-aware pricing under a generic DSO NEM tariff constraint that charges the energy-sharing platform different prices based on its aggregate net consumption. Secondly, the dynamic network-aware pricing of a platform that is subject to the DSO's fixed and exogenous NEM price gives rise to a market manager's profit/deficit that needs to be re-allocated to the coalition members. We shed light on a unique re-allocation rule that makes the prosumers' payment functions uniform, even if they are located on different buses and the network constraints are binding. Such a re-allocation rule is highly relevant when charging end-users, as it avoids 'undue discrimination', which is one of the key principles of rate design outlined by Bonbright [17].

In this paper, we present a network-aware and welfare-maximizing pricing policy for energy-sharing coalitions that aggregate BTM DERs downstream of a DSO's revenue meter that charges the energy-sharing platform based on a generic NEM tariff. The pricing policy announces an *ex-ante* locational, threshold-based, and dynamic price to induce a collective prosumer response that decentrally maximizes the social welfare, while abiding by the network voltage constraints. An *ex-post* charge/reward is then used to ensure the market operator's profit neutrality. If the network constraints are nonbinding, the *ex-post* charge component vanishes. We show that the market mechanism achieves an equilibrium to the Stackelberg game between the energy-sharing market operator and its prosumers. Although network constraints couple the decisions of the energy-sharing prosumers, which give rise to locational marginal prices (LMP), we show that by adopting a unique proportional re-allocation rule, the payment function becomes uniform for all prosumers, even if they are located at different buses in the energy-sharing network. Numerical simulations using the IEEE 13-bus test feeder and real BTM DER data shed more light on how the

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An online version of this paper containing remaining proofs is available in [1].

pricing policy influences prosumers' response to ensure safe network operation.

This paper extends our previous work on Dyanmic NEM (D-NEM) without OEs [4] and with OEs [11] by incorporating network constraints, which add substantial theoretical complexity, primarily due to coupling the DER decisions across network buses.

For the rest of the paper, when necessary, boldface letters denote column vectors, as in $\boldsymbol{x} = (x_1, \ldots, x_n)$. $\mathbf{1}$ and $\mathbf{0}$ are column vectors of all ones and zeros, respectively. $\boldsymbol{x}^\top$ represents the transpose of the vector $\boldsymbol{x}$. For a multi-variate function f of $\boldsymbol{x}$, we interchangeably use $f(\boldsymbol{x})$ and $f(x_1, \ldots, x_n)$. For vectors $\boldsymbol{x}, \boldsymbol{y}$, the element-wise inequality is $\boldsymbol{x} \preceq \boldsymbol{y}$. $x_i \leq y_i$ for every i, and $[\boldsymbol{x}]^+, [\boldsymbol{x}]^-$ represents the positive and negative elements of the vector $\boldsymbol{x}$, i.e. $[x_i]^+ = \max\{0, x_i\}$, $[x_i]^- = -\min\{0, x_i\}$ for all i, and $\boldsymbol{x} = [\boldsymbol{x}]^+ - [\boldsymbol{x}]^-$. To be concise, we use the notation $[x]_{\underline{x}}^{\overline{x}} = \max\{\underline{x}, \min\{x, \overline{x}\}$ Also, $[x]_{\underline{x}}^{\overline{x}}$ represents the projection of x into the closed and convex set $[\underline{x}, \overline{x}]$. Using the rule $[x]_{\underline{x}}^{\overline{x}} := \max\{\underline{x}, \min\{x, \overline{x}\}\}$. This notation is also used for vectors, *i.e.,* $[\boldsymbol{x}]_{\underline{\boldsymbol{x}}}^{\overline{\boldsymbol{x}}}$. Lastly, we denote by $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$ the set of non-negative real numbers.

II. Proposed Framework and Network Model

We consider the problem of designing a welfare-maximizing and network-aware pricing policy for an *energy sharing* system that bidirectionally transacts energy and money with the DSO under a general NEM tariff. Under NEM, the energy sharing platform, whose members may be a mixture of *consumers* and *prosumers*, imports from the DSO at the *import* rate $\pi^+ > 0$ if its aggregate consumption is higher than its aggregate generation, and collectively exports from the DSO at the *export* rate $\pi^- > 0$ if the aggregate generation exceeds the aggregate consumption needs. A budget-balanced market operator is responsible for announcing the market's pricing policy and administering its transaction with the DSO. The market operator uses spatially varying pricing signals to adhere to its network's operational constraints communicated by the DSO.[1]

A radial low voltage distribution network flow model is used to model the network power flow [18], [19]. Consider a radial distribution network described by $\mathcal{G} = (\mathcal{B}, \mathcal{L})$, with $\mathcal{B} = \{1, \ldots, B\}$ as the set of *energy sharing* buses, excluding bus 0, and $\mathcal{L} = \{(i,j)\} \subset \mathcal{B} \times \mathcal{B}$ as the set of distribution lines between the buses, with i, j as bus indices. The root bus 0 represents the secondary of the distribution transformer and is referred to as the slack bus (substation bus). The natural radial network orientation is considered, with each distribution line pointing away from bus 0.

For each bus $i \in \mathcal{B}$, denote by $\mathcal{L}_i \subseteq \mathcal{L}$ the set of lines on the unique path from bus 0 to bus i, and by Z_i, q_i the active and reactive power consumptions of bus i, respectively. The magnitude of the complex voltage at bus i is denoted by v_i, and we denote the fixed and known voltage at the slack bus by v_0. For each line $(i,j) \in \mathcal{L}$, denote by r_{ij} and x_{ij} its resistance and reactance. For each line, $(i,j) \in \mathcal{L}$, denote by P_{ij} and Q_{ij} the real and reactive power from bus i to bus j, respectively. Let ℓ_{ij} denote the squared magnitude of the complex branch current from bus i to bus j.

