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# Radiative Transport Models for Solar Thermal Receiver/Reactors

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## ABSTRACT

Modeling the behavior of solar-driven chemical reactors requires detailed knowledge of the absorbed solar flux throughout the calculation domain. Radiative transport models, which determine the radiative intensity field and absorbed solar flux, are discussed in this paper with special attention given to particular needs for the application of solar thermal receiver/reactors. The geometry of interest is an axisymmetric cylinder with a specified intensity field at one end, diffuse reflection at boundaries, and containing a participating medium. Participating media are of interest because receiver/reactors are expected to have one or more zones containing small particles or monoliths acting as absorbers or catalyst supports, either of which will absorb, emit, and scatter radiation. A general discussion of modeling techniques is given, followed by a more complete discussion of three models--the two-flux, discrete ordinate, and the Monte Carlo methods. The methods are compared with published benchmark solutions for simplified geometries--the infinite cylinder and plane slab--and for geometries more closely related to receiver/reactors. Conclusions are drawn regarding the applicability of the techniques to general receiver/reactor models considering accuracy, ease of implementation, ease of interfacing with solution techniques for the other conservation equations, and numerical efficiency.

## NOMENCLATURE

- $a_m$ - coefficient of the phase function expansion
- b - back scatter fraction
- I - intensity, $W/m^2sr$
- $I_b$ - total blackbody intensity, $W/m^2sr$
- M - number of directions for the discrete ordinate method
- p - scattering phase function
- $Q_l$ - dimensionless heat flux at z = 0
- $Q_m$ - dimensionless heat absorption by medium
- $Q_0'$ - dimensionless flux divergence at z = 0
- q - radiative flux vector, $W/m^2$
- r - position vector, m
- z - axial coordinate, m
- $Z_0$ - cylinder length or slab thickness, m
- $\beta$ - extinction coefficient - $\sigma + \kappa$, $m^{-1}$
- $\varepsilon$ - emissivity of surface
- $\theta_0$ - half angle of cone containing incident intensity, Fig. 2
- $\kappa$ - absorption coefficient, $m^{-1}$
- $\sigma$ - scattering coefficient, $m^{-1}$
- $\tau_0$ - optical thickness
- $\omega$ - single scatter albedo = $\sigma/\beta$
- $\Omega$ - direction of intensity

## INTRODUCTION

Historically, concentrated solar thermal energy has been applied primarily to produce electricity via heat engine cycles. There has been recent interest, however, in applying solar thermal energy to other processes, most notably those involving fuels and chemicals applications. A recent conference featured presentations on solar destruction of hazardous waste, solar detoxification of organics in water, solar-induced surface transformation of materials, solar treatment of carbon fibers, and photo-assisted solar thermal chemical reactions [Couch 1989].

Although the work mentioned above was primarily concerned with proof-of-concept experiments, ultimate commercialization of these technologies will require a capability to predict receiver/reactor behavior via detailed analytical models. The reasons for this are: (i) validated models can be used to complement and extrapolate costly experimental data and to give guidance to experimental programs, (ii) results from such models support systems studies aimed at estimating the economic potential of the processes, and (iii) the models can be used to design receiver/reactors.

Depending on the level of complexity needed, a receiver/reactor model will include some or all of the following: (i) a radiative transport model to predict the local volumetric absorption of solar energy within the receiver/reactor, (ii) a radiative transport model to predict the transport of infrared radiation within the receiver/reactor, (iii) convective and conductive heat transfer models to determine the coupling of the absorbed solar energy into a gas phase or into receiver/reactor components, (iv) a model of the conservation of mass with the pertinent chemical reactions and reaction rates, and (v) models of the conservation of momentum. Item (i) will be required for all receiver applications. Item (ii) will be required for those receiver/reactors operating at high temperature. Item (iv) will be required for receivers incorporating chemical reactions. The remaining two items may be necessary depending on the particular receiver/reactor of interest. For example, if the chemical reaction is carried out in the gas phase, convective heat transfer and conductive heat transfer are probably not important. Convection and conduction will be important mainly in cases where solar energy is absorbed on a solid surface and then transferred to a gas or to other surfaces. If the gas flow pattern is well defined in the receiver/reactor, modeling the conservation of momentum may not be needed.

In the process of developing a capability for modeling receiver/reactors, we chose to focus first on the radiative transport for solar wavelengths because it is common to all receiver/reactor configurations. This component of a receiver/reactor model is fundamental from the standpoint of giving the quantity that drives the chemical reaction--the local volumetric absorption of solar energy.

Although a significant body of literature describes various comparisons and applications of radiative transport models, very little of that literature deals specifically with concentrated solar energy as a boundary condition and properties and geometries relevant to receiver/reactors. The objective of this paper, therefore, is to determine the capabilities of several radiative transport modeling methods as applied to the receiver/reactor problem. The characteristics of importance include accuracy, reliability, robustness, numerical efficiency, and applicability to receiver/reactor models. This paper will first present a brief description of receiver/reactors currently under consideration within the solar research and development community. Then, an introduction to methods of solving the radiative transport equations will be covered in general terms. Next, validation of three of these methods will be covered. These three methods will then be compared in terms of their ability to solve problems of interest to the receiver/reactor designer, and conclusions regarding these capabilities will be given.

## BACKGROUND

Several receiver/reactors are currently under investigation. A brief description of these concepts will help to understand the scope of design parameters such as geometry, optical properties, operating temperatures, and reactions that must be considered when developing a receiver/reactor model.

Glatzmaier et al. [1989] are currently investigating the use of a dish-mounted high temperature (~1000 K) photoreactor to destroy hazardous organic vapor. The reactor employs a two-stage design (see Fig. 1). The first stage consists of an absorbing, porous, ceramic insert in which air containing the hazardous vapor is heated to a high temperature. The vapor is then photolytically decomposed in the second stage using available ultraviolet (UV) solar radiation in a gas phase reaction. The optical properties of the ceramic insert are such that much of the incoming radiation is absorbed while the UV radiation is diffusely reflected to enhance the photodestruction in the reactor second stage.

