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MASTER

ANALYTICAL MODELING OF LINE FOCUS SOLAR COLLECTORS

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ABSTRACT

Solar thermal electric power generation systems and industrial process heat systems generating steam through flash vaporization require a constant outlet temperature from the collector field. This constant temperature is most efficiently maintained by adjusting the circulating fluid flow rate. Successful design of analog controllers for this regulation requi res knowledge of system dynamics .and the nonlinear nature of the system. parameters. Simplified models relating deviations in outlet temperature to changes in inlet temperature, insolation, and fluid flow rate illustrate the basic responses and the distributedparameter nature of line focus collectors. Detailed models are used to develop transfer functions and frequency response curves useful for design.

INTRODUCTION

Several potential applications for line focus solar devices require that the outlet temperature of the collector field be controlled. For example, organic Rankine-cycle engines require a constant inlet Systems circulating pressurized water through the collector field and then flash vaporizing the water to generate steam need to maintain an outlet temperature high enough to produce steam; yet they must also prevent temperature excursions that would lead to boiling in the receiver tubes $(2,3)$.

Collector field outlet temperature may be regulated either through conventional feedback analog control or by more sophisticated digital schemes. Analog control does not have the flexibility inherent in digital methods, but it is attractive because of its simplicity, durability, familiarity, and relatively low cost. Therefore, illustrated in this paper are the basic characteristics of solar system dynamics. In addition, equations are derived describing frequency response that may be used in the design of analog controllers.

The simplest equation for describing the performance of a line focus collector is

$$
q = \dot{m}C_p(T_o - T_i) = \text{IMW}, \qquad (1)
$$

where q = heat rate, \hat{n} = flow rate, C_p = heat capacity, $(T_0 - T_i)$ = the difference between the outlet and inlet temperatures, I = direct insolation, $n =$ overall collector efficiency, $W =$ aperture width, and $L =$ collector length. To hold the outlet temperature constant through the daily variations in insolation

(solar radiation), we must manipulate either the coolant flow rate or the inlet temperature. The inlet temperature may be adjusted by having a portion of the flow bypass the process interface and then returning it to the main flow to achieve the desired collector inlet temperature. However, adjustments to the inlet temperature do not affect the outlet temperature until the fluid has passed completely through the collector. Also, because the inlet temperature is increased during periods of low 1nsolation, the average receiver temperature increases and thermal losses become greater. Changes in flow
rate produce immediate changes in outlet produce immediate temperature. Decreasing the flow rate during periods of low. insolation lowers the heat transfer coefficient between the fluid and receiver wall, thus increasing the thermal losses; however, this effect is not as great as the effect of an increase in inlet temperature. For this reason, the remainder of this paper concerns temperature control by flow rate manipulation (see Fig. l).

Fig. 1. Control of a line focus collector by flow rate manipulation.

Recause a line focus collector is both a nonlinear and distributed parameter system, field tuning of a controller under a given set of conditions may not provide good response at other conditions. Therefore, models of collector transient and frequency response are developed to aid in controller tuning.

STEADY-STATE MODELING

To design controllers, we must have a knowledge of the process gain. Where the manipulated variable is the flow rate and the controlled variable is the outlet temperature, Eq. (1) may be solved for $(T_0 T_t$) and differentiated with respect to flow rate, yielding the steady-state process gain K_n :

$$
K_p = \frac{d(T_0 - T_1)}{d\hat{\pi}} = -\frac{(T_0 - T_1)}{\hat{\pi}} \qquad (2)
$$

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The process nonlinearity is immediately apparent. If the inlet and outlet temperatures are fixed, the gain will be greatest during low-flow, low-insolation periods, such as morning and evening. A concentrator may operate at insolations ranging from 250 to 1000 watts/ m^2 ; therefore, the rain may vary by a factor greater than four. The gain may vary by a factor greater than four. gain may vary over a greater range than the insolation because of the decrease in collector thermal efficiencies at low insolation and because optical efficiency varies with the position of the sun; A more detailed analysis that accounts for the changes in collector efficiency with temperature and flow rate shows that the predictions of gain by this simplified model can be up to 20% too high for
typical line focus collectors. In this paper, typical line focus collectors. however, Eq. (2) is used to predict gain because it gives a rapid and conservative estimate of process gain.

