SERI/TR-211-1950
UC Categories: 60, 62c
DE84004431

A Theoretical Analysis of Solar-Driven Natural Convection Energy Conversion Systems

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January 1984

Prepared under Task No. 1067.10
FTP No. 448

Solar Energy Research Institute
A Division of Midwest Research Institute

1617 Cole Boulevard
Golden, Colorado 80401

Prepared for the
U.S. Department of Energy
Contract No. DE-AC02-83CH10093

Printed in the United States of America
Available from:
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price:
Microfiche A01
Printed Copy A03

PREFACE

The Solar Energy Research Institute (SERI), a division of the Midwest Research Institute, has been authorized by the U.S. Department of Energy to provide technical management services for the Advanced/Innovative Wind Energy Concepts (AIWEC) Program. The objective of the AIWEC Program is to assess the technical feasibility and support research and development of innovative wind energy concepts.

The use of natural convection to extract energy has often been proposed to the AIWEC Program. Thus, a theoretical study of Natural Convection Energy Conversion Systems (NCECS) was undertaken by this Program. The purpose of the study was to determine what performance potential might be expected from these systems, considering various environmental and operational parameters. This report summarizes the assumptions, analyses, and results of that study.

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SUMMARY

Using the phenomenon of natural convection, or buoyancy, has often been proposed as a way to utilize solar energy. The general approach is to generate a natural convection flow within a confining structure such as a tower. Natural convection can provide either a heated (rising) or a cooled (sinking) flow. Heated natural convection flow can be achieved by direct collection of incident solar radiation. This type of solar energy collection would occur at the base of the system, and the flow would exit at the top. With cooled natural convection flow, heat would be extracted at the top and the flow would exit at the base of the system. One of the more obvious methods of extracting heat utilizes the latent heat of vaporization of evaporating water. Because the heat content of the ambient air is essentially provided by solar energy, the cooled natural convection energy conversion system can also be considered a solar-powered device. Note that heat collection or extraction can also occur within the confining structure.

This report summarizes theoretical analyses of the ideal thermodynamic cycle efficiencies potentially attainable by natural convection energy conversion systems. Assuming ideal processes (i.e., no frictional or thermal losses), appropriate thermodynamic models could be developed for both heated and cooled systems. Tower configurations were assumed for the analyses. However, the results are applicable to all structural configurations intended to utilize the natural convection phenomenon. Other possible configurations include geodesic domes and natural or man-made channels utilizing mountainous or hilly terrain. Computer simulations of these models were used to facilitate calculation procedures.

Ambient atmospheric conditions, such as a very stable lapse rate and high relative humidity, can have significantly adverse effects on the performance of natural convection towers. The ideal conversion efficiencies of both heated and cooled towers decrease as the lapse rate, or vertical temperature gradient, decreases within its stable range. Maximum performance is achieved for an adiabatic atmosphere corresponding to the maximum or neutrally stable lapse rate. Increases in ambient relative humidity significantly affect cooled natural convection tower performance, both by limiting heat extraction from the intake airflow and by reducing the ideal conversion efficiency for absolutely stable lapse rates. Analysis also shows that ideally, natural convection towers would provide higher conversion efficiencies in cooler climates, yet would produce less power because of lower insolation levels for the heated tower and diminished heat extraction from the cooled tower.

To determine the economic potential of natural convection towers, a cost study would be needed. However, some conclusions can be drawn from this study regarding performance limitations with significant economic impacts. For example, the low conversion efficiencies of heated natural convection towers would necessitate a very large solar collector area for useful power output levels. Thus, solar collector costs and associated land requirements may severely limit the potential of heated natural convection towers. Similarly, the cooled natural convection tower would be most efficient in a low-humidity environment. However, except in coastal regions, low humidity and abundant water supplies usually are mutually exclusive, thus limiting the applicability

of cooled natural convection towers. Both the performance and the cost of natural convection towers increase with tower height. Further analysis, then, would be needed to ascertain the most cost-effective tower height.

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NOMENCLATURE

A	cross-sectional area of flow
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
g	gravitational constant
m	mixing ratio ($kg_{water\ vapor}/kg_{air}$)
$\dot{m}$	mass flow rate ($= \rho AV$)
p	pressure
q_{in}	heat added to flow at tower base
q_{out}	heat lost or extracted from flow at top of tower
r	relative humidity ($= m/m_s$)
R	gas constant
T	temperature
V	flow velocity
w_{out}	work or power extracted by turbine
w_{pump}	ideal water pumping requirements for evaporative cooling system
z	height

Greek Symbols

γ	ratio of specific heats (c_p/c_v)
Δm	increase of mixing ratio due to evaporative cooling
Δp_t	pressure drop across turbine
ΔT_c	temperature decrease due to q_{out}
ΔT_h	temperature increase due to q_{in}
ΔT_{tower}	temperature increase due to solar energy collection by tower

λ	vertical temperature gradient or lapse rate (= $-dT/dz$)
η	energy conversion efficiency
η_{Carnot}	Carnot efficiency
ρ	density

Subscripts

a	adiabatic
A,B,C, D,E	thermodynamic states
d	dry air
m	air/water vapor mixture
o	initial condition
s	saturated
t	turbine
v	water vapor

SECTION 1.0

INTRODUCTION

The phenomenon of natural convection, or buoyancy, has often been proposed for use in solar energy conversion to produce either heated (rising) or cooled (sinking) flow within a confining structure such as a tower (chimney), a geodesic dome, or a closed channel utilizing natural terrain. Heated natural convection can be achieved by direct collection of incident solar energy to an inflow at the base of the system. For cooled natural convection, heat would be extracted from the inflow at the top and the flow would exit at the base of the system. The heat can most easily be extracted by making use of the latent heat of vaporization for evaporating water.

This paper summarizes theoretical analyses of the ideal conversion efficiencies attainable by natural convection energy conversion systems (NCECS). Although the analyses employ the tower (chimney) configuration, the results are applicable to any open-cycle NCECS. Schematics of solar-powered heated and cooled natural convection towers are shown in Figures 1-1 and 1-2, respectively. Because the convective flow can be thought of as an induced wind, such systems are often considered wind energy conversion devices. However, an NCECS is more properly described as a heat engine with a corresponding thermodynamic cycle and a thermodynamic efficiency. In essence, an NCECS exploits the energy differential between the artificial atmosphere created within its confining structure and the ambient atmosphere.