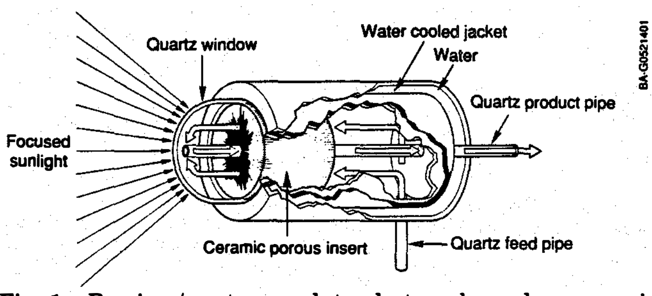


Fig. 1. Receiver/reactor used to destroy hazardous organic vapors [Glatzmaier 1989].

Fish and Hawn [1987] developed a dish-mounted receiver/reactor for use in the carbon dioxide reforming of methane. The reactor consists of a single-stage design in which methane and carbon dioxide are heated to a high temperature (~1000 K) and reformed to carbon monoxide and hydrogen through the use of an absorbing, porous, alumina insert. The absorber is impregnated with rhodium catalyst.

These two designs represent two-dimensional axisymmetric geometries in which the incident intensity entering the reactor is isotropic within the 45° cone angle characteristic of point focus concentrators.

Researchers are currently investigating the use of solar energy for the destruction of toxic organics in dilute solution in wastewater. Pruden and Ollis [1983] examined the degradation of trichloro-ethylene under illumination of concentrated UV radiation in the presence of a photoactive catalyst under laboratory conditions. The reaction takes place at low temperature, relying on the relatively low concentrations of UV radiation available from line focusing concentrators. In an actual system, the photocatalyst such as titanium dioxide would be impregnated upon a solid matrix inside a receiver tube. This system demonstrates a two-dimensional nonaxisymmetric geometry with a circumferential variation in the incident intensity. A similar concept incorporating a falling film reactor using radiation concentrated by a field of heliostats has been conceptualized for destruction processes requiring greater throughput of the waste material [Tyner 1989].

Each receiver described above uses solar energy to obtain a desired reaction between compounds. As a more general example, Skocypec et al. [1988] describe a dish-mounted receiver in which an air stream is heated to high temperature for downstream processes. Heating is accomplished by absorption of solar radiation onto a metal wire grid. Air flows through the absorber where energy is convectively transferred to the gas stream.

## ANALYSIS METHODS

To determine the local volumetric absorption of solar energy in the receivers described above, it is necessary to solve the radiative transport equation (RTE), which may be written in general for a gray medium as follows:

$$(\Omega\bullet\nabla)I(\mathbf{r},\Omega) = -(\kappa+\sigma)I(\mathbf{r},\Omega) + \kappa I_b(\mathbf{r}) + \frac{\sigma}{4\pi}\int_{\Omega'=4\pi} I(\mathbf{r},\Omega')p(\Omega'\rightarrow\Omega)d\Omega' \quad (1)$$

Equation 1 relates the gradient of radiant intensity in direction $\Omega$ (left side of equation) to absorption, out-scattering, emission, and in-scattering, respectively. With properties of the surface bounding the medium and appropriate boundary conditions, the solution of Eq. 1 gives the radiant intensity everywhere in the medium as a function of direction $\Omega$. The intensity field is then related to the volumetric absorption of radiant energy by the divergence of the radiative flux

$$\nabla\bullet\mathbf{q} = \kappa[4\pi I_b(\mathbf{r}) - \int_{\Omega=4\pi} I(\mathbf{r},\Omega)d\Omega] \quad (2)$$

We see from Eq. 2 that flux divergence is equal to the local difference in emitted and in-scattered radiation.

The solution of Eq. 1 has received considerable attention in the literature because of interest in a wide range of engineering applications. Viskanta [1966, 1986] and Howell [1988] review the state of the art in radiant heat transfer with emphasis on participating media. Viskanta [1986] categorized solution methods as analytical, approximate, and numerical. For solving practical problems, it appears that methods in the last two categories are more applicable.

Approximate methods include multi-flux, discrete ordinates, spherical harmonics, and others, all of which are approximations to Eq. 1 and can be applied to multidimensional geometries, an advantage for receiver/reactor applications. An example of a numerical method is the Monte Carlo approach, which also can be applied to multidimensional geometries.

Application of these various methods has concentrated for the most part on one-dimensional slab problems. Daniels et al. [1978, 1979] compared the two-flux, six-flux, and discrete ordinate methods to predict absorption and scattering in a layer of turbid water. Brewster and Tien [1982] compared results from the two-flux method with an exact solution of the RTE for pure scattering in a slab geometry. Menguc and Viskanta [1983] compared two-flux, spherical harmonics, and discrete ordinate methods for the slab geometry with an absorbing, highly forward-scattering medium. Incropera and Houf [1979] investigated the viability of a three-flux method for predicting radiative transfer in an aqueous suspension layer. Tong and Tien [1983] compared results from the two-flux model and a linear anisotropic scattering model for a slab geometry in radiative equilibrium. Houf and Incropera [1980] compared several methods of solving the RTE for the slab geometry with an absorbing, scattering medium. Azad and Modest [1981] solved the RTE exactly for an infinitely long cylinder containing an absorbing, emitting, scattering medium.

Howell and Perlmutter [1964] and Perlmutter and Howell [1964] used the Monte Carlo method to solve for radiant transfer between plane gray walls and between infinitely long concentric cylinders, respectively. In both cases, the medium was absorbing and emitting, but not scattering.

For multidimensional problems, Fiveland [1982, 1984] applied the discrete ordinate method to radiant transfer in a rectangular geometry and in an axisymmetric cylindrical geometry, respectively.

