Distributed Optimization for Infeasible Combined T&D Networks

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Part of a Bigger Team - CREATE

Combined T&D Interactions are Growing

- Future Grid will rely on approaches that enable joint operation and control of T&D resources

Source: VELCO

Combined T&D Interactions are Growing

- Consider a sunny Spring day in VT

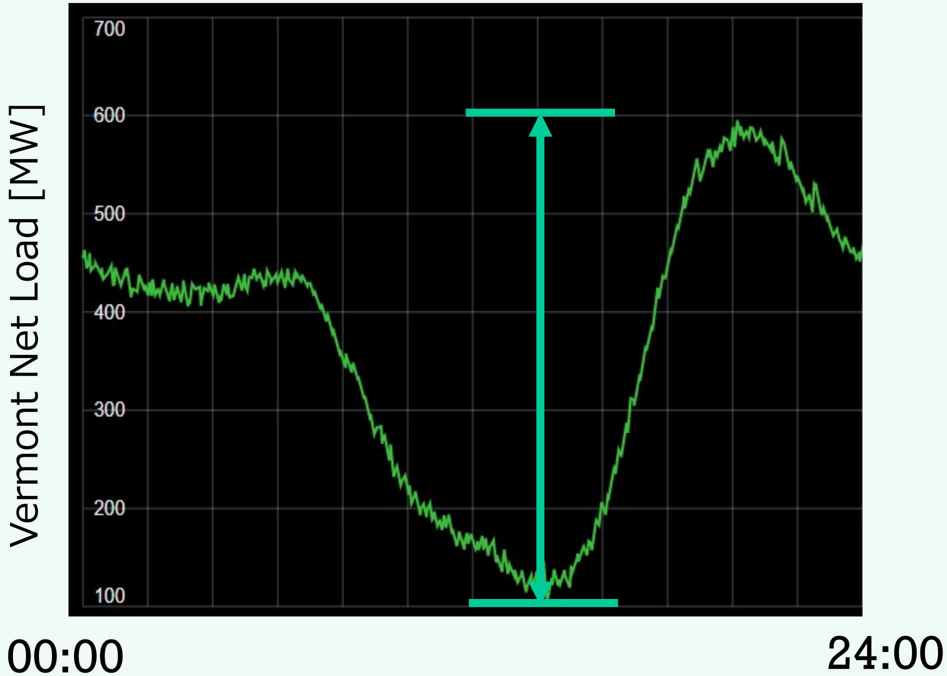


Figure: Load net load in VT during the afternoons on 5/14/2023

Source: VELCO

DER Majority Generation is a New Reality

- VT’s net load was only 100 MW on May’ 23
  - ~80% of net generation from DERs in distribution nets