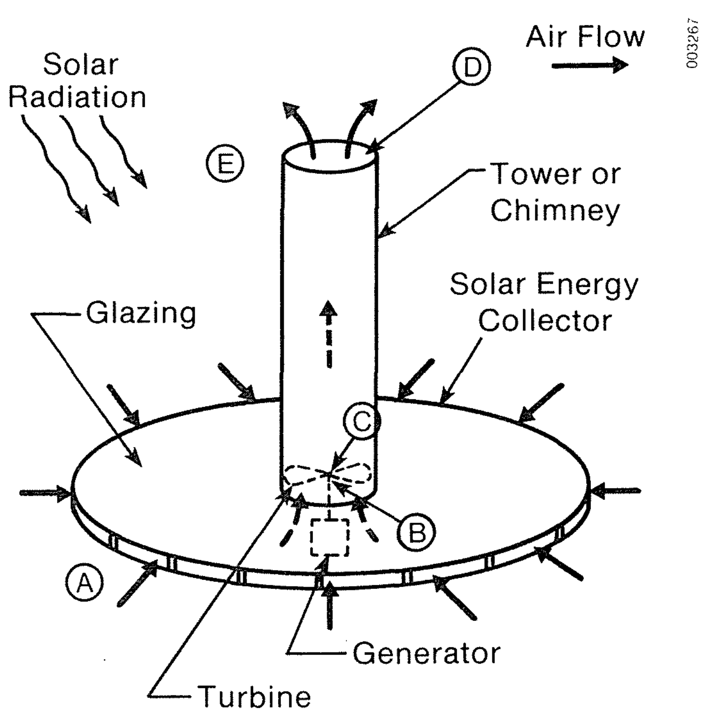

Figure 1-1. A Heated Natural Convection Tower (Chimney)

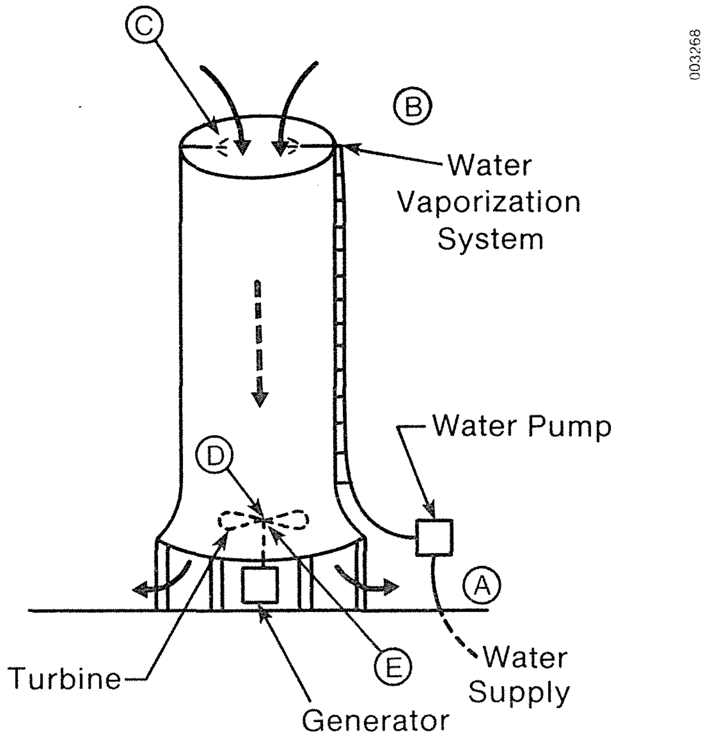

Figure 1-2. A Cooled Natural Convection Tower (Chimney)

SECTION 2.0

HEATED NATURAL CONVECTION TOWERS

The thermodynamic cycle representing an ideal heated natural convection tower in a dry adiabatic atmosphere is shown in Figure 2-1. The dashed lines depict processes occurring within the atmosphere outside the tower. The thermodynamic states A,B,C,D, and E correspond to the equivalently labeled positions in Figure 1-1. The five constituent processes of this cycle are as follows:

1. Isobaric heating of inlet air by solar energy collection. The temperature increase is isobaric because of the open-cycle configuration [Leg A-B].
2. Power extraction by adiabatic, isentropic expansion through a turbine. Turbine, transmission, and generator losses are neglected [Leg B-C].
3. Adiabatic, isentropic expansion of the flow within the tower downstream of the turbine [Leg C-B].
4. Isobaric cooling of the exhaust flow from the tower exit into ambient conditions [Leg D-E].
5. The variation of ambient air pressure and temperature as a function of height for a dry adiabatic atmosphere. Nonadiabatic temperature and pressure gradients are also discussed [Leg E-A].

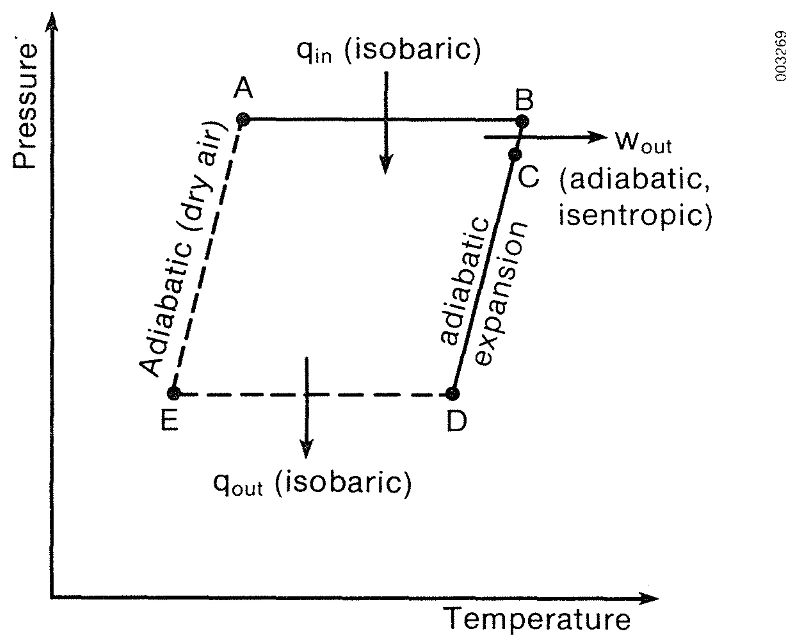

Figure 2-1. Schematic of Thermodynamic Cycle for a Heated Natural Convection Tower. (Dashed lines represent atmospheric processes.)

Barring saturated conditions, humidity effects would be insignificant; therefore, they can be neglected. In addition, dynamic pressures are neglected by presuming that the system can be configured so as to eliminate dynamic pressure effects on the thermodynamic state variables. All losses, such as friction, mechanical, and heat losses, are assumed to be zero, and all heat exchange and work processes are assumed to be 100% efficient. Any real system would incur these losses, however, and resulting efficiencies would be lower than the ideal case.

2.1 IDEAL CONVERSION EFFICIENCY OF A HEATED NATURAL CONVECTION TOWER

The energy conversion efficiency of a heated natural convection tower represented by the thermodynamic cycle shown in Figure 2-1 is given by

$$\eta = \frac{w_{out}}{q_{in}} , \tag{2-1}$$

where w_{out} is the work output (i.e., the power extracted by the turbine), and q_{in} is the heat input provided by solar energy collection. w_{out} is defined as the product of the volume flow rate and the pressure drop across the turbine:

$$w_{out} = A_t V_t \Delta p_t . \tag{2-2}$$

At 100% collection efficiency, q_{in} is equivalent to the increase in enthalpy of the air flow, or

$$q_{in} = \dot{m} c_p \Delta T_h , \tag{2-3}$$

where $\dot{m}$ is the mass flow rate, c_p is the specific heat at constant pressure, and ΔT_h is the increase in air temperature. Because of continuity, (i.e., conservation of mass), $\dot{m}$ can be defined as $\dot{m} = \rho_t A_t V_t$ where ρ_t is the mean air density through the turbine. Equation 2-1 can then be reduced to yield

$$\eta = \frac{\Delta p_t}{\rho_t c_p \Delta T_h} . \tag{2-4}$$

Equation 2-4 shows that the ideal conversion efficiency of a heated natural convection tower is limited by the maximum possible Δp_t attainable for a given ΔT_h and is independent of flow rate and tower cross-sectional area.