We adopt the distribution flow (DistFlow) model, introduced in [18], to model steady state power flow in a radial distribution network, as

$$P_{ij} = -Z_j + \sum_{k:(j,k)\in\mathcal{L}} P_{jk} + r_{ij}\ell_{ij} \tag{1a}$$

$$Q_{ij} = -q_j + \sum_{k:(j,k)\in\mathcal{L}} Q_{jk} + x_{ij}\ell_{ij} \tag{1b}$$

$$v_j^2 = v_i^2 - 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right) + \left(r_{ij}^2 + x_{ij}^2\right)\ell_{ij}, \tag{1c}$$

where $\ell_{ij} = (P_{ij}^2 + Q_{ij}^2)/v_i^2$ is the line losses, (1a)-(1b) are the active and reactive power balance equations, and (1c) is the voltage drop. We exploit a linear approximation of the DistFlow model above that ignores line losses, given that in practice $\ell_{ij} \approx 0$ for all $(i,j) \in \mathcal{L}$. Therefore, the linearized Distflow (LinDistFlow) equations are given by re-writing (1a)-(1c) to

$$P_{ij} = -\sum_{k\in\mathcal{O}(j)} Z_k \tag{2a}$$

$$Q_{ij} = -\sum_{k\in\mathcal{O}(j)} q_k \tag{2b}$$

$$v_j^2 = v_i^2 - 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right), \tag{2c}$$

where $\mathcal{O}(j)$ is the set of all descendants of node j including node j itself, i.e., $\mathcal{O}(j) := \{i : \mathcal{L}_j \subseteq \mathcal{L}_i\}$. This yields an explicit solution for v_i^2 in terms of v_0^2, given by

$$v_0^2 - v_i^2 = -2\sum_{j\in\mathcal{B}} \tilde{R}_{ij} Z_j - 2\sum_{j\in\mathcal{B}} \tilde{X}_{ij} q_j,$$

where

$$\tilde{R}_{ij} := \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} r_{hk}, \quad \tilde{X}_{ij} := \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} x_{hk} \tag{3}$$

The LinDistFlow can be compactly written as,

$$\boldsymbol{v} = -\boldsymbol{R}\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{q} + v_0^2\mathbf{1}, \tag{4}$$

where $\boldsymbol{v} := (v_1^2, \ldots, v_B^2), \boldsymbol{Z} := (Z_1, \ldots, Z_B), \boldsymbol{q} := (q_1, \ldots, q_B)$, and $\boldsymbol{R} := [2\tilde{R}_{ij}]_{B\times B}$ and $\boldsymbol{X} := [2\tilde{X}_{ij}]_{B\times B}$ are the resistance and reactance matrices, respectively. We treat the reactive power $\boldsymbol{q}$ as given constants rather than decision variables, which allows us to write (3) as

$$\boldsymbol{v} = -\boldsymbol{R}\boldsymbol{Z} + \hat{\boldsymbol{v}}, \tag{5}$$

where $\hat{\boldsymbol{v}} := -\boldsymbol{X}\boldsymbol{q} + v_0^2\mathbf{1}$. The voltage magnitude vector above is constrained as

$$\boldsymbol{v}_{\text{min}} \preceq \boldsymbol{v} \preceq \boldsymbol{v}_{\text{max}}, \tag{6}$$

where $\boldsymbol{v}_{\text{min}} := v_{\text{min}}^2\mathbf{1}$ and $\boldsymbol{v}_{\text{max}} := v_{\text{max}}^2\mathbf{1}$. Given that the second term in (5) is fixed, we re-write (6) to

$$\underline{\boldsymbol{v}} \preceq \boldsymbol{v} \preceq \overline{\boldsymbol{v}}, \tag{7}$$

where $\overline{\boldsymbol{v}} := \boldsymbol{v}_{\text{max}} - \hat{\boldsymbol{v}}$ and $\underline{\boldsymbol{v}} := \hat{\boldsymbol{v}} - \boldsymbol{v}_{\text{min}}$. We will impose (7) on the operation of the energy-sharing market.

[1] We posit that energy communities and DER aggregators are informed by the DSO about their networks' information, including OEs, line thermal limits, voltage limits, among others.

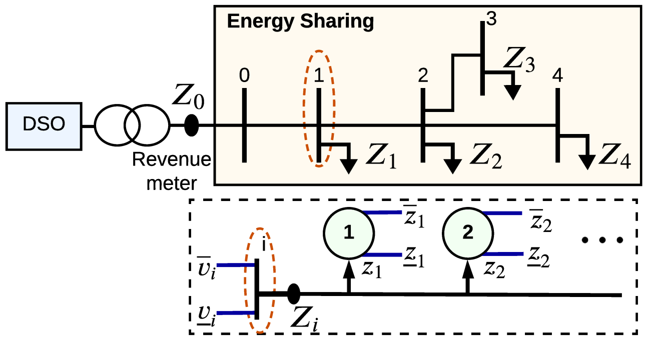

Fig. 1. A 4-bus energy sharing platform. $Z_0, Z_i, z_n \in \mathbb{R}$ are the net consumption of the whole energy sharing platform, net consumption of bus i, and net consumption of prosumer n, respectively. $\overline{z}_n \geq 0$ and $\underline{z}_n \leq 0$ are the prosumer's import and export OEs, respectively.

III. Energy Sharing Mathematical Model

Let $\mathcal{N} := \{1, \ldots, N\}$ denote the set of *energy sharing* system's prosumers. Every prosumer n is connected to one of the B buses in the considered radial network through its revenue meter that measures the prosumer's net consumption and BTM generation. Figure 1 shows an example 4-bus energy sharing platform. We denote the set of prosumers connected to bus $i \in \mathcal{B}$ by $\mathcal{N}_i$, therefore, $\mathcal{N} = \bigcup_{i \in \mathcal{B}} \mathcal{N}_i$. In this section, we model prosumers' DER in §III-A, and payment and surplus functions in §III-B, followed by a formulation of the proposed bi-level program representing the market operator's problem in §III-C.

A. DER Modeling

Prosumers' DER composition consists of BTM renewable distributed generation (DG), e.g., solar PV, and flexible loads (decision variables). The random *renewable DG* output of every prosumer $n \in \mathcal{N}$, denoted by $g_n \in \mathbb{R}_+$, is used primarily for self-consumption but gets exported back at the energy sharing price if the prosumer's generation is higher than its loads. The vector of prosumers' generation profiles is denoted by $\boldsymbol{g} := (g_1, \ldots, g_N)$, and the aggregate generation in the energy sharing platform is defined by $G_0 = \sum_{n \in \mathcal{N}} g_n$.