SIMPLIFIED DYNAMIC MODELING

A simplified energy balance around the receiver tube of a line focus collector illustrates system dynamics:

$$
\left(\rho C_p \lambda\right)_{12} \frac{\partial T}{\partial t} = -v \left(\rho C_p \lambda\right)_1 \frac{\partial T}{\partial x} + I W n \quad , \tag{3}
$$

where $(\rho C_n A)_1$ is the thermal mass of a unit length of fluid, $(\rho C_n A)_{12}$ is the thermal mass per unit length of the flufd and receiver tube, v is the circulating fluid velocity, t is time, and x is the distance down. the receiver tube,

After normalizing and subtracting the steadystate components, we obtain the following equation in deviation variables:

$$
\frac{\partial Y}{\partial \zeta} = - p \frac{\partial Y}{\partial z} + \frac{H}{\tau} , \qquad (4)
$$

where Y' is the normalized deviation of the temperature from steady state, p is the ratio $(\rho C_p \Lambda) / (\rho C_p \Lambda)_{12}$, H' is the normalized deviation in insolation, ζ and z are the normalized time and length, and T is the ratio of the capacitance to the solar energy gain. The deviation variabies represent small pertubations about the steady-state solution. In this small region the equation is linearized to allow the use of Laplace transform techniques. If we take the Laplace transfer with respect to time, solve the resulting linear first-order ordinary differential equation, and evaluate the solution at the inlet and outlet, we obtain a transfer function relating deviations in inlet temperature, outlet temperature, and insolation:

$$
\overline{Y}_{o} = \frac{\overline{H}}{\tau s} \left(1 - e^{-s/p} \right) + \overline{Y}_{1} e^{-s/p} \quad , \tag{5}
$$

where the variables with a bar over them are Laplacetransformed deviation variables and s is the Laplace variable,

To determine the transfer function relating fluid velocity and outlet temperature, we again begin
with Eq. (3). Considering temperature and velocity Considering temperature and velocity to be the variables, we obtain after normalization:

$$
\frac{\partial Y}{\partial \zeta} = - \text{ fp } \frac{\partial Y}{\partial z} + \frac{1}{\tau} \quad , \tag{6}
$$

where f is the normalized fluid velocity. The temperature and velocity terms are broken into steadystate and deviation components. are dropped, and the equation is separated into Second-order terms

steady-state and deviation equations:

$$
0 = - \tau p \frac{\partial Y_g}{\partial z} + 1 \tag{7}
$$

and

$$
\frac{\partial Y}{\partial \zeta} = - \tau p \frac{\partial Y}{\partial z} - f' \tau p \frac{\partial Y}{\partial z} , \qquad (8)
$$

where the subscript s refers to steady-state values and the prime mark refers to a deviation variable. The steady-state temperature profile from Eq. (7) is substituted into Eq. (8), and the resulting equation is Laplace transformed with respect to time. After integration with respect to z over the receiver length, we have the transfer function:

$$
\overline{Y}_0 = \frac{\overline{f}}{\tau_s} \left(e^{-s/p} - 1 \right) + \overline{Y}_1 e^{-s/p} \quad . \tag{9}
$$

An understanding of the dynamic response is gained by looking at the response of the collector outlet temperature to step increases in inlet temperature, insolation, and fluid velocity (Fig. 2). For the idealized case, where the thermal mass of the receiver wall is negligible $(p = 1)$, the physical significance of the solutions is easily determined. 8ecause this model neglects the variation of efficiency with temperature and velocity, and, we assume no wall effects, a step change in inlet conditions at

Fig. 2. Dynamic response of a line focus collector to stepchange inputs in insolation, velocity, and inlet temperature.

i; •, 0, propagates through· the receiver· unchanged and emerges one residence time later (at $\zeta = 1$). A step increase in insolation is a distributed input, and lts effect is immediately apparent throughout the receiver length. The fluid exiting the tube $at \zeta = 0$, $z = 1$, is unaffected by the increase in insolation, while the fluid entering the receiver at $\zeta = 0$, z • 0, feels the full effect of the new conditions. Therefore, we expect the output to be a ramp function between the old and new steady-state solutions, with a duration of one residence time and a slope of $1/\tau$. Inclusion of wall heat capacity (p < 1) simply decreases the slope by the factor p and.increases the ramp length by 1/p. The step response to velocity changes may be understood. in a **similar** manner.