2.1.1 The Carnot Efficiency

The heated natural convection tower is a heat engine; hence, it cannot exceed the efficiency of a Carnot cycle with the same hot and cold reservoirs (i.e., with the same temperatures as the ambient atmosphere at the tower inlet and exit heights). Note that the Carnot efficiency must be based on a dry adiabatic atmosphere. Thus,

$$\eta_{Carnot} = 1 - \frac{T_E}{T_A} . \tag{2-5}$$

For an idealized heated natural convection tower, Eqs. 2-4 and 2-5 furnish equivalent results for a dry adiabatic atmosphere. However, Eq. 2-4 provides greater insight into performance parameters and limitations and is, therefore, the basis of the present analysis.

2.1.2 Sample Calculation of Heated Natural Convection Tower Performance

The following is a sample calculation of the ideal conversion efficiency of a 304.8-m (1000-ft)-high heated natural convection tower employing the thermodynamic cycle represented in Figure 2-1. The initial conditions assumed are

$$p_A = p_B = 101{,}325 \text{ Pa } (2116.2 \text{ psf});$$
$$T_A = 288.15 \text{ K } (518.7 \text{ R});$$
$$z_D = z_E = 304.8 \text{ m } (1000 \text{ ft});$$
$$z_A = z_B = z_C = 0;$$
$$c_{pd} = 1005 \text{ Nt-m/kgK } (186.8 \text{ ft-lb}_f/\text{lb}_m\text{R})$$
$$\gamma_d = c_p/c_v = 1.4; \text{ and}$$
$$R_d = 287.05 \text{ Nt-m/kgK } (53.35 \text{ ft-lb}_f/\text{lb}_m\text{R}).$$

$R_d = c_p - c_v$ is the gas constant for dry air. The values chosen for p_A and T_A are the base conditions assumed by the NACA Standard Atmosphere [1].

The turbine exit pressure (p_C) is most easily derived by first determining the tower exit pressure (p_D). For a dry adiabatic atmosphere, p(z) and T(z) as functions of initial pressure (p_o) and temperature (T_o) at height z_o are given by

$$p(z) = \left[1 - \frac{(g/g_c)(\gamma_d - 1)(z - z_o)}{\gamma_d R_d T_o}\right]^{\gamma_d/(\gamma_d - 1)} p_o , \tag{2-6}$$

and

$$T(z) = \left[\frac{p(z)}{p_o}\right]^{(\gamma_d - 1)/\gamma_d} T_o , \tag{2-7}$$

respectively. Equations 2-6 and 2-7 are derived in Appendix A and yield, for $p_o = p_A$, $T_o = T_A$, and $z_o = z_A = 0$,

$$p_E = \left[1 - \frac{(9.8 \text{ m/s}^2)(1.4 - 1)(304.8 \text{ m})}{(1.4)(287.05 \text{ m}^2/\text{s}^2\text{K}))(288.15 \text{ K})}\right]^{1.4/(1.4-1)} 101{,}325 \text{ Pa}$$

$$= 97{,}713 \text{ Pa } (2040.8 \text{ psf}) ,$$

and

$$T_E = \left[\frac{97{,}713 \text{ Pa}}{101{,}325 \text{ Pa}}\right]^{(1.4-1)/1.4} 288.15 \text{ K}$$

$$= 285.18 \text{ K } (513.3 \text{ R}) .$$

T_D is found by assuming a value for ΔT_c, the decrease in temperature necessary to return the exhaust flow to the ambient air temperature at the tower exit height (z_D). ΔT_h is implicitly chosen by this assumption. For the present calculation, ΔT_c is assumed to be 11.0 K (19.8 R), giving T_D = 296.18 K (533.1 R). Because the cooling process is idealized as isobaric, $p_D = p_E$ = 97,713 Pa (2040.8 psf).

Assuming that the flow within the tower is adiabatic, Eqs. 2-6 and 2-7 can again be employed to give p_C = 101,189 Pa (2113.4 psf) and T_C = 299.15 K (538.5 R). With p_C and T_C known and $p_B = p_A$, Eq. 2-7 then yields T_B = 299.26 K (538.7 R). Thus, $\Delta T_h = T_A - T_B$ = 11.11 K (20.0 R), and from Eq. 2-4:

$$\eta = \frac{p_B - p_C}{\rho_{td} c_{pd} \Delta T_h} \tag{2-8}$$

$$= \frac{(101{,}325 - 101{,}189)\ \mathrm{Nt/m^2}}{(1.180\ \mathrm{kg/m^3})(1005\ \mathrm{Nt\text{-}m/kgK})(11.11\ \mathrm{K})}$$

$$= 1.03\%\ .$$

The Carnot efficiency as given by Eq. 2-5 is also 1.03%, thereby confirming the validity of the analysis. The density of ρ_{td} = 1.180 kg/m^3 is based on the perfect gas equation:

$$\rho_{td} = p_t / R_d T_t\ ,$$

where T_t and p_t are the mean temperature and pressure of the flow during the turbine expansion process.

2.2 TOWER HEIGHT EFFECTS ON HEATED NATURAL CONVECTION TOWER PERFORMANCE

Varying the height (z_E) of the heated natural convection tower in the previous analysis shows that the pressure drop across the turbine (Δp_t) and the conversion efficiency increase with tower height. As illustrated in Figure 2-2, the conversion efficiency is a linear function of height approximately equal to 3.4%/km (1.0%/1000 ft).

The tower in a heated natural convection system can also be used as a solar energy collector. The effectiveness of heat added to the tower can be analyzed by substituting

$$q_{in} = \rho_{dt} A_t V_t c_{pd} (\Delta T_h + \Delta T_{tower}) \tag{2-10}$$

for Eq. 2-3. The simple form of Eq. 2-10 is due to conservation of mass. Equation 2-4 then becomes

$$\eta = \frac{\Delta p_t}{\rho_{td} c_{pd} (\Delta T_h + \Delta T_{tower})}\ . \tag{2-11}$$

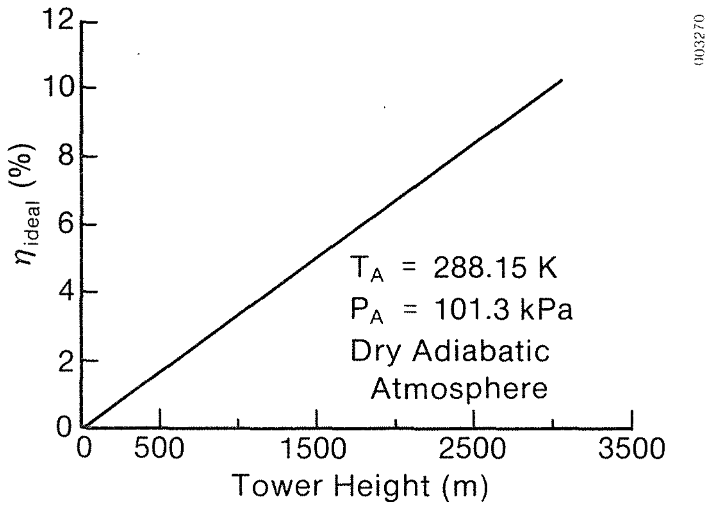

Figure 2-2. Heated Natural Convection Tower Performance as a Function of Tower Height

A plot of heated natural convection tower performance versus the fraction of total heat collection provided by the tower is shown in Figure 2-3. Solar energy collection by the tower is assumed to be constant with height.