The *flexible loads* are represented by devices $k \in \mathcal{K} := \{1, \ldots, K\}$ whose load consumption bundle is denoted by the vector $\boldsymbol{d}_n \in \mathbb{R}_+^K$, which is constrained by the devices' flexibility limits, as

$$\boldsymbol{d}_n \in \mathcal{D}_n := [\underline{\boldsymbol{d}}_n, \overline{\boldsymbol{d}}_n], \quad \forall n \in \mathcal{N}, \tag{8}$$

where $\underline{\boldsymbol{d}}_n$ and $\overline{\boldsymbol{d}}_n$ are the device bundle's lower and upper consumption limits of $n \in \mathcal{N}$, respectively.

The *net consumption* $z_n \in \mathbb{R}$ of each prosumer is the difference between its gross consumption and BTM generation, hence $z_n = \mathbf{1}^\top \boldsymbol{d}_n - g_n$.[2] The *aggregate energy sharing net consumption* is simply $Z_0 = \sum_{n \in \mathcal{N}} z_n = \sum_{i \in \mathcal{B}} Z_i$.

[2] The proposed pricing policy can be generalized to incorporate OEs, *i.e.,* export and import limits on prosumers' net consumption, with only little mathematical complication. We show this in the appendix.

B. Payment, Surplus, and Profit Neutrality

Here, we show the payment and surplus functions of every prosumer, in §III-B.1, and the payment of the energy sharing operator to the DSO, in §III-B.2, which is subject to the profit neutrality axiom that the operator must abide by

1) Prosumer Payment and Surplus: The energy sharing operator designs a pricing policy χ for its members, which specifies the payment function for each prosumer $n \in \mathcal{N}$ under χ, denoted by $C_n^\chi(z_n)$.

Energy-sharing prosumers are assumed to be rational and self-interested. Therefore, they schedule their DER based on surplus maximization. For every bus $i \in \mathcal{B}$, the *surplus* of every prosumer $n \in \mathcal{N}_i$ is given by

$$S_n^\chi(\boldsymbol{d}_n, g_n) := U_n(\boldsymbol{d}_n) - C_n^\chi(z_n), \quad z_n = \mathbf{1}^\top \boldsymbol{d}_n - g_n, \tag{9}$$

where for every $n \in \mathcal{N}$, the *utility of consumption function* $U_n(\boldsymbol{d}_n)$ is assumed to be additive, concave, non-decreasing, and continuously differentiable with a *marginal utility function* $\boldsymbol{L}_n := \nabla U_n = (L_{n1}, \ldots, L_{nK})$. We denote the *inverse marginal utility* vector by $\boldsymbol{f}_n := (f_{n1}, \ldots, f_{nK})$ with $f_{nk} := L_{nk}^{-1}, \forall n \in \mathcal{N}, k \in \mathcal{K}$.

2) Energy Sharing Payment: The operator transacts with the DSO under the *NEM X tariff*, introduced in [20], which charges the energy sharing coalition based on whether it is *net-importing* ($Z_0 > 0$) or *net-exporting* ($Z_0 < 0$) as

$$\pi^{\text{NEM}}(Z_0) = \begin{cases} \pi^+, Z_0 \geq 0 \\ \pi^-, Z_0 < 0 \end{cases}, C^{\text{NEM}}(Z_0) = \pi^+[Z_0]^+ - \pi^-[Z_0]^-, \tag{10}$$

where $(\pi^+, \pi^-) \in \mathbb{R}_+$ are the *buy* (retail) and *sell* (export) rates, respectively. We assume $\pi^+ \geq \pi^-$, in accordance with NEM practice [21], which also eliminates risk-free price arbitrage, since the retail and export rates are deterministic and known *apriori*. The operator of the energy sharing regime is *profit-neutral*, a term we define next.

Definition 1 (Profit neutrality): The operator is profit-neutral if its pricing policy to every member achieves the following

$$\sum_{n \in \mathcal{N}} C_n^\chi(z_n) = C^{\text{NEM}}\Big(\sum_{n \in \mathcal{N}} z_n\Big).$$

The *profit neutrality* condition requires the operator to match aggregate prosumers' payments to the payment it submits to the DSO. The challenging question we ask is how can the operator design the payment C_n^χ, for every $n \in \mathcal{N}$, to achieve network-awareness, profit neutrality and equilibrium to the energy sharing market, which we define next.

C. Energy Sharing Stackelberg Game

A Stackelberg game involves a leading agent making an initial move that affects the optimal subsequent moves made by its followers ultimately affecting the outcome for the leader. We formulate this game as a bi-level mathematical program with the upper-level optimization being the operator's pricing problem, and the lower-level optimizations representing prosumers' optimal decisions.

Denote the consumption policy of the nth prosumer, given the pricing policy χ, by $\psi_{n,\chi}$. Formally,

$$\psi_{n,\chi} : \mathbb{R}_+ \to \mathcal{D}_n,\ g_n \overset{C_n^\chi}{\mapsto} \psi_{n,\chi}(g_n),$$

with $\boldsymbol{\psi}_\chi := \{\psi_{1,\chi}, \ldots, \psi_{N,\chi}\}$ as the vector of prosumers' policies. The operator strives to design a network-aware and welfare-maximizing pricing policy $\chi^\sharp_{\boldsymbol{\psi}}$ (given $\boldsymbol{\psi}$), where $\chi^\sharp_{\boldsymbol{\psi}} : \mathbb{R}^N_+ \to \mathbb{R}^N,\ \boldsymbol{g} \mapsto \boldsymbol{C}^\chi := (C_1^\chi, \ldots, C_N^\chi)$, and the welfare is defined as the sum of total prosumers' surplus, as

$$W^{\chi,\boldsymbol{\psi}_\chi} := \sum_{n\in\mathcal{N}} S_n^\chi(\psi_{n,\chi}(g_n), g_n),$$

where $\boldsymbol{\psi}_\chi := \{\psi_{1,\chi}, \ldots, \psi_{N,\chi}\}$. The bi-level program can be compactly formulated as