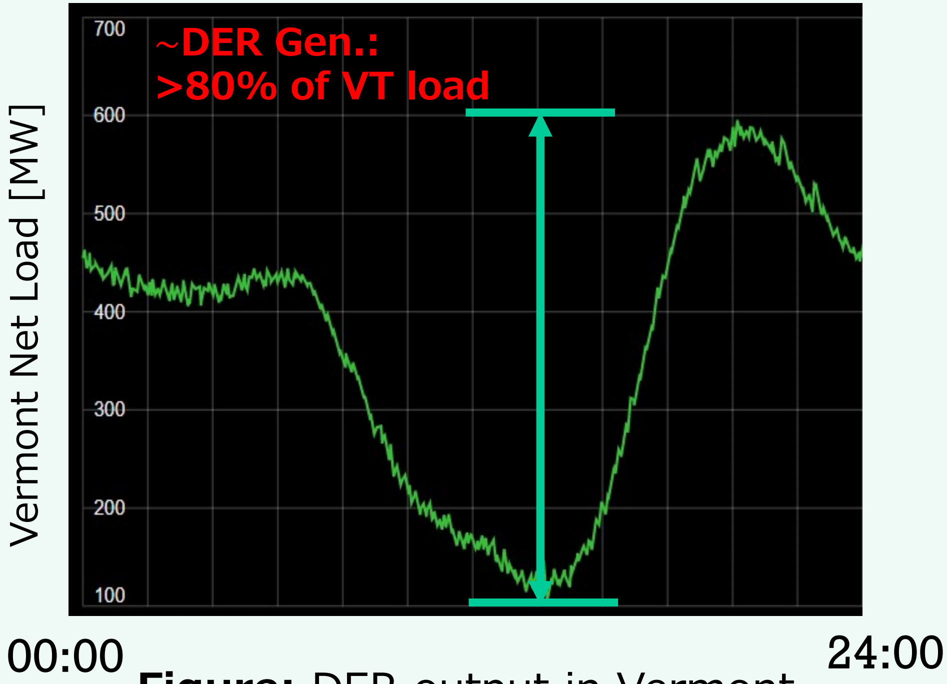
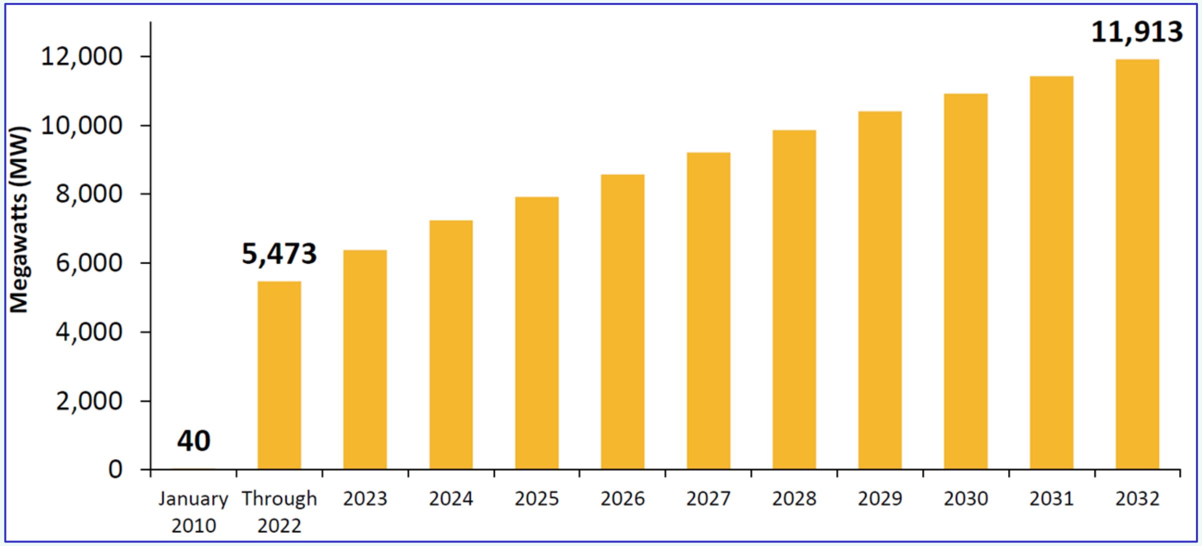


Figure: DER output in Vermont greater than 500 MW.

Source: VELCO

DER Growth in New-England is Accelerating

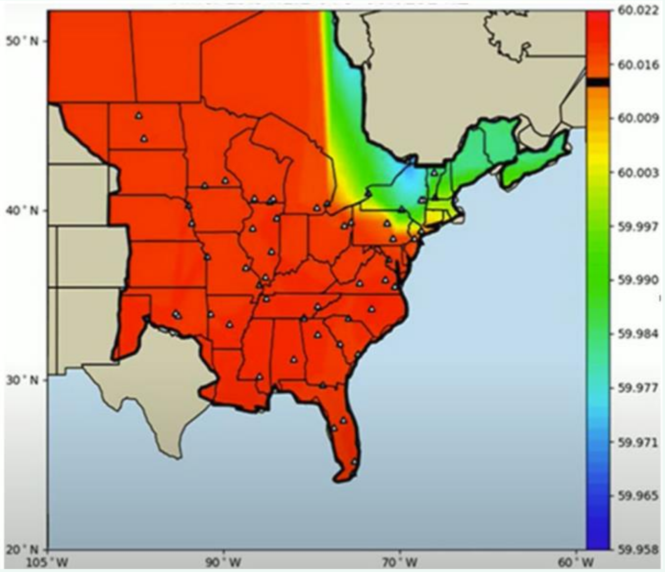


31630 MW is the expected peak summer demand for ISO-NE

Figure: Projected growth in PV within ISO-NE is accelerating.

Source: ISO-new England

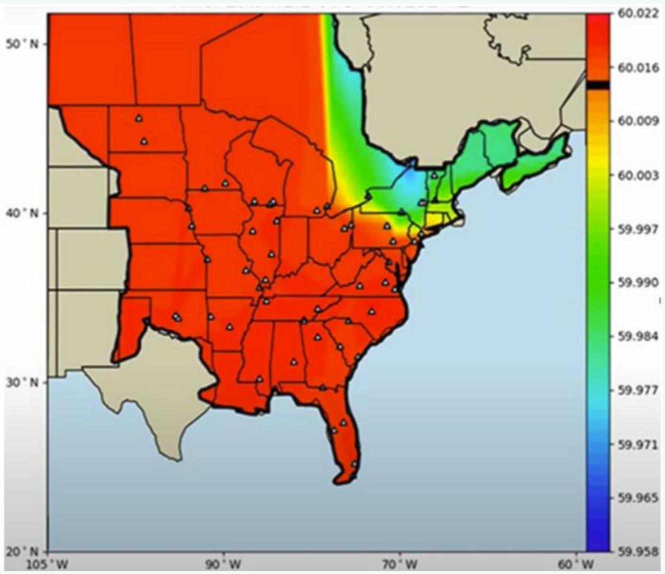
Example: Improved Modeling DER is Critical