The results indicate that the heat input to the tower is less effective and adversely affects overall conversion efficiency. As shown above, heated natural convection tower performance is strongly dependent on height. Hence, any heat input to the system above base level would be less effective, and, consequently, overall conversion efficiency decreases with the use of the tower as a solar collector. Although using the tower as a solar collector would increase heat input and, hence, power output, the projected area of the tower would normally be small compared with the solar collector area at the base of the system, thus limiting the relative collection capacity of the tower. Depending on the tower as the dominant or sole energy collection system would not only reduce conversion efficiency, it would also severely limit power output.

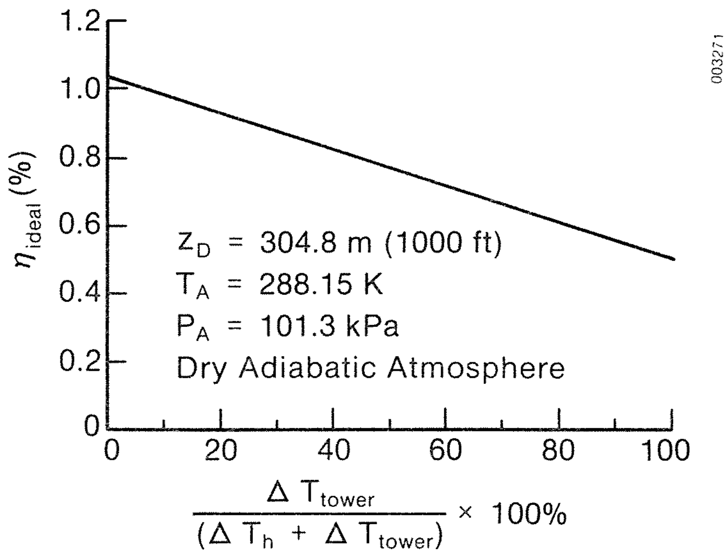

Figure 2-3. Heated Natural Convection Tower Performance vs. Percentage of Total Solar Energy Collection Provided by the Tower

2.3 ENVIRONMENTAL PARAMETERS AFFECTING HEATED NATURAL CONVECTION TOWER PERFORMANCE

Because a natural convection tower exploits the energy differential between the flow within the tower and the ambient atmosphere, the performance of the system would be sensitive to atmospheric conditions. Thus, analysis of a heated natural convection tower requires modelling differing atmospheric conditions and resulting effects on performance. Relevant parameters include the vertical temperature gradient or lapse rate ($-dT/dz$), initial pressure (p_A) and temperature (T_A), and relative humidity (r_A). More detailed treatments of the atmospheric thermodynamics employed by this analysis are available in Refs. [1] and [2].

2.3.1 Lapse Rate Effects

Although the earth's atmosphere is a complex, dynamic environment, a simplified dry air atmospheric model, based on constant temperature lapse rates ($\lambda = -dT/dz$ = constant), illustrates the effects on performance of varying the vertical gradients of atmospheric temperature and pressure. The resulting temperatures and pressures are given by

$$T(z) = T_A - \lambda z \tag{2-12}$$

and

$$p(z) = (1 - \lambda z/T_A)^{1/\lambda R_d}\ p_A\ , \tag{2-13}$$

respectively. Equations 2-12 and 2-13 are derived in Appendix B and can be substituted for Eqs. 2-6 and 2-7 in calculating ideal heated natural convection tower performance. The conversion efficiency in a dry adiabatic atmosphere can be obtained with Eqs. 2-12 and 2-13 by employing the dry adiabatic lapse rate, λ_{ad} = 9.76 K/km (5.36 R/1000 ft). Note that the NACA Standard Atmosphere [1] has a lapse rate of 6.49 K/km (3.56 R/1000 ft). A dry adiabatic atmosphere is neutrally stable; i.e., λ_{ad} is the maximum stable lapse rate. Lapse rates greater than λ_{ad} evidence an unstable atmosphere and are, therefore, precluded from the present study, although the tower could conceivably produce without the addition or extraction of heat in an unstable atmosphere.

Heated natural convection tower performance for differing lapse rates in dry air is shown as a function of temperature increase in the solar collector (ΔT_h) in Figure 2-4. These results demonstrate that conversion efficiency is maximized and independent of ΔT_h for a dry adiabatic atmosphere. In contrast, the efficiencies obtained for absolutely stable atmospheres ($\lambda < \lambda_{ad}$) are dependent on ΔT_h, going to zero as $\Delta T_h \to (1/2)z_E\ (\lambda_{ad} - \lambda)$, and asymptotically approaching the dry adiabatic result as $\Delta T_h \to \infty$. $\Delta T_\lambda = z_E(\lambda_{ad} - \lambda)$ represents the heat "loss" in the atmospheric leg (E-A) of the thermodynamic cycle. For $\Delta T_h = -\Delta T_c = (1/2)\Delta T_\lambda$, $\Delta p_t = 0$ and $\eta = 0$, while for $\Delta T_h = \Delta T_\lambda$, $\Delta T_c = 0$ and η is one-half of the adiabatic conversion efficiency for all λ. As $\Delta T_h \to \infty$, $\Delta T_\lambda/\Delta T_h \to 0$, thus minimizing the adverse effect of heat loss in the nonadiabatic atmosphere. However, since the heat loss is directly proportional to z_E, this effect increases with tower height, as shown in Figure 2-5.

2.3.2 Inlet Atmospheric Conditions (p_A and T_A)

All of the results presented previously are based on an atmospheric pressure p_A of 101,325 Pa (2116.2 psf) and a temperature T_A of 288.15 K (518.7 R) at the ground level inlet to the solar energy collection system. Repeating the calculations with varying inlet atmospheric conditions leads to the conclusion that heated natural convection tower performance is independent of p_A but dependent on T_A. Inspection of Eqs. 2-6, 2-7, 2-12, and 2-13 shows that $T(z)/T_A$, $p(z)/p_A$, and thus Δp_t are functions of T_A but independent of p_A. Ideal conversion efficiency versus inlet temperature (T_A) in a dry adiabatic atmosphere is shown in Figure 2-6 and indicates that the heated natural convection tower would achieve a higher performance in a cooler climate. However, the increase is relatively insignificant, and a high insolation level would be more important in siting the system.

2.3.3 Humidity Effects

The presence of ambient humidity or water vapor requires modification of the physical constants used in the analysis. The necessary equations are

$$c_{pm} = (1 + 0.87\ m)c_{pd}\ , \tag{2-14}$$

$$R_m = (1 + 0.61\ m)R_d\ , \tag{2-15}$$

and

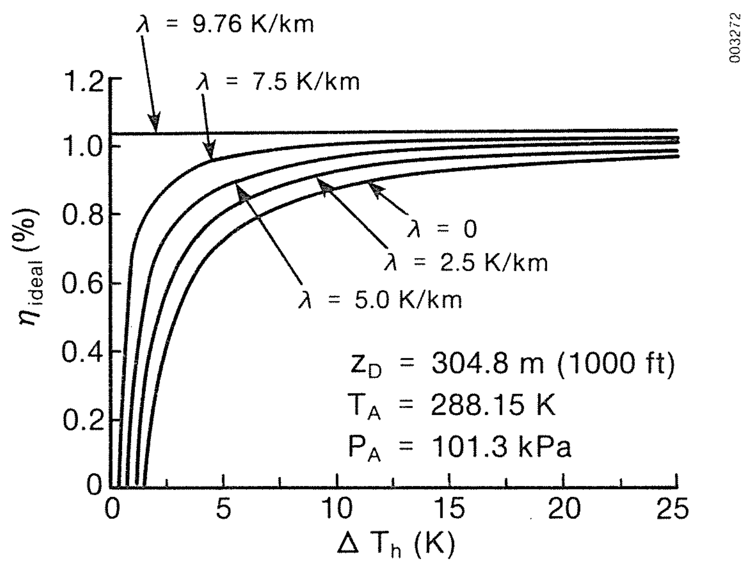

Figure 2-4. Heated Natural Convection Tower Performance vs. Air Flow Rate ($\alpha l/\Delta T_h$) for Different Lapse Rates. (λ = 9.76 K/km is adiabatic lapse rate.)