$$\underset{\boldsymbol{C}(\cdot)}{\text{maximize}} \left(W^{\chi_{\boldsymbol{\psi}}} = \sum_{n\in\mathcal{N}} U_n(\boldsymbol{d}_n^{\psi_\chi^\sharp}) - C^{\text{NEM}}(Z_0^{\psi_\chi^\sharp}) \right) \quad (11a)$$

$$\text{subject to} \quad \sum_{n\in\mathcal{N}} C_n^\chi(z_n^{\psi_\chi^\sharp}) = C^{\text{NEM}}(Z_0^{\psi_\chi^\sharp}) \quad (11b)$$

$$Z_0^{\psi_\chi^\sharp} = \sum_{n\in\mathcal{N}} \left(\boldsymbol{1}^\top \boldsymbol{d}_n^{\psi_\chi^\sharp} - g_n\right) \quad (11c)$$

$$(\underline{\boldsymbol{\eta}}, \overline{\boldsymbol{\eta}}) \quad \underline{\boldsymbol{v}} \preceq -\boldsymbol{R}\boldsymbol{Z}^{\psi_\chi^\sharp} \preceq \overline{\boldsymbol{v}} \quad (11d)$$

$$\text{for all } i = 1, \ldots, B, n \in \mathcal{N}_i \quad (11e)$$

$$\boldsymbol{d}_n^{\psi_\chi^\sharp} := \underset{\boldsymbol{d}_n\in\mathcal{D}_n}{\text{argmax}}\ S_n^\chi(\boldsymbol{d}_n, g_n) := U_n(\boldsymbol{d}_n) - C_n^\chi(z_n) \quad (11f)$$

$$\text{subject to} \quad z_n = \boldsymbol{1}^\top \boldsymbol{d}_n - g_n, \quad (11g)$$

where

$$\boldsymbol{Z}^{\psi_\chi^\sharp} := \Big(\sum_{n\in\mathcal{N}_1} \boldsymbol{1}^\top \boldsymbol{d}_n^{\psi_\chi^\sharp} - g_n, \ldots, \sum_{n\in\mathcal{N}_B} \boldsymbol{1}^\top \boldsymbol{d}_n^{\psi_\chi^\sharp} - g_n\Big).$$

In the following, we will assume that problem (11) is feasible, i.e., a solution meeting all the constraints exists.

The bi-level optimization above defines the Stackelberg strategy. Specifically, $(\chi^*, \boldsymbol{\psi}^*)$ is a Stackelberg equilibrium since (a) for all $\chi \in \mathcal{X}$ and $n \in \mathcal{N}$, $S_n^\chi(\psi_n^*(g_n), g_n) \geq S_n^\chi(\psi_n(g_n), g_n)$ for all $\boldsymbol{\psi} \in \Psi$; (b) for all $\boldsymbol{\psi} \in \Psi, W^{\chi^*, \boldsymbol{\psi}^*} \geq \sum_n S_n^\chi(\psi_n^*(g_n), g_n)$. Specifically, the Stackelberg equilibrium is the optimal community pricing when community members optimally respond to the community pricing.

IV. Network-Aware Pricing and Equilibrium

In the proposed market, at the beginning of each billing period, the operator sets the pricing policy and communicates the price to each prosumer under each bus. Given the announced price, the prosumers simultaneously move to solve their own surplus maximization problem. At the end of the billing period, and given the resulting Z_0, the DSO charges the energy sharing operator based on the NEM X tariff in (10). We propose the network-aware pricing policy and delineate its structure, in §IV-A, followed by solving the optimal response of prosumers in §IV-B. We discuss the operator's profit/deficit redistribution in §IV-C and §IV-E. Lastly, in §IV-D, we establish the market equilibrium result.

A. Network-Aware Dynamic Pricing

The operator uses the renewable DG vector $\boldsymbol{g}$ to dynamically set the price taking into account network constraints. That is, the dynamic price is used to satisfy network constraints in a decentralized way by internalizing them into prosumers' private decisions.

Network-aware pricing policy 1: For every bus $i \in \mathcal{B}$, the pricing policy charges the prosumers based on a two-part pricing

$$\chi^* : \boldsymbol{g} \mapsto C_n^{\chi^*}(z_n) = \underbrace{\pi_i^*(\boldsymbol{g})}_{\text{ex-ante price}} \cdot z_n - \underbrace{A_n^*}_{\text{ex-post allocation}}, \forall n \in \mathcal{N}_i, \quad (12)$$

where the *ex-ante* bus price $\pi_i^*(\boldsymbol{g})$ abides by a two-threshold policy with thresholds

$$\begin{aligned} \sigma_1(\boldsymbol{g}) &= \sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \boldsymbol{1}^\top [\boldsymbol{f}_n(\boldsymbol{1}\chi_i^+)]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n}, \\ \sigma_2(\boldsymbol{g}) &= \sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \boldsymbol{1}^\top [\boldsymbol{f}_n(\boldsymbol{1}\chi_i^-)]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n} \geq \sigma_1(\boldsymbol{g}), \end{aligned} \quad (13)$$

as

$$\pi_i^*(\boldsymbol{g}) = \begin{cases} \chi_i^+(\boldsymbol{g}) & , G_0 < \sigma_1(\boldsymbol{g}) \\ \chi_i^z(\boldsymbol{g}) & , G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})] \\ \chi_i^-(\boldsymbol{g}) & , G_0 > \sigma_2(\boldsymbol{g}), \end{cases} \quad (14)$$

and the price χ_i^κ, where $\kappa := \{+, -, z\}$, is given by

$$\chi_i^\kappa = \pi^\kappa - \sum_{j\in\mathcal{B}} R_{ji}(\overline{\eta}_j^* - \underline{\eta}_j^*) \quad (15)$$

where $\overline{\eta}_j^*$ and $\underline{\eta}_j^*$ are the dual variables of the upper and lower voltage limits in (11d), respectively, and the price $\pi^z := \mu^*$ is the solution of

$$\sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \boldsymbol{1}^\top [\boldsymbol{f}_n(\boldsymbol{1}\mu - \boldsymbol{1} \sum_{j\in\mathcal{B}} R_{ji}(\overline{\eta}_j^* - \underline{\eta}_j^*))]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n} = G_0. \quad (16)$$

For every bus $i \in \mathcal{B}$, the prosumer's ex-post charge/reward is denoted by A_n^*, which we delineate in §IV-C and §IV-E.