## MODELS UTILIZED

The literature provides a good basis for understanding how to apply the various methods of solving the RTE, but it is difficult to determine which methods can be applied successfully to the receiver/reactor problem. Subtle interactions between the problem parameters (radiative properties, geometry, boundary conditions) and the solution methods can lead to convergence problems, poor accuracy, and other difficulties. For this reason, we chose to investigate in detail three RTE solution methods: the two-flux method, the discrete ordinate method, and the Monte Carlo method. Although several other RTE solution methods could have been chosen for investigation, these three cover a reasonably broad spectrum. The two-flux method is a widely used, relatively simple differential method applicable to one-dimensional problems. The discrete ordinate method is a more detailed differential method that has seen widespread use, often as a baseline against which other methods are compared and which has recently seen application to multidimensional problems. Finally, the Monte Carlo method differs significantly from the other two in that it does not seek a solution of Eq. 1 but rather approaches the problem statistically. Development of these three methods has been discussed in detail in the references cited and will not be repeated here.

## CODE VALIDATION

Each of the three methods was used to develop computer codes. To ensure proper operation of these codes, they were compared with published calculations, and the results will be discussed in this section. For any calculation, the parameters of importance include problem geometry, boundary conditions (heated surfaces or specified surface intensity or flux), surface emissivities, and medium properties (optical thickness, single scatter albedo, and scattering phase function). Results can be presented in terms of surface heat transfer or volumetric absorption of radiant energy. Finally, the solution may be carried out by assuming radiative equilibrium. With this assumption, there is no net volumetric absorption of energy because the medium temperature is assumed to rise sufficiently so that remitted radiation balances absorbed radiation. If radiative equilibrium is not assumed, some means other than medium re-emission is assumed to be removing the absorbed radiation.

Tong and Tien [1983] solved for the rate of heat transfer from surface to surface in the one-dimensional slab problem, assuming radiative equilibrium, black surfaces, and an isotropic source on one plane of the slab. Figure 2 shows the geometry and nomenclature for this problem. For a range of albedo 0 to 1.0, a range of optical thickness 0.2 to 6.0, and a range of backward scatter fraction 0.025 to 0.5, we found that our two-flux code agreed with their results to better than four decimal places.

For the case of not assuming radiative equilibrium, Menguc and Viskanta [1983] give surface heat transfer and flux divergence at both surfaces of a one-dimensional slab calculated with the two-flux method. An albedo of 0.8, a range of backscattering parameter from 0.345 to 0.039, and a range of optical thickness from 0.1 to 10 were chosen. A comparison of our two-flux calculations with those of Menguc and Viskanta again showed very close agreement.

The previous two comparisons show that our two-flux code operates correctly over a wide range of problem parameters when compared to other two-flux codes.

Siegel and Howell [1981] present the solution for the slab problem with an isotropically scattering medium, no absorption, black surfaces, and one surface heated, in terms of the surface heat flux. In comparing the results of the present two-flux, discrete ordinate, and Monte Carlo codes with Siegel and Howell [1981], we found that the two-flux method is somewhat less accurate than the other two methods over the range of optical thickness of interest (0.1 to 3.0). The two-flux method differed by 17% from the Siegel and Howell results for optical thickness of 3.0. The discrete ordinate and Monte Carlo results agreed to within 2% of the Siegel and Howell results. However, pure scattering is not as interesting as is absorption with scattering. For this reason, more detailed validation runs comparing against Menguc and Viskanta [1983] will be discussed shortly.

Azad and Modest [1981] solved for radiative transport in an infinitely long cylinder with black walls containing an absorbing and scattering medium with albedo of 0.5. A constant temperature medium and linear anisotropic scattering (LAS) were assumed. In this problem, radial heat transport is emphasized, and the two-flux model is not applicable. Heat flux at the cylinder surface was presented. We found satisfactory performance of both the discrete ordinate and Monte Carlo codes, which agreed with the published results to within ±5%.

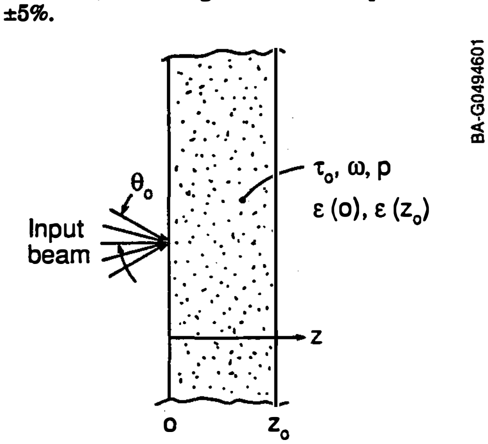


Fig. 2. Nomenclature for the one-dimensional radiative transport problem.

The final set of validation runs involved further comparisons with Menguc and Viskanta [1983]. They compared solutions of the one-dimensional slab problem from the two-flux, discrete ordinate, and spherical harmonics methods. We compared the results of our two-flux, discrete ordinate, and Monte Carlo codes with their results from the discrete ordinate and two-flux methods. They used two scattering phase functions, MVI and MVII, which represent moderate forward scatter (b = 0.345) and strong forward scatter (b = 0.075), respectively. A single scatter albedo of 0.8 and optical thickness of 1.0 were assumed. The boundary condition corresponded to an isotropic source at z = 0. Dimensionless heat flux at z = 0, $Q_l$, and total energy absorbed by the media, $Q_m$, were presented. For both scattering phase functions, all five models give fairly close results. The two-flux models tend to slightly overpredict $Q_m$, by about 6%. The discrete ordinate method did not converge for the highly forward scatter phase function, MVII, because of the inadequate number of ordinate directions used in this study, M = 24.

The following conclusions may be made based on the above results. Comparisons have been made with published results for conditions as close as possible to those expected for receiver/reactors, i.e., absorbing and scattering medium, forward scattering phase function, and moderate optical thickness. The comparisons show that all three solution methods adequately predict surface heat transfer and overall volumetric heat absorption, with the two-flux method perhaps slightly less accurate. It should be noted, however, that the two-flux code is about three orders of magnitude faster than either the discrete ordinate or Monte Carlo methods.