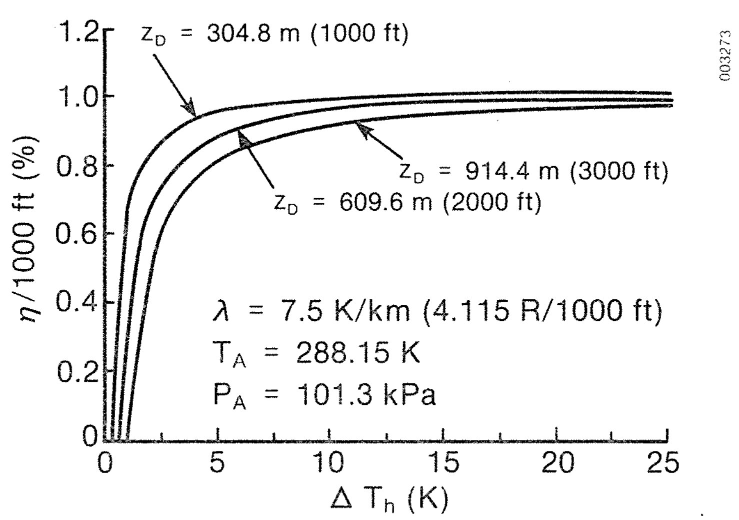

Figure 2-5. Heated Natural Convection Tower Performance vs. Air Flow Rate ($\alpha l/\Delta T_h$) for Different Tower Heights and a Stable, Nonadiabatic Lapse Rate of 7.5 K/km

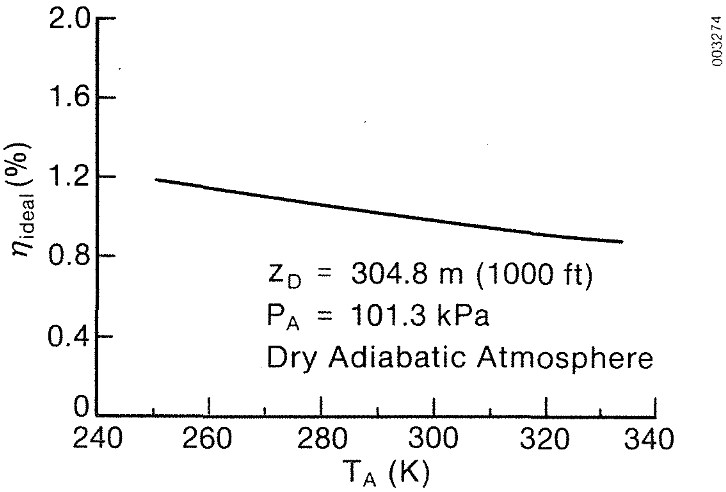

Figure 2-6. Heated Natural Convection Tower Performance vs. Inlet Temperature of the Solar Energy Collection System

$$\gamma_m = \frac{(1 + 0.87\ m)}{(1 + 0.97\ m)}\ \gamma_d\ , \tag{2-16}$$

where m is the mixing ratio:

$$m = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{\rho_v}{\rho_d}\ . \tag{2-17}$$

For unsaturated moist air in an adiabatic atmosphere, dm/dz = 0. Relative humidity is related to the mixing ratio by

$$r = \frac{m}{m_s}\ , \tag{2-18}$$

where m_s is the saturation mixing ratio. Equations 2-14 through 2-17 are derived in Ref. [2].

Substituting the physical constants for an air/water vapor mixture in the analysis of heated natural convection tower performance yields little variation from the dry air results. As m is normally much less than 0.04, the effects of humidity on performance are insignificant. For example, reiterating the previous sample calculation assuming m_A = 0.005, which is sufficiently low to prohibit saturation, produces no significant change in the ideal performance predicted for a heated natural convection tower. Saturated conditions imply low insolation levels (cloudy conditions); thus, they are precluded from this analysis.

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SECTION 3.0

COOLED NATURAL CONVECTION TOWERS

The thermodynamic cycle modelling an ideal cooled natural convection tower in an adiabatic atmosphere is shown in Figure 3-1. As for the heated tower, all losses are assumed to be zero, all heat exchange and work processes are assumed to occur at 100% efficiency, and dynamic pressure effects are neglected. However, with the use of evaporative cooling, humidity effects are significant.

The thermodynamic states A,B,C,D, and E in Figure 3-1 correlate with the identically labeled positions in Figure 1-2. Dashed lines denote thermodynamic processes taking place within the atmosphere external to the tower. The five processes forming the thermodynamic cycle are

1. Variation of ambient temperature and pressure with altitude for an unsaturated ($dm/dz = 0$) adiabatic atmosphere [Leg A-B].
2. Evaporative cooling of inlet air at constant wet-bulb temperature to a saturated state. Assuming that the injected water is at the wet-bulb temperature of the inlet air, the cooling process is isenthalpic as well as isobaric. Thus, the total heat remains constant, but the intake air temperature is reduced, resulting in a downward buoyant force [Leg B-C].

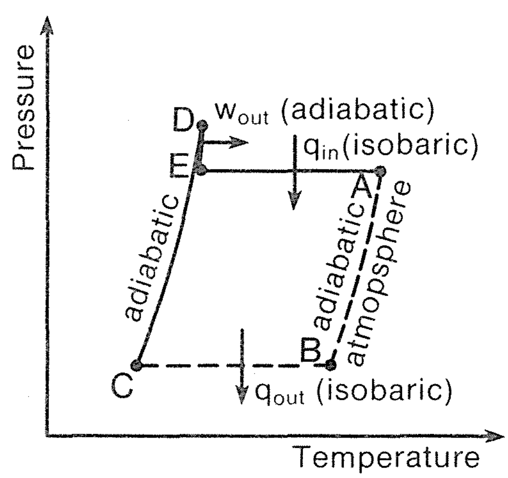

Figure 3-1. Schematic of the Thermodynamic Cycle for an Ideal Cooled Natural Convection Tower. (Dashed lines represent atmospheric processes.)

3. Adiabatic, isentropic compression of the flow within the tower upstream of the turbine. The dry-bulb air temperature increases and the relative humidity decreases in the descending flow [Leg C-D].
4. Power extraction by adiabatic, isentropic expansion through a turbine. Turbine, transmission, and generator losses are neglected [Leg D-E].
5. Isobaric heating of the flow exiting at the bottom of the tower [Leg E-A].

A discussion of evaporative cooling can be found in Ref. [3]. Assuming that the injected water is at the wet-bulb temperature, necessary for purely evaporative cooling, simplifies the analysis. Other water temperatures would have negligible effects on conversion efficiencies, although the reduction in air temperature, and hence the power output (w_{out}), would be affected. Note, however, that the wet-bulb temperatures obtained in this study are only slightly above freezing; thus, they essentially provide the upper limit to heat extraction from the intake air flow. In general, the relative effects on both power conversion efficiency (η) and magnitude (w_{out}) must be differentiated in analyzing cooled natural convection tower performance.

