Distribution Grid Incentive Design with Unknown Agent Behavior

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Abstract

Motivation: During extreme events, traditional grid regulation methods (e.g., energy prices, net power injection limits) may be insufficient. While system operators typically lack control over end-user grid interactions, (e.g., energy demand), *incentives* can influence behavior -- for example, a user that receives a grid-driven incentive may adjust their consumption or expose relevant control variables in response.

Problem: Optimize for the best incentive subject to system stability constraints. However, user behavior is unknown to the SO – i.e., for a given incentive, the amount of curtailed load or control variables exposed is unknown.

Problem Formulation

Our approach: We propose a general incentive mechanism in the form of a constrained optimization problem -- our approach is distinguished from prior work by modeling human behavior *(e.g., reactions to an incentive)* as an arbitrary unknown function.

incentive $\mathbf{i}$, cost given by e.g., $c(\mathbf{i}) = \|\mathbf{i}\|_1$

$$\begin{aligned} \min_{\mathbf{i}\in\mathbb{R}^n} \quad & c(\mathbf{i}), && (\textit{cost of incentive}) \\ \text{s.t.,} \quad & h(g(\mathbf{u},\mathbf{i})) \preccurlyeq \mathbf{0}. && (\textit{system constraints}) \end{aligned} \tag{1}$$

$g(\cdot,\ \cdot)$ arbitrary & unknown operating point $\mathbf{u}' = g(\mathbf{u},\mathbf{i})$

SO's desired setpoint $\mathbf{u}$ s.t. $h(\mathbf{u}) \leq \mathbf{0}$

For an application to **distribution grid voltage control**, we define a *constraint function* $h(\cdot)$, based on the LinDistFlow[1] linearized power flow model:

system voltages ← $\mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + \tilde{\mathbf{v}}$, → linearization point

where $\mathbf{p}$ and $\mathbf{q}$ are the *active and reactive power demand*, and $\mathbf{R}, \mathbf{X}$ are *symmetric positive definite matrices*.

For system stability, we will require that voltages $\mathbf{v}$ are within upper and lower bounds given by $[\,\underline{\mathbf{v}}, \overline{\mathbf{v}}\,]$:

$$h(\mathbf{u}') := \max\{\underline{\mathbf{v}} - (\mathbf{R}\mathbf{p}' + \mathbf{X}\mathbf{q}' + \tilde{\mathbf{v}}),\ (\mathbf{R}\mathbf{p}' + \mathbf{X}\mathbf{q}' + \tilde{\mathbf{v}}) - \overline{\mathbf{v}}\},$$

where $\mathbf{u}'$ indicates the tuple $(\mathbf{p}', \mathbf{q}')$. $(h(\mathbf{u}') \preccurlyeq \mathbf{0} \rightarrow \mathbf{v} \in [\,\underline{\mathbf{v}}, \overline{\mathbf{v}}\,])$

Algorithm Design

We propose *feedback-based optimization algorithms* to solve (1). Each algorithm leverages different amounts of information about the problem (or measurements).

First-order primal-dual technique

When the gradient $\nabla_{\mathbf{i}} g(\mathbf{u}, \mathbf{i})$ is known *or can be estimated*, we consider an iterative first-order optimization algorithm – since (1) is constrained, we start by introducing the Lagrangian:

$$\mathcal{L}(\mathbf{i}, \boldsymbol{\lambda}) = c(\mathbf{i}) + \boldsymbol{\lambda}^\top (h(g(\mathbf{u}, \mathbf{i}))), \qquad \boldsymbol{\lambda} \geq \mathbf{0},$$

where $\boldsymbol{\lambda}$ is a vector of *dual variables*. The algorithm is as follows:

1: **input:** Time-varying step sizes $\{\epsilon_k > 0\}_{k\geq 1}$.
2: **initialize:** Initialize primal and dual guesses $\mathbf{i}^{(0)}, \boldsymbol{\lambda}^{(0)} \succcurlyeq \mathbf{0}$..
3: **for** steps $k = 1, 2, \ldots$ **do**
4: Incentive (primal) update: $\mathbf{i}^{(k)} = \mathbf{i}^{(k-1)} - \epsilon_k \nabla \mathcal{L}^{(k)}$;
5: Dual update: $\boldsymbol{\lambda}^{(k)} = \left[\boldsymbol{\lambda}^{(k-1)} + \epsilon_k \cdot h(g(\mathbf{u}, \mathbf{i}^{(k)}))\right]_{\mathbb{R}^n_+}$.
6: **end for**