The pricing policy is a two-part and two-threshold one, with both thresholds $(\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g}))$ being DG-independent. The two policy parts are composed of a locational dynamic price that is announced *ex-ante* and a charge (reward) that is distributed *ex-post*.

The locational *ex-ante* price $\pi_i^*(\boldsymbol{g})$ for every $i \in \mathcal{B}$ is used as a mechanism to induce a collective prosumer response at each bus so that the network constraints are satisfied and the energy sharing social welfare is maximized. The energy sharing price has a similar structure to the celebrated LMP in wholesale markets [22] in the sense that it takes into account demand, generation, location, and network physical limits. Also, like *congestionless* LMP, the energy sharing price is uniform across all buses if the network constraints are nonbinding, as described in (15).

Similar to D-NEM without network constraints [4], the price obeys a two-threshold policy and it is a monotonically decreasing function of the system's renewables $\boldsymbol{g}$. As shown in (15), the thresholds partition G_0 and the price at each bus is the D-NEM price, adjusted by the shadow prices of violating network voltage limits. When $G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})]$ the community is energy balanced, and the price $\chi_i^z(\boldsymbol{g})$ is the sum of the dual variables for energy balance and network voltage limits.

The thresholds and locational prices can be computed while preserving prosumers' privacy. The operator do not need the functional form of prosumers' utilities or marginal utilities but rather asks the prosumers to submit a value for every device k at a given price.

B. Optimal Prosumer Decisions

Given $\boldsymbol{g}$, and under every bus $i \in \mathcal{B}$, the *ex-ante* dynamic price is announced and prosumers simultaneously move to solve their own surplus maximization problem to determine their optimal decisions policy $\psi^*_{n,\chi^*} : \mathbb{R}_+ \to \mathcal{D}_n,\ g_n \overset{C_n^{\chi^*}}{\mapsto} \boldsymbol{d}_n^{\psi^*} := \psi_n^*(g_n), \forall n \in \mathcal{N}$. Therefore, from the surplus definition in (9), each prosumer solves

$$\begin{aligned}\boldsymbol{d}_n^{\psi^*} =& \underset{\boldsymbol{d}_n \in \mathcal{D}_n}{\operatorname{argmax}}\ \ S_n^{\chi^*}(\boldsymbol{d}_n, g_n) := U_n(\boldsymbol{d}_n) - \pi_i^*(\boldsymbol{g}) \cdot z_n \\ & \text{subject to} \quad z_n = \boldsymbol{1}^\top \boldsymbol{d}_n - r_n, \end{aligned} \tag{17}$$

where A_n^* was omitted because it is announced after the consumption decisions are performed. Lemma 1 formalizes the optimal consumption of each prosumer.

Lemma 1 (Prosumer optimal consumption): Under every bus $i \in \mathcal{B}$, given the pricing policy χ^*, the prosumer's optimal consumption is

$$\boldsymbol{d}_n^{\psi^*}(\pi_i^*) = [\boldsymbol{f}_n(\boldsymbol{1}\pi_i^*)]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n}, \quad \forall n \in \mathcal{N}_i. \tag{18}$$

By definition, the aggregate net consumption is

$$Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g})) = \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} \begin{cases} \boldsymbol{d}_n^{\psi^*}(\chi_i^+) - g_n\, , G_0 < \sigma_1(\boldsymbol{g}) \\ \boldsymbol{d}_n^{\psi^*}(\chi_i^z) - g_n\ , G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})] \\ \boldsymbol{d}_n^{\psi^*}(\chi_i^-) - g_n\, , G_0 > \sigma_2(\boldsymbol{g}), \end{cases} \tag{19}$$

where $\boldsymbol{\pi^*} := (\pi_1^*, \dots, \pi_B^*)$, and $Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g})) > 0$ if $G_0 < \sigma_1(\boldsymbol{g})$, $Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g})) = 0$ if $G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})]$, and $Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g})) < 0$ if $G_0 > \sigma_2(\boldsymbol{g})$.

Proof: We drop the prosumer subscript n for brevity. The objective in (17) is concave and differentiable. The Lagrangian function of the surplus maximization problem, for a prosumer under bus i, is

$$\mathcal{L}(\boldsymbol{d}, \overline{\boldsymbol{\gamma}}, \underline{\boldsymbol{\gamma}}) = \pi_i^*(\boldsymbol{g}) \cdot z - U(\boldsymbol{d}) + \overline{\boldsymbol{\gamma}}^\top(\boldsymbol{d} - \overline{\boldsymbol{d}}) - \underline{\boldsymbol{\gamma}}^\top(\boldsymbol{d} - \underline{\boldsymbol{d}}),$$

where $\overline{\boldsymbol{\gamma}} \in \mathbb{R}_+^K$ and $\underline{\boldsymbol{\gamma}} \in \mathbb{R}_+^K$ are the Lagrangian multipliers of the upper and lower consumption limits. From the KKT conditions we have

$$\nabla_{\boldsymbol{d}}\mathcal{L} = \boldsymbol{1}\pi_i^*(\boldsymbol{g}) - \boldsymbol{L}(\boldsymbol{d}^{\psi^*}) + \overline{\boldsymbol{\gamma}} - \underline{\boldsymbol{\gamma}} = \boldsymbol{0},$$

therefore, for each device $k \in \mathcal{K}$, we have

$$\begin{aligned} d_k^{\psi^*} &= \begin{cases} f_k(\pi_i^*) & , \overline{\gamma}_k = \underline{\gamma}_k = 0 \\ \overline{d}_k & , \overline{\gamma}_k > 0, \underline{\gamma}_k = 0 \\ \underline{d}_k & , \overline{\gamma}_k = 0, \underline{\gamma}_k > 0 \end{cases} \\ &=: [f_k(\pi_i^*)]_{\underline{d}_k}^{\overline{d}_k}, \end{aligned}$$

where $f_k := L_k^{-1}$.