3.1 IDEAL CONVERSION EFFICIENCY OF A COOLED NATURAL CONVECTION TOWER

The thermodynamically rigorous definition of cooled natural convection tower performance is given by Eq. 2-1, with a modification to account for the power needed to pump the required water to the top of the tower. However, the efficiency of greater interest is based on the heat extraction (q_{out}) from the inlet air at the top of the tower; i.e.,

$$\eta = \frac{w_{out} - w_{pump}}{q_{out}} \; . \tag{3-1}$$

As for the heated tower, w_{out}, the turbine power output is given by Eq. 2-2:

$$w_{out} = A_t V_t \Delta p_t \; .$$

The ideal or minimum pumping requirements can be related to the intake air flow rate by

$$w_{pump} = \Delta m \text{ (pumping required/kg}_{water}) \; \dot{m}_{air} , \tag{3-2}$$

where $\dot{m}_{air} = \rho_{tm} A_t V_t$; and Δm, the mixing ratio increase during the evaporative cooling process ($m_C - m_B$), is the amount of water per kg of air necessary to achieve saturation. ρ_{tm} is based on the mixing ratio of the inlet air; i.e.,

$$\rho_{tm} = (1 + m_B)\rho_{td} \; . \tag{3-3}$$

m_B is used in Eq. 3-3, because $\dot{m}_{air}$ does not include water vapor added during the evaporation process. The heat extracted by the evaporative cooling is given by

$$q_{out} = \rho_{tm} A_t V_t c_{pm} \Delta T_c \; . \tag{3-4}$$

Only the heat loss of the intake air needs to be calculated, because the water injected into the flow is assumed to be at the wet-bulb temperature. Thus, for Eq. 3-4, ρ_{tm} is provided by Eq. 3-3 and c_{pm} is given by Eq. 2-14, with $m = m_B$. Combining Eqs. 2-2, 3-2, 3-3, and 3-4 with Eq. 3-1 yields

$$\eta = \frac{\Delta p_t - (g/g_c)(m_C - m_B)(z_B - z_A)(1 + m_B)\rho_{td}}{(1 + m_B)\rho_{td}(1 + 0.87\ m_B)c_{pd}\Delta T_c} . \qquad (3\text{-}5)$$

The calculation of cooled natural convection tower performance, based on the ideal thermodynamic cycle shown in Figure 3-1, proceeds similarly to that of the heated tower. The initial conditions and physical constants used in the analysis are

$p_A = p_E = 101{,}325$ Pa (2116.2 psf);

$T_A = 288.15$ K (518.7 R);

$z_B = z_C = 304.8$ m (1000 ft);

$z_A = z_D = z_E = 0$;

$r_A = 20\%$;

$c_{pd} = 1005$ Nt-m/kgK(186.8 ft-lb_f/lb_mR);

$\gamma_d = c_{pd}/c_{vd} = 1.40$; and

$R_d = 287.05$ Nt-m/kgK (53.35 ft-lb_f/lb_mR),

where p_A and T_A are based on the NACA Standard Atmosphere [1].

For the initial conditions chosen, the mixing ratio of $m_A = 0.0021$ can be obtained from a psychrometric chart [3] developed for the base level pressure of 101,325 Pa (2116.2 psf). Equations 2-15 and 2-16 then yield $R_m = 287.4$ Nt-m/kgK (53.4 ft-lb_f/lb_mR) and $\gamma_m = 1.400$, respectively. From Eq. 2-6, with $p_o = p_A$, $T_o = T_A$, and $z_o = z_A = 0$, p_B is given by

$$p_B = \left[1 - \frac{(g/g_c)(\gamma_m - 1)(z_B - z_A)}{\gamma_m R_m T_A}\right]^{\gamma_m/(\gamma_m - 1)} p_A$$

$$= 97{,}717 \text{ Pa } (2040.9 \text{ psf}) . \qquad (3\text{-}6)$$

T_B, the dry-bulb temperature at height z_B, is obtained from Eq. 2-7; i.e.,

$$T_B = \left[\frac{p_B}{p_A}\right]^{(\gamma_m - 1)/\gamma_m} T_A$$

$$= 285.2 \text{ K } (513.4 \text{ R}) . \qquad (3\text{-}7)$$

With an unsaturated atmosphere, $m_B = m_A = 0.0021$.

For a dry-bulb temperature of 285.2 K (513.4 R) and a mixing ratio of 0.0021, the wet-bulb temperature is 277.4 K (499.3 R). This result is obtained by interpolation between the base-level psychrometric chart and a psychrometric chart developed for an altitude of ~610 m (2000 ft) [4]. However, using only the base-level chart would introduce little error into the calculations for tower heights less than ~610 m (2000 ft).

Isobaric, isenthalpic, evaporative cooling to saturation would yield T_C = 277.4 K (499.3 R) with $p_C = p_B$ = 97,717 Pa (2040.9 psf) and m_C = 0.0053. R_m and γ_m for a mixing ratio of 0.0053 are 288.0 Nt-m/kgK (53.5 ft-lb_f/lb_mR) and 1.399, respectively. The flow is saturated only at the top of the tower, becoming increasingly unsaturated while descending within the tower. Hence, $m_D = m_C$, and Eqs. 2-6 and 2-7 can again be used to obtain p_D = 101,420 Pa (2118.1 psf), T_D = 280.4 K (505.1 R), and T_E = 280.3 K (504.9 R).

From Eq. 3-5, the ideal conversion efficiency based on the heat extraction at the top of the tower is

$$\eta = \frac{(p_D - p_E) - (g/g_c)(m_C - m_B)(z_B - z_A)(1 + m_B)\rho_{td}}{(1 + m_B)\rho_{td}(1 + 0.87\ m_B)c_{pd}(T_B - T_C)}$$

$$= \frac{(101{,}420 - 101{,}325)\ \text{Nt/m}^2 - 12\ \text{Nt/m}^2}{(1{,}258\ \text{kg/m}^3)(1006.8\ \text{Nt-m/kgK})(7.8\ \text{K})}\ (100\%)$$

$$= 0.84\%,$$

where $$w_{pump} = (0.0032)(9.8\ \text{m/s}^2)(304.8\ \text{m})(1.258\ \text{kg/m}^3)$$

$$= 12\ \text{Nt/m}^2\ .$$

The mean dry air density of 1.255 kg/m^3 (0.0786 lb_m/ft^3) for the flow through the turbine is obtained from the perfect gas equation; i.e., $\rho_{td} = p_t/R_dT_t$, where p_t and T_t are the mean turbine air pressure and temperature, respectively.

Neglecting the pumping requirements and basing the efficiency on q_{in}, the heat added to the exhausted flow, yields a conversion efficiency of 0.95%. The Carnot efficiency, given by Eq. 2-5 with T_B substituted for T_E, is 1.03%. Thus, the addition of water vapor during the evaporative cooling process has a small but adverse effect on the ideal cooled natural convection tower performance.

3.2 TOWER HEIGHT EFFECTS ON COOLED NATURAL CONVECTION TOWER PERFORMANCE

Cooled natural convection tower performance is also strongly dependent on tower height. As shown in Figure 3-2, the ideal conversion efficiency increases as a linear function of height equal to ~2.75%/km (0.85%/1000 ft). The lower result relative to the heated natural convection tower is due to the addition of water vapor in the evaporative cooling process and associated water pumping losses. Without the pumping losses, the ideal conversion efficiency is ~3.1%/km (0.95%/1000 ft).