## CODE COMPARISONS FOR PROBLEMS OF INTEREST

As discussed in previous sections, numerous comparisons of RTE solution methods appear in the literature. Many comparisons of interest to receiver/reactor modeling are not available in the literature, however. For this reason, we compared the three codes described in the previous section for several configurations specific to the receiver/reactor problem.

The first such comparison sought to extend the slab-geometry comparison presented earlier to other scattering phase functions. Specifically, the codes were compared for an isotropic, LAS ($a_1$ = 1) and for phase function MVI and MVII, with single scattering albedo of 0.5 and 0.8 and optical thickness of 1.0 and 2.0. All three methods gave good agreement (for all phase functions except MVII where the discrete ordinate code did not converge) for $Q_l$; the two-flux results for $Q_m$ were about 8% high relative to the discrete ordinate and Monte Carlo codes. For scattering phase function MVII, the Monte Carlo and two-flux codes were compared on the basis of $Q_l$, $Q_m$, and $Q_o$. The comparisons were again very close with the two-flux code overpredicting $Q_m$ by 6% relative to the Monte Carlo code.

For a more detailed look at the performance of the two codes for this problem, Fig. 3 shows the local dimensionless flux divergence as calculated by the two-flux and Monte Carlo codes. The runs were for optical thickness 3.0, single scattering albedo 0.5, and the MVII scattering phase function. This comparison is perhaps the most important from the standpoint of receiver/reactor applications because it is this local absorption of solar energy that drives the chemical reaction and therefore determines reactor performance. Figure 3 shows that the two methods agree within about 10% over the entire range of z and would therefore be expected to give approximately the same reactor performance.

These comparisons of the codes for a wide variety of scattering phase functions show that viability of these codes does not depend on this problem parameter, i.e., strong forward scattering does not preclude the use of either the two-flux or Monte Carlo methods. The discrete ordinate method did not converge for the strongly forward-scattering phase function.

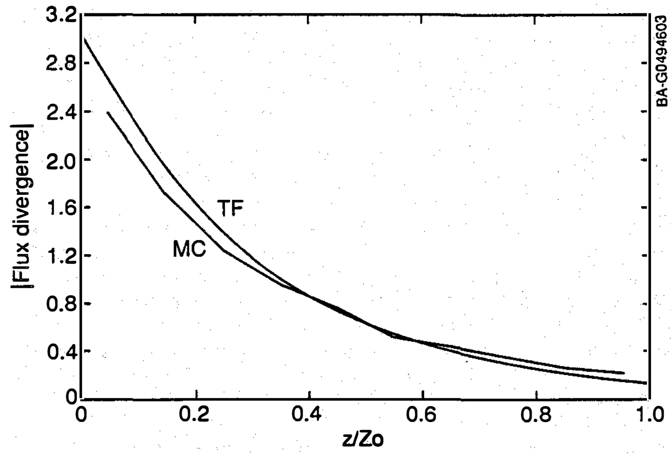


Fig. 3. Local flux divergence for two-flux (TF) and Monte Carlo (MC) codes, optical thickness = 3.0, albedo = 0.5, MVII phase function, isotropic intensity distribution at z = 0.

The next comparisons sought to add more realism to the problem parameters by more nearly modeling the boundary conditions to be expected with a receiver/reactor. Specifically, the intensity distribution at z = 0 for a dish-type collector is expected to be isotropic within a cone half angle of about 45° and zero outside this cone, see Fig. 2. For the two-flux method, this cone angle does not appear as a parameter in the solution. Therefore, the two-flux method is not sensitive to the distribution of intensity at a boundary. Calculations were performed for isotropic, LAS ($a_1$ = 1), MVI and MVII phase functions, single scattering albedo 0.5, and optical thickness of 1.0 to 3.0. Comparisons of $Q_l$, $Q_m$, and $Q_o'$, are given in Table 1.

The data presented in Table 1 show that the two-flux model grossly overpredicts the flux divergence at z = 0 compared with the discrete ordinate and Monte Carlo models; the discrepancy is in the range of 48% to 63%. The two-flux model also overpredicts the overall absorption of energy, $Q_m$, by as much as 35%. Figure 4 shows this behavior in more detail. For the same conditions as those used to produce Fig. 3, except that for the Monte Carlo code the intensity at z = 0 was confined to a 45° half angle, the graph shows that the two-flux model overpredicts the flux divergence at z = 0 and underpredicts it at z = $Z_0$.

To help clarify this behavior, Fig. 5 shows the effect of incident intensity cone angle, $\theta_0$, on flux divergence at z = 0 for the three methods. An optical thickness of 2.0, single scatter albedo of 0.5, and LAS ($a_1$ = 1) were used. As discussed previously, the two-flux method is not sensitive to $\theta_0$ and gives a dimensionless flux divergence of -2.135.

The discrete ordinate method gives two values of flux divergence depending on whether cos $\theta_0$ is greater or less than 0.29. This is related to the direction set chosen for the discrete ordinate method. For M = 24 as used in this work, there are 12 directions in the positive z direction. However, for these 12 directions, there are only two unique angles with the z axis with cosines of 0.2959 (8 each) and 0.9082 (4 each). To impose an isotropic boundary condition at z = 0 requires that intensities in all 12 directions be set to unity. If $0 \leq \cos\theta_0 < 0.2959$, all 12 directions will be also be set to unity. If $0.2959 \leq \cos\theta_0 < 0.9082$, only the four directions with cosine of 0.9082 will be set to unity; the remaining eight intensities will be set to zero. Finally, if $\cos\theta_0 \geq 0.9082$, no solution is possible because none of the directions lie within the specified cone. The two values produced by the discrete ordinate method in Fig. 5 (-1.270 and -2.097) correspond to these two ranges of $\theta_0$.