Zero-order gradient estimation technique

When the gradient $\nabla_{\mathbf{i}} g(\mathbf{u}, \mathbf{i})$ is *not known and cannot be estimated* (e.g., due to lack of data) we consider *zero-order two function eval*[2,3] estimation of the Lagrangian gradient. The following approximation is used in a tweaked primal update (line 4):

$$\hat{\nabla}\mathcal{L}^{(k)} := \frac{\zeta^{(k)}}{2\sigma}\left[\hat{\mathcal{L}}(\mathbf{i}_+^{(k)}, \boldsymbol{\lambda}^{(k)}) - \hat{\mathcal{L}}(\mathbf{i}_-^{(k)}, \boldsymbol{\lambda}^{(k)})\right],$$

with *perturbed incentives* $\mathbf{i}_\pm^{(k)} := \mathbf{i}^{(k)} \pm \sigma \boldsymbol{\zeta}^{(k)}$, where $\boldsymbol{\zeta}^{(k)} \in \mathbb{R}^n$ is a random signal, and $\sigma > 0$ controls the magnitude of perturbation.

Theoretical guarantees

Under standard conditions on problem (1) *(e.g., convexity, Slater's conditions, Lipschitz continuity)* we can show that both first-order and zero-order converge to an asymptotically stable and (near-)optimal incentive.

Case Study: Voltage Control

We evaluate the first-order and zero-order algorithms in a voltage control simulation on the IEEE 33-bus radial distribution network,[1] with real loads.[4] To define a "realistic" $g(\mathbf{u}, \mathbf{i})$, we define a *step function* for each PQ (i.e., load) bus.

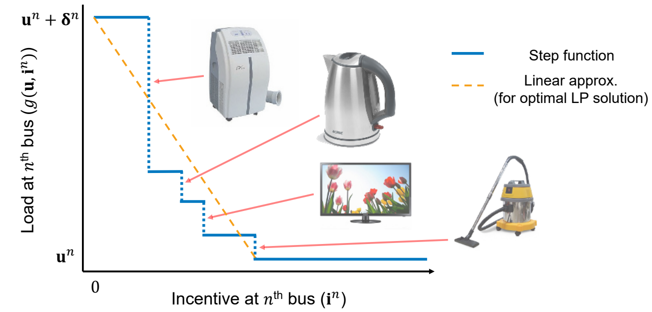


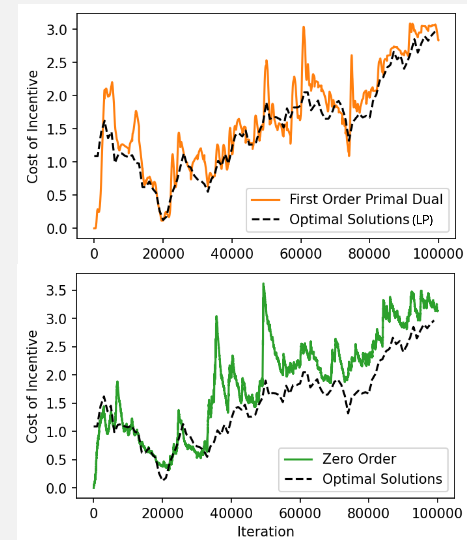
Figure 1. An annotated example of a step g function. Each step corresponds to a "controllable device".

We construct *time-varying* instances where each bus has an average of 6 controllable devices. Devices are added and removed over time in a Poisson process. Our proposed algorithms track the time-varying optimal solution in an iterative fashion, while keeping voltages within the bounds $[\,\underline{\mathbf{v}}, \overline{\mathbf{v}}\,]$.

Figure 2. Iterates of the first-order and zero-order algorithms on time-varying problem (1).

The g step function is updated every "minute", where 1000 iterations ≈ 1 minute.

A linear approx. of the g step function is used to construct and solve an LP that serves as a time-varying benchmark.



References

1. Baran and Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing." *IEEE Trans. on Power Delivery* 4(2), 1989.
2. Cavraro et al., "Feedback Optimization of Incentives for Distribution Grid Services." *arXiv:2403.19616*, 2024.
3. Chen et al., "Model-Free Primal-Dual Methods for Network Optimization with Application to Real-Time Optimal Power Flow." *American Control Conference*, 2020.
4. Barker et al., "Smart*: An open data set and tools for enabling research in sustainable homes." *ACM SustKDD*, 2012.

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