The frequency response is derived easily from the transfer functions. For inlet temperature forcing, we obtain the amplitude ratio AR and phase angle θ of a pure delay as a function of the frequency ω :

$$
AR = 1, \text{ and } \theta = \omega \tau_A / p \qquad (10)
$$

For both insolation and velocity forcing, the frequency response is

AR =
$$
\frac{1}{\tau \omega}
$$
 $\sqrt{2 - 2 \cos(\omega/p)}$ (11)

and

$$
9 = -90^{\circ} + \tan^{-1}\left(\frac{\sin(\omega/p)}{1 - \cos(\omega/p)}\right)
$$
 (12)

(see Fig. 3).

From these expressions we can see the major features that distinguish the frequency response of line focus collectors: a resonance superimposed on a lag. The resonance phenomenon arises from the distributed nature of the process and forcing functions and is most easily visualized by considering the history of parcels of fluid moving through a receiver tube with no thermal capacitance in the wall. For insolation forcing when the forcing frequency is a multiple of 2π rad/t_d, the fluid passing through the tube is exposed to above-average insolation for half its life and to below-average insolation the other half. The affects cancel and the amplitude ratio has a $minimum$. If the frequency is a multiple of 3π rad/ τ_d , some fluid elements spend two-thirds of their life in above-average insolation while others spend two-thirds of their life in below-average insolation. The effects tend to reinforce each other and peaks are found in the amplitude ratio. Inclusion of the thermal mass of the wall in the model simply shifts the peak and minimum frequencies to lower frequencies by a constant multiple p.

It is interesting to note that not only are the frequency response curves for insolation and velocity forcing the same, but for any given collector the dimcnsionless frequency response curves are identical for all operating conditions. This is because **pis a** constant for any given collector, and the value of T must always be equal' to $1/p$.

OETAILED DYNAMIC MODELING

In the design of a feedback controller, a more complete nodel of collector dynamics is used (4). By assuming that fluid properties are constant, axial conduction is unimportant, and there are no radial temperature gradients within either the fluid or re-

ceiver tube, we can write the following normalized **equations** in **deviation variables:**

$$
\frac{\partial Y_1}{\partial \zeta} = -\frac{\partial Y_1}{\partial z} + \frac{1}{T_{211}} (Y_2 - Y_1)
$$
 (13)

and

$$
\frac{\partial Y_2^i}{\partial \zeta} = -\frac{1}{\tau_{212}} \left(Y_2^i - Y_1^i \right) - \frac{1}{\tau_{322}} \left(Y_2^i - Y_3^i \right) + \frac{H'}{\tau_{\infty}}, \quad (14)
$$

where Y_1 and Y_2 are the normalized temperatures of the fluid and receiver tube, $T_{ijk} =$ the ratio of the

thermal capacitance of region k to the heat transfer potential between regions i and j, and τ_{∞} = wall capacitance/radiation heat transfer, Region l is the fluid, region 2 the receiver wall, and region 3 the surrounding air, Equations (13) and (14) are Laplace transformed with respect to time and comblned to eliminate the wall temperature. Integrating with respect to z from the inlet to outlet yields the transfer functions relating deviations in inlet temperature, ambient temperature, and insolation to deviations in outlet temperature:

$$
\overline{Y}_{1,0} = \overline{Y}_{1,1} e^{-a(s,\tau)} + \overline{Y}_{3} \frac{b(s,\tau)}{a(s,\tau)} \left(1 - e^{-a(s,\tau)}\right)
$$

$$
+ \overline{N} \frac{c(s,\tau)}{a(s,\tau)} \left(1 - e^{-a(s,\tau)}\right) \qquad (15)
$$

To obtain the transfer function relating fluid ·velocity and outlet temperature, we begin with

$$
\frac{\partial \mathbf{Y}_1}{\partial \zeta} = - \mathbf{f} \frac{\partial \mathbf{Y}_1}{\partial z} + \frac{1}{\tau_{211}} (\mathbf{Y}_2 - \mathbf{Y}_1)
$$
 (16)

and

$$
\frac{\partial Y_2}{\partial \zeta} = -\frac{1}{\tau_{212}} \left(Y_2 - Y_1 \right) - \frac{1}{\tau_{322}} \left(Y_2 - Y_3 \right) + \frac{1}{\tau_{\infty}} . \quad (17)
$$