Give the aggregate net consumption definition $Z_0 = \sum_{n\in\mathcal{N}}(\boldsymbol{1}^\top \boldsymbol{d}_n - g_n)$ and the dynamic price in (14), one can easily get (19). Finally, from (13), we can re-formulate (19) as

$$Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g})) = \begin{cases} \sigma_1(\boldsymbol{g}) - G_0 & , G_0 < \sigma_1(\boldsymbol{g}) \\ 0 & , G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})] \\ \sigma_2(\boldsymbol{g}) - G_0 & , G_0 > \sigma_2(\boldsymbol{g}), \end{cases}$$

which proves the sign of $Z_0^{\psi^*}(\boldsymbol{\pi^*}(\boldsymbol{g}))$ under each piece. ■

C. Ex-Post Allocation

Unlike the *ex-ante* price, the *ex-post* allocation is distributed after the prosumers schedule their DER. The operator may choose to accrue the *ex-post* charge amount of each prosumer to be distributed after multiple billing periods rather than at every billing period. The *ex-post* fee is essentially levied to compensate for the fact that the *ex-ante* volumetric charge is insufficient to ensure profit neutrality. Indeed, using Def.1, we have

$$\begin{aligned} \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} C_n^{\chi}(z_n) - C^{\text{NEM}}(\sum_{n\in\mathcal{N}} z_n) = 0 \overset{(10),(12)}{\Longrightarrow} \\ \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i}(\pi_i^*(\boldsymbol{g}) \cdot z_n - A_n^* - \pi^{\text{NEM}}(Z_0) \cdot z_n) = 0 \\ \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i}(\pi_i^*(\boldsymbol{g}) \cdot z_n - \pi^{\text{NEM}}(Z_0) \cdot z_n) = \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} A_n^* \\ \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i}(\pi_i^*(\boldsymbol{g}) - \pi^{\text{NEM}}(Z_0)) \cdot z_n = A^*(\boldsymbol{g}), \end{aligned}$$

where $A^*(\boldsymbol{g}) := \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} A_n^*$ is the profit/deficit that the operator accumulates after the price is announced and the transaction with the DSO is settled. One can see that the larger the price differential between the energy sharing price and NEM price ($\delta_i := \pi_i^*(\boldsymbol{g}) - \pi^{\text{NEM}}(Z_0), \forall i \in \mathcal{B}$), the larger the profit/deficit $|A^*(\boldsymbol{g})|$. Note that if the network constraints are non-binding, *i.e.,* $\overline{\eta}_i^* = \underline{\eta}_i^* = 0, \forall i \in \mathcal{B}$, then $A^*(\boldsymbol{g}) = 0$, and the pricing policy becomes one-part; see D-NEM in [4].

There is no unique way to re-allocate the operator's profit/deficit $A^*(\boldsymbol{g})$. A profit-sharing coalitional game can be established to fairly re-allocate the operator's profit/deficit [23]. In §IV-E, we propose a proportional re-allocation rule that makes the payment function uniform for all prosumers.

D. Stackelberg Equilibrium

Before we present the main market equilibrium theorem, we need the following Lemma 2 that establishes the profit neutrality of the pricing policy under the optimal prosumer response in Lemma 1.

Lemma 2: Under the solution (χ^*, ψ^*), the operator is profit-neutral.

Proof: The proof is straightforward. Using Def.1 under the pricing policy and the prosumer response, we have

$$\sum_{n\in\mathcal{N}} C_n^{\chi^*}(z_n^{\psi^*}) - C^{\text{NEM}}(\sum_{n\in\mathcal{N}} z_n^{\psi^*})$$
$$= \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} (\pi_i^* \cdot z_n^{\psi^*} - A_n^* - \pi^{\text{NEM}}(Z_0^{\psi^*})\cdot z_n^{\psi^*}) = 0$$
$$= A^*(\boldsymbol{g}) - \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} (\pi_i^*(\boldsymbol{g}) \cdot z_n^{\psi^*} - \pi^{\text{NEM}}(Z_0^{\psi^*})\cdot z_n^{\psi^*}) = 0$$

where, for every $n \in \mathcal{N}$, $z_n^{\psi^*} = \mathbf{1}^\top \boldsymbol{d}_n^{\psi^*} - g_n$. ■

Given the prosumer response to the pricing policy in Lemma 1 and profit neutrality in Lemma 2, it remains to show that the network-aware pricing policy achieves a Nash equilibrium to the leader-follower game in \$III-C.

Theorem 1: The solution (χ^*, ψ^*) is a Stackelberg equilibrium that also achieves social optimality, *i.e.,*

$$\begin{aligned}(\boldsymbol{d}_1^{\psi^*}, \ldots, \boldsymbol{d}_N^{\psi^*}) = \underset{(\boldsymbol{d}_1,\ldots,\boldsymbol{d}_N)}{\text{argmax}} \quad & \sum_{i\in\mathcal{B}}\sum_{n\in\mathcal{N}_i} U_n(\boldsymbol{d}_n) - C^{\text{NEM}}(Z_0)\\ \text{subject to} \quad & Z_0 = \sum_{n\in\mathcal{N}} \left(\mathbf{1}^\top \boldsymbol{d}_n - g_n\right)\\ & \boldsymbol{d}_n \in \mathcal{D}_n \ \forall n \in \mathcal{N}\\ & \underline{\boldsymbol{v}} \preceq -\boldsymbol{R}\boldsymbol{Z} \preceq \overline{\boldsymbol{v}}.\end{aligned}$$

Proof: See the appendix in [1]. ■

The proof of Theorem 1 solves an upper bound of (11) that relaxes the profit-neutrality constraint (11b).

E. Energy Sharing Payment Uniformity

We propose here a unique way to reallocate the operator's profit/deficit $A^*(\boldsymbol{g})$. For every bus $i \in \mathcal{B}$, the re-allocation to every prosumer is given by

$$A_n^*(\boldsymbol{g}) = (\pi_i^*(\boldsymbol{g}) - \pi^{\text{NEM}}(Z_0))\cdot z_n, \forall n \in \mathcal{N}_i, \quad (20)$$

which has three favourable features. First, it redistributes the profit/deficit proportionally to the prosumers based on how far the price they face from the DSO NEM price, which is basically how much they paid (got paid) for voltage correction. Second, it makes prosumer payment functions $C_n^{\chi^*} \forall n \in \mathcal{N}$ uniform. Indeed, plugging (20) into (12) cancels out the locational dynamic price $\pi_i^*(\boldsymbol{g})$, and yields a simple, uniform payment function that charges customers based on the NEM price, *i.e.,* for every bus $i \in \mathcal{B}$,

$$C_n^{\chi^*}(z_n) = \pi^{\text{NEM}}(Z_0)\cdot z_n, \forall n \in \mathcal{N}_i.$$

Third, unlike the computationally expensive coalitional-game-based profit allocation schemes such as the Shapley value [24] and nucleolus [25], the allocation rule in (20) is straightforward and directly links the allocation to the energy sharing price and the prosumer's net consumption. The decentralization argument may not hold under the allocation in (20), as it compensates prosumers explicitly based on their own net consumption, which may influence their consumption decisions resulting in deviations from the welfare-maximizing decisions. It might be, however, too difficult for prosumers to anticipate if the operator performs the re-allocation at every multiple billing periods rather than at every single billing period.