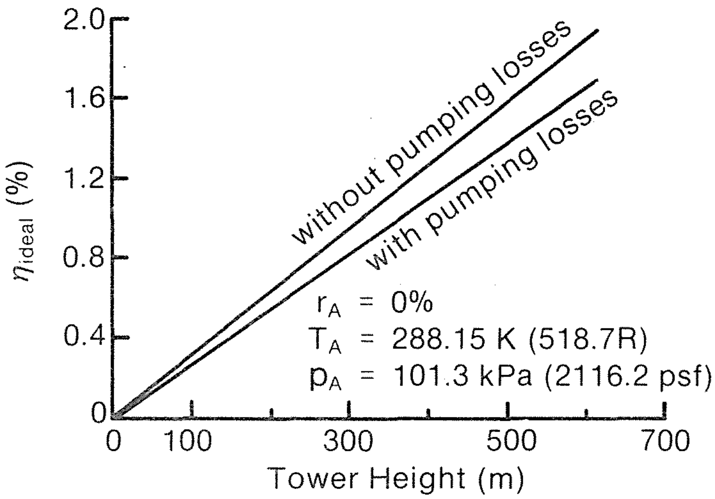

Figure 3-2. Cooled Natural Convection Tower Performance as a Function of Height (z_B)

Evaporative cooling of the flow within the tower would decrease the overall conversion efficiency yet increase the total heat extraction from the airflow and, hence, the power output of a cooled natural convection system. Because the conversion efficiency for any increment of heat extraction is dependent on the height of the extraction point, evaporative cooling within the tower adds to the downward buoyant force, but less effectively than the evaporative cooling at the tower top inlet. However, the increase in power output due to evaporative cooling within the tower would be relatively small. For the previous sample calculation, the power increase would be less than 10%. This result is obtained by replacing T_D with its corresponding wet bulb temperature in the previous sample calculation.

3.3 ENVIRONMENTAL PARAMETERS AFFECTING COOLED NATURAL CONVECTION TOWER PERFORMANCE

Analysis of a cooled natural convection tower also requires modelling different atmospheric conditions to determine the resulting effects on performance. The relevant parameters again include lapse rate ($-dT/dz$), initial pressure (p_A) and temperature (T_A), and ambient relative humidity (r_A).

3.3.1 Initial Atmospheric Conditions (p_A and T_A)

Varying the initial pressure and temperature used in the cooled natural convection tower analysis shows that performance is independent of p_A but dependent on T_A. As for the heated tower, $T(z)/T_A$, $p(z)/p_A$, and Δp_t are functions of T_A but independent of p_A. A plot of cooled natural convection tower performance versus T_A for an adiabatic atmosphere is shown in Figure 3-3. Figure 3-3 indicates that the cooled natural convection tower would also achieve better efficiencies in a cooler climate. However, power output would decrease as T_A decreases because of diminished heat extraction.

3.3.2 Lapse Rates and Humidity Effects

The combined effects of the vertical temperature gradient or lapse rate and the relative humidity are of particular importance for cooled natural convection towers. As can be seen in Figure 3-4, the ideal conversion efficiency of a cooled tower is independent of ambient relative humidity in an adiabatic atmosphere. However, humidity effects are increasingly adverse as the lapse rate decreases in the absolutely stable range ($\lambda < \lambda_a$). Also note that, for any lapse rate, power output must necessarily go to zero as the ambient relative humidity at the tower inlet approaches 100%. Thus, cooled natural convection tower performance is adversely affected by high ambient relative humidity and/or low lapse rates.

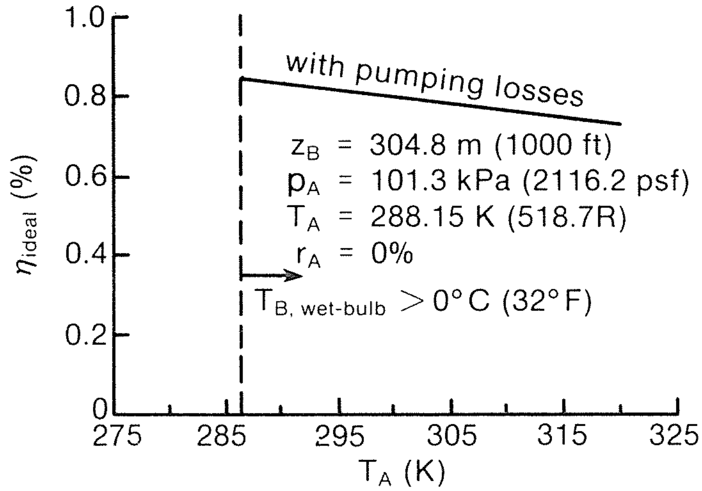

Figure 3-3. Cooled Natural Convection Tower Performance vs. Initial Temperature at Ground Level (T_A)

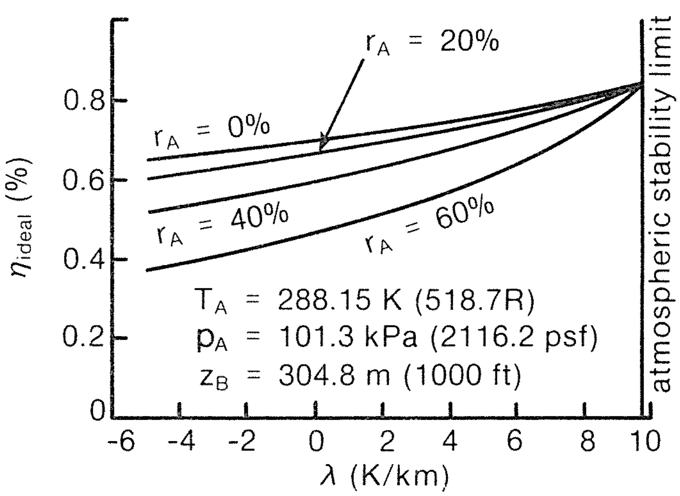

Figure 3-4. Cooled Natural Convection Tower Performance vs. Atmospheric Temperature Gradient ($\lambda = -dT/dz$) with Varying Ambient Relative Humidity

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SECTION 4.0

SUMMARY AND CONCLUSIONS

Use of the natural convection phenomenon for energy conversion has been analyzed for both heated (ascending) and cooled (descending) flows within a tower or chimney. However, the results are applicable to any open-cycle configuration. The heated tower flow can be produced by direct collection of incident solar energy. Evaporative cooling with water can be employed as a cooling mechanism for the cooled natural convection tower.

Natural convection tower performance is most strongly affected by tower height. For the heated tower, the ideal conversion efficiency increases at ~3.4%/km (1.0%/1000 ft). Cooled tower performance increases at the lower rate of ~2.75%/km (0.85%/1000 ft) because of the addition of water vapor in the evaporative cooling process and water pumping losses.

Ambient atmospheric conditions such as a very stable lapse rate and a high relative humidity can have significantly adverse effects on natural convection tower performance. The ideal conversion efficiencies of both heated and cooled towers decrease as the lapse rate, or vertical temperature gradient, decreases within its stable range. Maximum performance is achieved for an adiabatic atmosphere corresponding to the maximum or neutrally stable lapse rate. Increases in ambient relative humidity significantly affect cooled natural convection tower performance, both by limiting heat extraction from the intake airflow and by reducing the ideal conversion efficiency for absolutely stable lapse rates. Analysis also shows that ideally, natural convection towers would provide higher conversion efficiencies in cooler climates, yet would produce less power because of lower insolation levels for the heated tower and diminished heat extraction from the cooled tower.