The heat transfer coefficient between the fluid and the inner wall is a function of velocity; it can be expanded in a Taylor series with respect to velocity and the higher order terms may be dropped. The temperature and velocity terms are broken into steadystate and deviation variables. The linearized deviation equations are Laplace transformed with respect to time, and the steady-state equations are solved to provide the steady-state temperature profiles, Finally, the deviation equations are solved simultaneously to eliminate wall temperature. The transfer function thus derived is

$$
\frac{\overline{Y}}{\overline{r}} = \frac{E}{a(s,\tau) - a(o,\tau)} \left[e^{-a(s,\tau)} - e^{-a(o,\tau)} \right] \quad . \tag{18}
$$

The inversion of the detailed transfer functions for even the simple case of a step-change forcing function is time consuming and yields little new insight. Fortunately, the frequency response may be obtained without the inversion.

The frequency response to inlet temperature forcing is that of a modified time delay, The response to insolation forcing is that of a secondorder lag with a superimposed damped sinusoid con-
tributed by the term [1 - e^{-a(s,†)}] (see Fig, 4). The frequency response curves for insolation forcing are useful for determining the ability of a downstream heater to compensate for temperature variations from an uncontrolled collector field,

To design an analog feedback controller we require the frequency response of the open-loop transfer function. For this design we need the frequency response for velocity forcing. This response has the form of a resonant term contributed by $(e^{-a(s,\tau)} - e^{-a(o,\tau)})$ superimposed over a first-order lead and a second-order lag. The frequency response is expressed by the following equations:

$$
AR = \frac{C_1 C_2}{C_3} \left(1 + (\omega/c_2)^2 \right)^{1/2} \left(1 + (\omega/c_3)^2 \right)^{-1/2} \omega^{-1}
$$

Fig, 4, Frequency response of a line focus collector to insolation forcing,

$$
\times \left(\left(-e^{-a(\omega, \tau)} + e^{q(\omega)} \cos[\omega_p(\omega)] \right)^2 + \left(-e^{q(\omega)} \sin[\omega_p(\omega)] \right)^2 \right)^{1/2}
$$
\n(19)

and

$$
\Lambda = -90^{\circ} + \tan^{-1}(\omega/c_2) + \tan^{-1}(-\omega/c_3)
$$

$$
+ \tan^{-1}\left(\frac{-e^{q(\omega)}\sin[\omega p(\omega)]}{-e^{-a(o,\tau)} + e^{q(\omega)}\cos[\omega p(\omega)]}\right).
$$
 (20)

Fig. 5. Frequency response of a line focus collector to velocity forcing.

The detailed dimensionless frequency response curves for velocity and insolation forcing are extremely similar. The low-frequency asymptotes of the amplitude ratio may vary by a few percent while the phase angle curves differ only at high frequencies, where the response would be difficult to detect in any case. Variations in flow rate and insolation also have only minor effects on the dimensionless curves. This is not unexpected in light of the results of the simplified model. Because the dimensionless frequency response to velocity forcing changes very little over the range of operating conditions, it may be possible to relate optimum controller settings for any particular operating conditions directly to the collector gain and the fluid residence time.

CONCLUSIONS

The outlet temperature of a row of line focus collectors may be controlled by manipulating the flow rate. Intelligent design of controllers for such a nonlinear and distributed parameter process requires an understanding of system dynamics. Simple transfer functions, derived from a collector model incorporating only energy transport by the circulating fluid and energy gain due to insolation, may be used to illustrate the nature of the transient and frequency responses. In the design of feedback controllers, a more detailed model is required. Transfer functions relating changes in outlet temperature to changes in fluid flow rate, insolation, and ambient temperature can be derived from a model that includes the interaction between the receiver tube and the circulating fluid, as well as energy gains due to insolation, losses to the surroundings, and transport by the circulating fluid. Inversion of the transfer functions to yield transient response is cumbersome, but the frequency response is more readily obtained. To design a feedback controller we need the frequency response curves relating the manipulated (flow rate) and output (outlet temperature) variables. When plotted in dimensionless form, the curves for different operating conditions are essentially iden-
tical, suggesting that feedback controller settings may be simply related to easily determined quantities such as collector gain and fluid residence time.