Table 1. Comparison of two-flux (TF), discrete ordinate (DO), and Monte Carlo (MC) methods for one-dimensional slab problem, single scatter albedo = 0.5 and specified boundary intensity at z = 0 consisting of isotropic distribution within a half cone angle of 45° and zero outside this cone.

| Scattering Phase Optical | Optical Thickness | | ========Method======== TF | DO | MC |
|---|---|---|---|---|---|
| isotropic | 1 | $Q_i$ | 0.8383 | 0.8859 | 0.8868 |
| | | $Q_m$ | 0.6022 | 0.4807 | 0.4917 |
| | | $-Q_0'$ | 1.1212 | 0.6690 | 0.7560 |
| | 2 | $Q_i$ | 0.8290 | 0.8775 | 0.8777 |
| | | $Q_m$ | 0.7729 | 0.7184 | 0.7291 |
| | | $-Q_0'$ | 2.1874 | 1.3400 | 1.4220 |
| | 3 | $Q_i$ | 0.8285 | 0.8771 | 0.8703 |
| | | $Q_m$ | 0.8176 | 0.8153 | 0.8085 |
| | | $-Q_0'$ | 3.1787 | 1.9880 | 2.0500 |
| LAS ($a_1$=1) | 1 | $Q_i$ | 0.8707 | 0.9266 | 0.9390 |
| | | $Q_m$ | 0.6093 | 0.4859 | 0.5080 |
| | | $Q_0'$ | 1.0924 | 0.6350 | 0.6720 |
| | 2 | $Q_i$ | 0.8617 | 0.9261 | 0.9299 |
| | | $Q_m$ | 0.7933 | 0.7379 | 0.7472 |
| | | $Q_0'$ | 2.1354 | 1.2700 | 1.3700 |
| | 3 | $Q_i$ | 0.8611 | 0.9264 | 0.9272 |
| | | $Q_m$ | 0.8453 | 0.8475 | 0.8510 |
| | | $Q_0'$ | 3.1093 | 1.8900 | 2.0320 |
| MVI | 1 | $Q_i$ | 0.8791 | 0.9101 | 0.9181 |
| | | $Q_m$ | 0.8703 | 0.9036 | 0.9088 |
| | | $Q_0'$ | 1.0849 | 0.6420 | 0.6860 |
| | 2 | $Q_i$ | 0.8703 | 0.9036 | 0.9088 |
| | | $Q_m$ | 0.7984 | 0.7219 | 0.7320 |
| | | $Q_0'$ | 2.1216 | 1.2850 | 1.3440 |
| | 3 | $Q_i$ | 0.8696 | 0.9033 | 0.9031 |
| | | $Q_m$ | 0.8524 | 0.8281 | 0.8325 |
| | | $Q_0'$ | 3.0907 | 1.9090 | 2.0300 |
| MVII | 1 | $Q_i$ | 0.9692 | * | 0.9817 |
| | | $Q_m$ | 0.6275 | * | 0.4637 |
| | | $Q_0'$ | 1.0040 | * | 0.6440 |
| | 2 | $Q_i$ | 0.9656 | * | 0.9785 |
| | | $Q_m$ | 0.8494 | * | 0.7267 |
| | | $Q_0'$ | 1.9636 | * | 1.2120 |
| | | $Q_i$ | 0.9651 | * | 0.9805 |
| | | $Q_m$ | 0.9271 | * | 0.8531 |
| | | $Q_0'$ | 2.8740 | * | 1.7840 |

*discrete ordinate code did not converge

In contrast to the two-flux and discrete ordinate methods, the Monte Carlo method produces a continuous variation in the flux divergence as $\theta_0$ is increased. At cos $\theta_0$ = 0, all three models agree reasonably well (TF = -2.135, DO = -2.097, MC = -1.993). At cos $\theta_0$ = 0.8, the Monte Carlo and discrete ordinate methods agree closely, mainly by happenstance. At cos $\theta_0$ = 0.7071 we see as before that the two-flux method overpredicts the flux divergence by about 56% relative to the Monte Carlo and discrete ordinate methods, which agree fairly closely.

The reason for this behavior can be thought of in terms of the paths taken by bundles of energy. Bundles that originate in a direction more nearly parallel to the z axis are more likely to reach z = $Z_0$ than are bundles that originate at a large angle to the z axis. The latter are more likely to be absorbed by the media, especially near the surface z = 0.

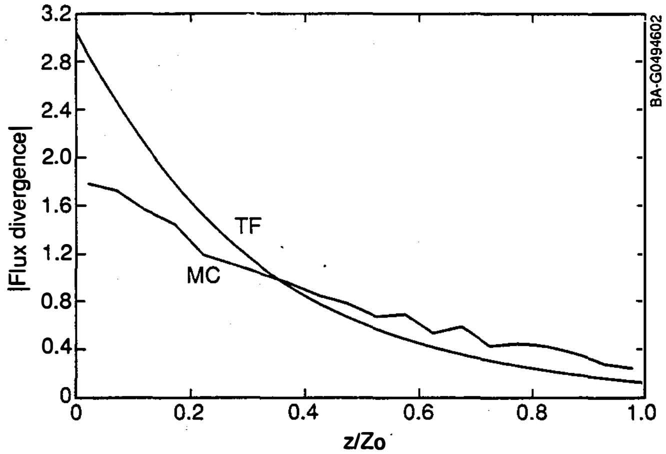


Fig. 4. Local flux divergence for two-flux (TF) and Monte Carlo (MC) codes, same conditions as in Fig. 2 except intensity distribution at z = 0 is not isotropic.