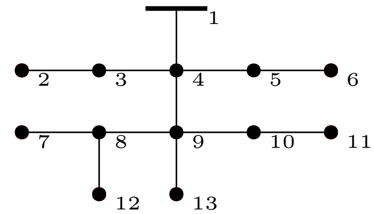

Fig. 2. The IEEE 13-bus test feeder.

V. NUMERICAL STUDY

Our network-aware market mechanism was validated on the IEEE 13-bus feeder converted to a single-phase equivalent [26], see Figure 2. Bus 1 is the substation and represents the network slack bus. Buses 2 to 13 instead host twenty-three prosumers. For every $n \in \mathcal{N}$ was chosen to be the concave and non-decreasing function

$$U_n(d_n) = \begin{cases} \alpha_n d_n - \frac{1}{2}\beta_n d_n^2, & 0 \le d_n \le \frac{\alpha_n}{\beta_n}\\ \frac{\alpha_n^2}{2\beta_n}, & d_n > \frac{\alpha_n}{\beta_n}, \end{cases} \quad (21)$$

where the parameters α_n, β_n were learned and calibrated using historical retail prices, consumptions and by assuming an elasticity of 0.21 taken from [27] (see appendix D in [21])[3] The minimum demand was set to $\underline{\boldsymbol{d}}_n = \mathbf{0}$ for every $n \in \mathcal{N}$, whereas the maximum demands $\overline{\boldsymbol{d}}_n$ and the DER generations were obtained using data from the PecanStreet dataset. We set $\boldsymbol{v}_{\text{min}} = 0.95$ p.u. and $\boldsymbol{v}_{\text{max}} = 1.05$ p.u.

In our simulations, we considered four scenarios, described in the following, that differ for the DER generation levels. For each scenario, we used the exact AC power flow solver MATPOWER to obtain the bus voltages, whereas we solved the optimization problems relying on the power flow equation linearization (4). The results are shown in Figure 3.

Scenario 1: the DER generation here is zero for each prosumer. Hence, $G_0 = 0$ and $G_0 < \sigma_1(\boldsymbol{g})$. The energy-sharing system is importing energy. In this case, the energy sharing optimization problem solutions are such that $\underline{\eta}_j^* \neq 0$, i.e., some voltages are on the lower bound $\boldsymbol{v}_{\text{min}}$. The resulting prices are in general higher than π^+.

Scenario 2: the DER generation G_0 is non-zero but still not enough to cover the demand, i.e., $G_0 < \sigma_1(\boldsymbol{g})$. Hence, the energy-sharing system is importing energy. However, the optimum demands are such that all the voltages are within the desired bounds and the energy prices equal π^+.

Scenario 3: the DER generation was further increased in this scenario and $\sigma_1(\boldsymbol{g}) \le G_0 \le \sigma_2(\boldsymbol{g})$. That is, the energy-sharing platform did not exchange active power with the external network. The energy sharing platform optimization problem provides an energy price within π^+ and π^-; voltage limits are satisfied at the optimal consumption.

Scenario 4: here, we increased the DER generations until $G_0 \ge \sigma_2(\boldsymbol{g})$. The platform exports power to the external grid. The energy sharing platform optimization problem solution

[3]The retail prices were taken from Data.AustinTexas.gov historical residential rates in Austin, TX. For the demands, we used pre-2018 PecanStreet data for households in Austin, TX.

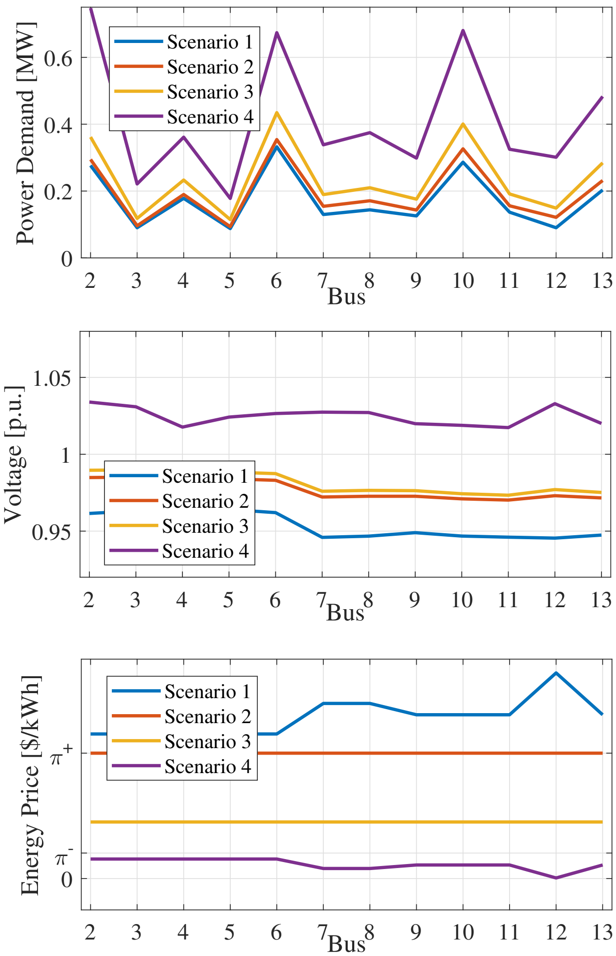

Fig. 3. Summary of the numerical tests on the four considered scenarios. The lower panel reports the ex-ante energy prices obtained after solving the energy sharing platform optimization problem (11). The upper panel shows the cumulative power demand at each bus obtained after the energy sharing operator dispatched the energy prices. The middle panel reports the resulting bus voltage magnitudes.

is such that the voltages in some locations are exactly $\boldsymbol{v}_{\max}$ and the Lagrange multipliers vector $\overline{\eta}_j^*$ is different from zero. The energy prices are smaller than π^- and close to zero, i.e., consumption is incentivized to take full advantage of DER generation.