NOMENCLATURE

$$
a(s,\tau) = s + \frac{1}{\tau_{211}} - \frac{\tau_{322}}{\tau_{211} (\tau_{212} \tau_{322} s + \tau_{212} + \tau_{322})}
$$

\n
$$
b(s,\tau) = \frac{\tau_{212}}{\tau_{211} (\tau_{212} \tau_{322} s + \tau_{212} + \tau_{322})}
$$

\n
$$
c(s,\tau) = \frac{\tau_{212} \tau_{322}}{\tau_{211} \tau_{\infty} (\tau_{212} \tau_{322} s + \tau_{212} + \tau_{322})}
$$

\n
$$
c_1 = (1 - n) a(o,\tau) \left(Y_{3,0} - Y_{1,0} + \frac{\tau_{322}}{\tau_{\infty}} \right)
$$

\n
$$
c_2 = \frac{(\tau_{212} + \tau_{322}) (1 - n) + n\tau_{322}}{\tau_{212} \tau_{322} (1 - n)}
$$

\n
$$
c_3 = \frac{\tau_{211} (\tau_{212} + \tau_{322})^2 + \tau_{212} \tau_{322}^2}{\tau_{211} \tau_{212} \tau_{322} (\tau_{212} + \tau_{322})}
$$

\n
$$
c_p = heat capacity
$$

\n
$$
E = (1 - n + n \frac{\tau_{211} \tau_{322}}{\tau_{212} \tau_{322} s + \tau_{212} s_{22} s_{21})
$$

 $f = normalized velocity v/v_s$

h_{i.i} = heat transfer coefficient between regions i and j

- $H = normalized insolution = I/I_c$
- I= insolation (direct beam)
- $K = gain$
- $L = collector length$
- $m = mass$.flow rate
- n = exponent describing the velocity
- dependence of h $\mathbf{1}$
- $p = (pC_pA)_1/(pC_pA)_{12}$

$$
p(\omega) = 1 + \frac{\tau_{212} \tau_{322}}{\tau_{211} [(\tau_{212} \tau_{322} \omega)^2 + (\tau_{212} + \tau_{322})^2]}
$$

 $\overline{}$

 q = heat rate

$$
q(w) = -\frac{1}{\tau_{211}} + \frac{\tau_{322}(\tau_{212} + \tau_{322})}{\tau_{211}[(\tau_{212} - \tau_{322}w)^2 + (\tau_{212} + \tau_{322})^2]}
$$

- s Laplace variable
- t = time
- T temperature
- v = velocity
- $W =$ aperture width
- $x =$ distance down collector row
- $Y =$ normalized temperature = T/(T₀ T_i)_s
- $z =$ normalized length = x/L
- n collector efficiency

$$
\tau = \frac{{\binom{p}{\binom{n}{a}}}_{12}}{\frac{1}{s}^{n/L}} \, v_{0} (T_{0} - T_{1})_{g}
$$

 τ_d = fluid residence time

$$
\tau_{1jk} = \frac{(\rho c_{\rho}A)_{k}}{h_{1j}\pi d_{1j}} \frac{v_{s}}{L}
$$
\n
$$
\tau_{\infty} = \frac{(\rho c_{\rho}A)_{2}}{I_{s}(\rho \tau d)TKW} \frac{v_{s}(r_{o} - r_{1})_{s}}{L}
$$

 $(PC_pA)_j$ = thermal mass of section j ζ = normalized time = tv/L *w* = normalized frequency (rad/residence time) $(p \tau \alpha)$ IK = optical efficiency x spillover **x cosine losses**

Superscripts

= **deviation variable** = **Laplace-transformed deviation variable**

Subscripts

- **p** = **process**
- **s steady state**
- 1 fluid in **receiver**
- 2 receiver walls
- 3 ambient air i • inlet
- o outlet

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 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

 $\tilde{\mathcal{E}}$

 $\frac{1}{\sqrt{2}}\sum_{i=1}^{n} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2.$

Quincia de la