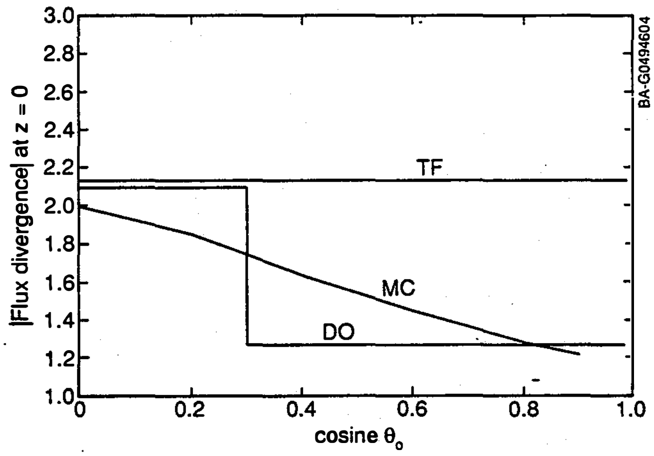


Fig. 5. Flux divergence at z = 0 for two-flux (TF), discrete ordinate (DO), and Monte Carlo (MC) codes as a function of half angle containing intensity at z = 0. Optical thickness = 2.0, albedo = 0.5, LAS $a_1$ = 1 phase function.

For this reason, the more collimated is the incident solar beam, the less will be the local absorption of energy near z = 0. Because the two-flux method cannot account for this behavior, it predicts the results as if the incident beam were isotropic ($\theta_0$ = 90°), significantly overpredicting the flux divergence when $\theta_0$ = 45°. Like the two-flux method, the discrete ordinate method deals with a finite number of directions and exhibits poor resolution in terms of the effect of $\theta_0$, albeit slightly better than the two-flux method. The number of directions M could be increased for the discrete ordinate method to improve this resolution. However, our experience indicates that this would give the Monte Carlo method a clear advantage over the discrete ordinate method in terms of execution speed in addition to the advantage of better resolution with respect to $\theta_0$.

Figure 6 shows that this behavior is not sensitive to single scatter albedo. For a cone half angle of 45°, optical thickness of 3.0, and scattering phase function MVI, Fig. 6 shows the flux divergence at z = 0 plotted as a function of single scatter albedo for all three models. The two-flux model consistently overpredicts the flux divergence, by 48% for large albedo and 61% for small albedo relative to the Monte Carlo solution. The discrete ordinate model and the Monte Carlo model agree to within about 5% over the range of albedo tested.

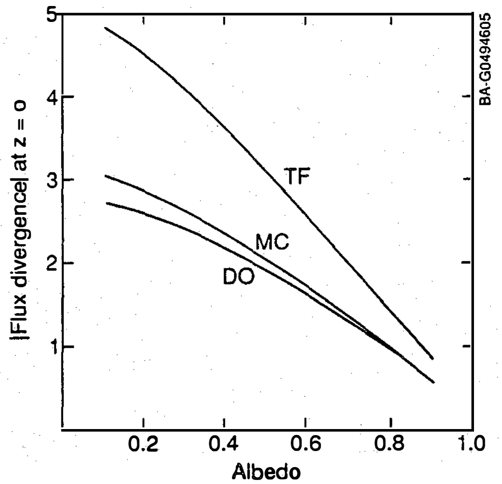


Fig. 6. Flux divergence at $z = 0$ for two-flux (TF), discrete ordinate (DO), and Monte Carlo (MC) codes as a function of single scattering albedo. Optical thickness = 3.0, intensity at $z = 0$ confined to cone of half angle = 45°, MVI phase function.

If the absorption of solar energy near $z = 0$ is significantly overpredicted, one would expect the reaction rate, which typically increases exponentially with temperature, to be significantly overpredicted there also. For a purely thermal receiver, used to heat a gas for example, overpredicting volumetric heat absorption would lead to an overprediction of local absorber temperatures near $z = 0$, overstate reradiation losses, and prematurely indicate the potential for absorber damage.

The behavior described above typically has not been covered in the literature. This is because almost without exception, radiant transfer problems have dealt with specified surface temperature boundary conditions that give isotropic surface intensity distributions rather than specified nonisotropic intensity distributions as found in concentrated solar energy applications. One exception is the work of Daniels et al. [1979] in which the absorption of (nonconcentrated) solar flux in a horizontal layer of water was studied. In that work, the boundary condition consisted of the collimated (direct normal) solar beam plus a diffuse component. They found that the two-flux model significantly overpredicted volumetric absorption near the upper surface of the layer similar to the effect seen in this work. These results, in addition to experimental data [Daniels et al. 1978], led Incropera and Houf [1979] to develop a three-flux model to circumvent these problems.

This suggests that a three-flux approach may alleviate some of the problems found with the two-flux method in the present work. One flux would be contained within a cone half angle from zero to $\theta_0$, i.e., the cone containing the incident beam; a second flux would be contained from $\theta_0$ to 90°; and the third flux would be contained in the back hemisphere. Although it is not certain that this approach would work, the exceptional simplicity and numerical efficiency of the two-flux method relative to the other two methods suggest that the three-flux approach or other methods to improve the two-flux method should be investigated.

The final sequence of comparison runs involved a geometry more closely related to a receiver/reactor than the one-dimensional slab or infinite cylinder described previously. The number of receiver/reactor configurations currently under consideration indicates that several possible geometries would be worth investigating. Here we have chosen to work with one reasonably simple geometry that appears to have application to several receiver/reactor configurations. This geometry is a right circular cylinder with radius-to-length ratio of 6:1. Optical properties are albedo of 0.5, optical thickness of 3.0, scattering phase function MVI, and specified intensity distribution at one cylinder face. Such a cylinder could represent the reaction zone of the methane reforming reactor or the preheat section of the hazardous vapor destruction reactor as seen in Fig. 1.

Due to excessive computer memory requirements, the discrete ordinate model could not be used to solve this problem, except for a very coarse radial and axial grid. Therefore, discrete ordinate results will not be presented.