To determine the economic potential of natural convection towers, a cost study would be needed. However, some conclusions can be drawn from this study regarding performance limitations with significant economic impacts. For example, the low conversion efficiencies of heated natural convection towers would necessitate a very large solar collector area for useful power output levels. Thus, solar collector costs and associated land requirements may severely limit the potential of heated natural convection towers. Similarly, the cooled natural convection tower would be most efficient in a low-humidity environment. However, except in coastal regions, low humidity and abundant water supplies usually are mutually exclusive, which limits the applicability of cooled natural convection towers. Note also that both performance and cost for natural convection towers increase with tower height. Further analysis would be needed, then, to ascertain the most cost-effective tower height.

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SECTION 5.0

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APPENDIX A

VARIATION OF PRESSURE AND TEMPERATURE WITH HEIGHT FOR A DRY ADIABATIC ATMOSPHERE

The variation of pressure and temperature with height for a dry adiabatic atmosphere can be derived from the ideal equation of state for dry air,

$$pv = R_d T \ , \tag{A-1}$$

the Poisson equation for an ideal adiabatic process using air,

$$pv^{\gamma} = k = \text{constant} \ , \tag{A-2}$$

and the hydrostatic equation,

$$dp = -\rho_d \left[\frac{g}{g_c}\right] dz \ . \tag{A-3}$$

R_d, the specific gas constant for dry air, is given by

$$R_d = \frac{R}{M_d} = 287.05 \text{ J/kg/K} \ , \tag{A-4}$$

where M_d = 28.97 kg/mol is the molecular weight of dry air.

Using the relation $v = 1/\rho$, Eq. A-2 can be modified to provide

$$\rho_d = \frac{p^{1/\gamma_d}}{k^{1/\gamma_d}} \ . \tag{A-5}$$

Substituting Eq. A-5 into Eq. A-3 and integrating, gives

$$\int_{p_A}^{p(z)} p^{-1/\gamma d} dp = -\frac{g}{g_c k^{1/\gamma d}} \int_{z_A}^{z} dz \ ,$$

or

$$\frac{\gamma_d}{\gamma_d - 1}\left[p(z)^{\frac{\gamma_d - 1}{\gamma_d}} - p_A^{\frac{\gamma_d - 1}{\gamma_d}}\right] = \frac{-g}{g_c k^{1/\gamma_d}} (z - z_A) \ . \tag{A-6}$$

Combining Eqs. A-1 and A-2 yields

$$k^{1/\gamma d} = p_A^{1/\gamma d}(R_d T_A / p_A) = p_A^{\frac{1-\gamma d}{\gamma d}} R_d T_A \ . \tag{A-7}$$

Substituting Eq. A-7 into Eq. A-6 and rearranging then gives

$$p_A^{1-\gamma_d/\gamma_d}\left(p(z)^{\gamma_d-1/\gamma_d} - p_A^{\gamma_d-1/\gamma_d}\right) = \frac{-(\gamma_d-1)g}{\gamma_d g_c R_d T_A}(z - z_A) ,$$

or

$$p(z) = \left[1 - \frac{(\gamma_d-1)g(z-z_A)}{g_c \gamma_d R_d T_A}\right]^{\gamma_d/(\gamma_d-1)} p_A \quad . \qquad \text{(A-8)}$$

With p(z) determined, the corresponding T(z) is derived from Eq. A-7; i.e.,

$$p_A^{-1-\gamma_d/\gamma_d} - R_d T_A = p(z)^{1-\gamma_d/\gamma_d} R_d T(z) ,$$

or

$$T(z) = \left[\frac{p(z)}{p_A}\right]^{\gamma_d-1/\gamma_d} T_A \quad . \qquad \text{(A-9)}$$

APPENDIX B

VARIATION OF PRESSURE AND TEMPERATURE WITH HEIGHT AS A FUNCTION OF LAPSE RATE ($\lambda = -dT/dz$)

The change of pressure and temperature with height for varying lapse rates ($\lambda = -dT/dz$) can be derived by employing the ideal equation of state for dry air,

$$pv = R_d T \ , \tag{B-1}$$

and the hydrostatic equation,

$$dp = -\rho_d \left[\frac{g}{g_c}\right] dz \ . \tag{B-2}$$

For a given lapse rate, T(z) is given by

$$T(z) = T_A - \lambda z \ , \tag{B-3}$$

where T_A is the temperature at $z = z_A$. Combining Eq. B-2 with Eq. B-3 gives

$$dp = \frac{\rho_d g}{\lambda g_c} dT \ . \tag{B-4}$$

Using Eq. B-1 and the relationship $\rho = 1/v$, Eq. B-4 then becomes

$$dp = \frac{gp}{g_c \lambda R_d T} dT \ ,$$

or

$$\frac{dp}{p} = \frac{g}{g_c \lambda R_d} \frac{dT}{T} \ . \tag{B-5}$$

Integrating both sides of Eq. B-5 yields

$$\ln \left(p(z)/p_A\right) = \frac{g}{g_c \lambda R_d} \ln \left(T(z)/T_A\right) \ ,$$

or (B-6)

$$p(z) = \left(\frac{T(z)}{T_A}\right)^{g/g_c \lambda R_d} p_A \ .$$

Substituting Eq. B-3 for T(z) then gives

$$p(z) = \left(1 - \frac{\lambda z}{T_A}\right)^{g/g_c \lambda R_d} p_A \ . \tag{B-7}$$

Document Control Page	1. SERI Report No. SERI/TR-211-1950	2. NTIS Accession No.	3. Recipient's Accession No.

4. Title and Subtitle A Theoretical Analysis of Solar-Driven Natural Convection Energy Conversion Systems	5. Publication Date January 1984 6.
7. Author(s) Eric W. Jacobs and David D. Lasier	8. Performing Organization Rept. No.
9. Performing Organization Name and Address Solar Energy Research Institute 1617 Cole Boulevard Golden, Colorado 80401	10. Project/Task/Work Unit No. 1067.10 11. Contract (C) or Grant (G) No. (C) (G)
12. Sponsoring Organization Name and Address	13. Type of Report & Period Covered Technical Report 14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

This report presents a theoretical study of solar-powered natural convection tower (chimney) performance. Both heated and cooled towers are analyzed; the latter uses evaporating water as the cooling mechanism. The results, which are applicable to any open-cycle configuration, show that the ideal conversion efficiencies of both heated and cooled natural convection towers are linear functions of height. The performance of a heated tower in an adiabatic atmosphere ideally approaches the Carnot efficiency limit of ≈3.4%/km (1.0%/1000 ft). Including water pumping requirements, the ideal limit to cooled tower performance is ≈2.75%/km (0.85%/1000 ft). Ambient atmospheric conditions such as vertical temperature gradient (lapse rate) and relative humidity can have significantly adverse effects on natural convection tower performance. The combined effects of lapse rate and ambient relative humidity are especially important to cooled natural convection towers.

17. Document Analysis

a. Descriptors Convection ; Cooling ; Energy Conversion ; Heating ; Natural Convection ; Performance ; Thermodynamics

b. Identifiers/Open-Ended Terms

c. UC Categories

60 and 62c

18. Availability Statement National Technical Information Service U.S. Department of Commerce 5285 Port Royal Road Springfield, Virginia 22161	19. No. of Pages 37 20. Price A03

Form No. 0069 (3-25-82)