Some observations are in order. In general, we observe that increasing the DER generation G_0 results in the decrease of energy prices. The energy prices can in principle be bigger than π^+, see Scenario 1. This is to ensure that the voltage constraints are satisfied by decreasing the power demand. Finally, we note a slight difference between the true and the expected (i.e., the ones computed by the energy-sharing platform optimization problem) voltage magnitudes. Indeed, we see that the voltages in Scenario 4 are all strictly lower than $\boldsymbol{v}_{\max}$ even though we obtained $\overline{\eta}_j^* \neq 0$, see the middle panel of Figure 3. This can be explained by the fact that (11) was solved relying on the linearized equations (4) rather than on the true power flow equations. Note, however, that using the true equation would result in a nonconvex energy sharing optimization problem possibly displaying multiple local minima.

VI. Conclusion

In this work, we propose a network-aware and welfare-maximizing market mechanism for energy-sharing coalitions that aggregate small but ubiquitous BTM DER downstream of a DSO's revenue meter, charging the energy-sharing systems using a generic NEM tariff. The proposed pricing policy has *ex-ante* and *ex-post* pricing components. The *ex-ante* locational and threshold-based price that decreases as the energy-sharing supply-to-demand ratio increases is used to induce a collective prosumer reaction that decentrally maximizes social welfare while being network-cognizant. On the other hand, the *ex-post* charge is used to enforce the market operator's profit-neutrality condition. We show that the market mechanism achieves an equilibrium to the Stackelberg game between the operator and its prosumers. We show that a unique proportional rule to re-allocate the operator's profit/deficit can make the payment function of all energy-sharing prosumers uniform, even when the network constraints are binding. Our simulation results leverage real DER data on an IEEE 13-bus test feeder system to show how the dynamic pricing drives the energy sharing's flexible consumption to abide by the network voltage limits.

VII. Acknowledgment

The work of Ahmed S. Alahmed and Lang Tong was supported in part by the National Science Foundation under Award 2218110 and and the Power Systems Engineering Research Center (PSERC) under Research Project M-46. This work was authored in part by the National Renewable Energy Laboratory, operated by Alliance for Sustainable Energy, LLC, for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. Funding is provided by the U.S. Department of Energy Office of Energy Efficiency and Renewable Energy Building Technologies Office, United States. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

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Appendix
Incorporating Operating Envelopes

Here, we present the pricing policy under OEs at the prosumer's revenue meter, as shown in Fig.1.

OEs limit the net consumption of every prosumer $n \in \mathcal{N}$, as

$$z_n \in \mathcal{Z}_n := [\underline{z}_n, \overline{z}_n], \tag{22}$$

where $\underline{z}_n \leq 0$ and $\overline{z}_n \geq 0$ are the export and import envelopes at the prosumers' meters, respectively. From the analysis in [11], the network-aware pricing policy generalizes as in the following policy.

Network-aware pricing policy 2: For every bus $i \in \mathcal{B}$, the pricing policy charges the prosumers based on a two-part pricing

$$\chi^* : \boldsymbol{g} \mapsto C_n^{\chi^*}(z_n) = \underbrace{\pi_i^*(\boldsymbol{g})}_{\text{ex-ante price}} \cdot z_n - \underbrace{A_n^*}_{\text{ex-post allocation}}, \forall n \in \mathcal{N}_i,$$

where the *ex-ante* bus price $\pi_i^*(\boldsymbol{g})$ abides by a two-threshold policy with thresholds

$$\sigma_1(\boldsymbol{g}) = \sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \left[\mathbf{1}^\top [\boldsymbol{f}_n(\mathbf{1}\chi_i^+)]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n} \right]_{\underline{z}_n+g_n}^{\overline{z}_n+g_n},$$

$$\sigma_2(\boldsymbol{g}) = \sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \left[\mathbf{1}^\top [\boldsymbol{f}_n(\mathbf{1}\chi_i^-)]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n} \right]_{\underline{z}_n+g_n}^{\overline{z}_n+g_n} \geq \sigma_1(\boldsymbol{g}),$$

as

$$\pi_i^*(\boldsymbol{g}) = \begin{cases} \chi_i^+(\boldsymbol{g}) & , G_0 < \sigma_1(\boldsymbol{g}) \\ \chi_i^z(\boldsymbol{g}) & , G_0 \in [\sigma_1(\boldsymbol{g}), \sigma_2(\boldsymbol{g})] \\ \chi_i^-(\boldsymbol{g}) & , G_0 > \sigma_2(\boldsymbol{g}), \end{cases} \tag{23}$$

and the price χ_i^κ, where $\kappa := \{+, -, z\}$, is given by

$$\chi_i^\kappa = \pi^\kappa - \sum_{j\in\mathcal{B}} R_{ji}(\overline{\eta}_j^* - \underline{\eta}_j^*) \tag{24}$$

where $\overline{\eta}_j^*$ and $\underline{\eta}_j^*$ are the dual variables of the upper and lower voltage limits in (11d), respectively, and the price $\pi^z := \mu^*$ is the solution of

$$\sum_{i\in\mathcal{B}} \sum_{n\in\mathcal{N}_i} \left[\mathbf{1}^\top [\boldsymbol{f}_n(\mathbf{1}\mu - \mathbf{1}\sum_{j\in\mathcal{B}} R_{ji}(\overline{\eta}_j^* - \underline{\eta}_j^*))]_{\underline{\boldsymbol{d}}_n}^{\overline{\boldsymbol{d}}_n} \right]_{\underline{z}_n+g_n}^{\overline{z}_n+g_n} = G_0.$$

For every bus $i \in \mathcal{B}$, the prosumer's ex-post charge/reward is denoted by A_n^*, which we delineate in §IV-C and §IV-E.