Although the two-flux method applies strictly to one-dimensional problems, Skocypec et al. [1988] extended the method to axisymmetric problems. Applying their technique to the present geometry, we divide the cylinder into concentric rings and assume each ring is one-dimensional. A single two-flux calculation is performed for the optical properties specified and for the length of the cylinder. The incident flux distribution over the face of the cylinder is partitioned into the concentric rings, and the solution for each ring is then the two-flux solution scaled by the flux specified for the ring. It appears that such a procedure, which assumes each ring is independent, would neglect the radial component gradient term in Eq. 1. One purpose of the comparison of this approach with the Monte Carlo method is to determine if this term can be neglected.

Three boundary conditions were explored for the geometry. For the first boundary condition, denoted BCI, the intensity at $z = 0$ was assumed to be uniformly distributed over the cylinder face and isotropic. This boundary condition is equivalent to the one-dimensional problems discussed earlier where the cone half angle, $\theta_0$, is 90°.

The second boundary condition, BCII, was similar to the first except that a radial distribution of intensity was assumed, simulating the actual performance at the focal plane of a dish collector and providing a test of the validity of dropping the radial gradient term in Eq. 1.

The third boundary condition, BCIII, used the same radial distribution as the second boundary condition but added the realism of a cone of half angle, $\theta_0$, of approximately 45°. Input to the Monte Carlo code for this third boundary condition was calculated with a dish analysis code [Balch et al. 1989]. This code uses a Monte Carlo technique to determine the distribution and direction of bundles crossing the focal plane of a dish with given dimensions and properties. A 10-m diameter, glass/metal dish with focal length of 6 m was assumed. Gaussian slope, specularity, and sun shape errors of 1.5, 1.5, and 2.7 mr half angle, respectively, were assumed. In the focal plane the bundles were found to be contained within a radius of 0.1264 m with a root mean square radius of 0.0433 m. The cosine of the angle with the z axis ranged from 0.6976 to 0.9999, e.g. very nearly a cone half angle of 45°. Consistent with the range of bundle radii, the cylinder radius was assumed to be 0.127 m and the specified radius-to-length ratio of 6 gave a cylinder length of 0.021 m.

Results are presented in Table 2 in terms of the percent difference between the local flux divergence determined by the two-flux and Monte Carlo codes. A positive difference indicates the two-flux code overpredicted the volumetric heat absorption relative to the Monte Carlo code. The data are presented in an array, one for each boundary condition. The node nearest to $r = z = 0$ is the lower left corner of the array with radius increasing vertically upward and axial distance increasing to the right in each table. Only the first 10 axial nodes are shown because at this depth, the volumetric absorption has decreased to approximately 25% of the $z = 0$ value, so percent differences are not very meaningful. For BCII and BCIII, nodes near $r = R_0$ also exhibit small volumetric absorption, so large percent differences at large radii are not significant.

The results of Table 2 can be summarized as follows. For boundary condition BCI (Table 2a), with uniform radial and isotropic intensity, the two methods agree within roughly

**Table 2.** Comparison of the two-flux and Monte Carlo methods for a cylindrical absorber with albedo of 0.5, optical thickness of 3.0, scattering phase function MVI, and for three boundary conditions at z = 0, BCI--Table 2a, BCII--Table 2b, BCIII--Table 2c. The tables give percent differences between the local flux divergence calculated by the two methods at each radial and axial node.

**Table 2a.**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 12 | 23 | 38 | 42 | 45 | 41 | 46 | 50 | 40 | 30 |
| | 10 | 11 | 11 | 13 | 15 | 16 | 10 | 11 | -7 | 9 |
| | 3 | 11 | 12 | 11 | 11 | 11 | 9 | 2 | 1 | -6 |
| | 1 | 10 | 4 | 8 | 8 | 11 | 8 | 6 | -4 | -11 |
| | 3 | 11 | 13 | 14 | 5 | 6 | 2 | 0 | 3 | -9 |
| | 5 | 17 | 7 | 12 | 15 | 8 | 5 | -6 | -4 | -2 |
| | 5 | 19 | 12 | 4 | 22 | 6 | 1 | 14 | -1 | -10 |
| | 4 | 15 | 15 | 6 | 9 | 8 | 5 | -7 | -6 | -15 |
| | 9 | 13 | 2 | 10 | 9 | 9 | 9 | -1 | -12 | 4 |
| | 4 | 12 | 16 | 5 | -5 | -4 | -4 | 10 | -9 | -21 |
| | 7 | 10 | 22 | 6 | 12 | 18 | 15 | -4 | -0 | -15 |
| | 6 | 24 | 9 | 14 | 7 | 11 | 17 | 5 | 2 | -15 |
| | 15 | 20 | 5 | 9 | 20 | 19 | -8 | -7 | -13 | -13 |
| ↑ | 4 | 14 | -5 | -5 | 15 | -20 | -11 | 14 | -9 | -34 |
| r | 7 | 27 | 26 | -3 | 21 | 18 | 21 | -9 | 43 | -12 |
| z→ | | | | | | | | | | |

**Table 2b.**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 53 | -21 | -26 | 115 | -65 | -27 | 20 | - | 57 | -33 |
| | -17 | 18 | -23 | 33 | 65 | -26 | -16 | -31 | 51 | -53 |
| | 29 | -2 | -33 | -22 | -25 | 29 | -1 | -32 | 25 | 65 |
| | 26 | 21 | -1 | 9 | -11 | -10 | -20 | 6 | -23 | -43 |
| | -10 | 17 | 19 | 12 | -8 | -28 | -17 | -26 | -29 | -25 |
| | -0 | -4 | 4 | -8 | -8 | -16 | -20 | -5 | -31 | -35 |
| | 9 | 18 | 22 | 16 | 4 | -7 | -9 | -5 | -13 | 7 |
| | 1 | 14 | 10 | 7 | 3 | 2 | -4 | -10 | -15 | -13 |
| | 8 | 11 | 21 | 13 | 10 | 1 | 8 | -11 | -6 | -10 |
| | 4 | 13 | 8 | 14 | 7 | 10 | 3 | -4 | 4 | -6 |
| | 4 | 14 | 12 | 12 | 10 | 8 | -2 | -1 | 7 | -3 |
| | 4 | 7 | 12 | 11 | 11 | 9 | 3 | 5 | 8 | -1 |
| | 7 | 12 | 12 | 6 | 10 | 11 | 10 | -2 | 1 | 2 |
| ↑ | 3 | 19 | 19 | 17 | 17 | 21 | 8 | 14 | 7 | -5 |
| r | 12 | 9 | 5 | 20 | 8 | 9 | -9 | -5 | -2 | -2 |
| z→ | | | | | | | | | | |

**Table 2c.**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 53 | -42 | 31 | -28 | 17 | 45 | -60 | -2 | 57 | -56 |
| | 54 | 11 | -19 | 51 | -10 | 16 | -16 | 83 | -25 | -38 |
| | 19 | 23 | 47 | 21 | 15 | -14 | 35 | -28 | -28 | -8 |
| | 29 | 21 | 15 | -3 | -16 | -12 | -26 | -13 | -43 | -13 |
| | 59 | 9 | 1 | -10 | -4 | -23 | -12 | -24 | -22 | -21 |
| | 29 | 26 | 30 | -2 | 6 | 3 | -10 | -21 | -19 | -25 |
| | 39 | 27 | 12 | 14 | 9 | 0 | -18 | -24 | -21 | -34 |
| | 43 | 32 | 38 | 11 | 2 | -8 | -19 | -15 | -19 | -35 |
| | 49 | 32 | 23 | 6 | -3 | -5 | -5 | -19 | -22 | -22 |
| | 52 | 31 | 19 | 11 | 5 | -2 | -12 | -14 | -18 | -24 |
| | 47 | 29 | 21 | 14 | 9 | -2 | -5 | -11 | -14 | -26 |
| | 47 | 31 | 21 | 14 | 11 | -0 | -10 | -10 | -14 | -20 |
| | 56 | 44 | 23 | 13 | 9 | -0 | -6 | -13 | -16 | -19 |
| ↑ | 54 | 41 | 21 | 21 | 7 | 4 | -13 | -12 | -24 | -25 |
| r | 50 | 34 | 31 | 20 | 9 | 4 | 1 | -18 | -7 | -16 |
| z→ | | | | | | | | | | |

10%. This is consistent with Fig. 3, the equivalent one-dimensional problem. For BCII with isotropic intensity but with a radial distribution (Table 2b), agreement is still about 10%. Thus, for the properties and geometry tested, it seems permissible to drop the radial gradient term in Eq. 1, i.e., make the independent-ring assumption for the two-flux model. For BCIII, with nonisotropic intensity and radial distribution (Table 2a), the two-flux model overpredicts the volumetric absorption by about 50%. This is consistent with Fig. 4, the equivalent one-dimensional problem. In other words, it is only the presence of nonisotropic specified intensity at z = 0 that creates problems for the two-flux model for this receiver/reactor absorber.

## CONCLUSIONS

Three methods for solving the radiative transport equation have been compared for conditions typical for solar thermal receiver/reactors. Two-flux, discrete ordinate, and Monte Carlo methods were validated against published solutions for simple geometries and then compared against each other for one-dimensional problems and two-dimensional axisymmetric problems similar to a dish-mounted receiver/reactor.

The two-flux method is by far the most computationally efficient method, generally executing three orders of magnitude faster than the other two methods. In addition it is simple, easily implemented, and adaptable to two-dimensional axisymmetric problems, and it gave reasonably accurate results for all conditions with the exception of one important boundary condition. The method does not accurately predict volumetric absorption of radiant energy if the incident intensity is nonisotropic. For conditions typical of a dish collector the method overpredicts volumetric absorption by about 50% near the front absorber face. This is expected to lead to large errors in local chemical reaction rate calculations. Significant modifications and extensions of the two-flux method would be needed to overcome this shortcoming and also to apply it to more complex receiver geometries.

The discrete ordinate method also suffers when the incident intensity is not isotropic. This can be overcome by using a large number of ordinate directions, but the method will then be significantly slower than the Monte Carlo method. For isotropic boundary intensity, $\theta_0 = 90°$, and for $\theta_0 = 45°$, typical of dish collectors, the method compared very closely with the Monte Carlo method and can be used with confidence for these two conditions. However, execution time is comparable to the Monte Carlo code, and memory requirements are much greater. A large number of ordinate directions are needed to accommodate other values of $\theta_0$ and also to accommodate strong forward scattering. The requirement for many ordinate directions leads to an even greater memory requirement. The method is complex, especially in the two-dimensional axisymmetric configuration, so code development, maintenance, and debugging are difficult.

The Monte Carlo method proved to be reliable and accurate under all conditions tested and provides the analyst with an excellent physical appreciation of the problem. In addition the nature of the method simplifies development, testing, and debugging of the computer code used to implement the method. Relative to the two-flux method, the Monte Carlo method is much slower. This would be important if the Monte Carlo code were to be used for both the solar and infrared wavelength spectra in a full receiver/reactor model. Although the two-flux model cannot be recommended for calculating the local absorption of solar energy, it appears that it is satisfactory for solving for the infrared transport where the boundary intensities are isotropic. This is fortunate because, although determination of the solar absorption is only required once for a receiver/reactor model, solving for infrared transport must be carried out iteratively or simultaneously with the other conservation equations, a process that is awkward and inefficient with the Monte Carlo method. Thus, the Monte Carlo method can be used to accurately determine the local absorption of solar energy, and the two-flux method can be used with confidence to determine the infrared transport simultaneously with the other conservation equation algorithms needed in the full receiver/reactor model.

The above considerations strictly apply only to the axisymmetric cylindrical geometry considered in detail in this paper. Further comparisons should be performed if alternate receiver/reactor geometries are considered.

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