Event: Transmission line trip in NY resulted in a disturbance

Source: VELCO

Example: Improved Modeling DER is Critical



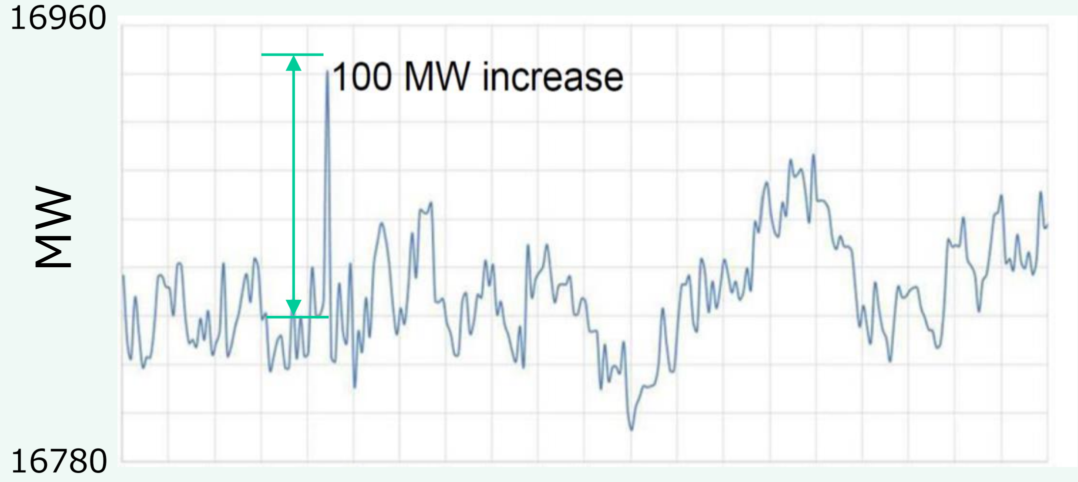


Figure: New England net load July 6,2022.

Event: Transmission line trip in NY resulted in a disturbance

Consequence: 100 MW of DERs (~15% of net VT load) tripped

Source: VELCO

Need: Combined T&D Steady-state Analysis

- To evaluate impact of high-penetration of DERs
  - Emergency operation – similar to NY line trip event
  - Normal operation – asset health when continuous backflow from distribution grids is present
- Initialization of combined T&D dynamic analysis

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Aggregating the resources may not work due to heterogenous type or control of various DER devices

SoA - Steady-state Combined T&D Analysis

- Co-simulation
  - Disparate models for T&D networks
  - Pros: Can mix-n-match various established tools
  - Cons: Lack of convergence, and robustness

Authors	Co-simulation	Co-modeling	Approach
K. Kalsi *et al.*	✓	×	PowerWorld + GridLab-D
K. Anderson *et al.*	✓	×	MATPOWER + GridLab-D

SoA - Steady-state Combined T&D Analysis

- Co-simulation
- Co-modeling
  - Unified models and algorithms for joint T&D networks

Authors	Co-simulation	Co-modeling	Approach
K. Kalsi *et al.*	✓	✗	PowerWorld + GridLab-D
K. Anderson *et al.*	✓	✗	MATPOWER + GridLab-D
Q. Huang *et al.*	✗	✓	Mixed sequence three phase
A. Pandey *et al.*	✗	✓	Equivalent Circuit Formulation

and many more!

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None of these approaches work when the combined T&D network has no solution

Solving Infeasible Combined T&D Nets

- System planners require clear indications regarding why power flow simulations fail
  - Where are the system weaknesses
  - Where to consider adding new assets

Solving Infeasible Combined T&D Nets

- System planners require clear indications regarding why power flow simulations fail

Preliminaries

- For the positive sequence transmission net; concept of missing power[a], introducing slack current[b] using circuit simulation
- For the three-phase distribution network; identifying the weak spots[c], identifying the problematic power flow constraints

[a]T. J. Overbye [2]
[b]M.Jereminov, Pandey, *et al.* [3]
[c]E.foster, Pandey, *et al.* [4]

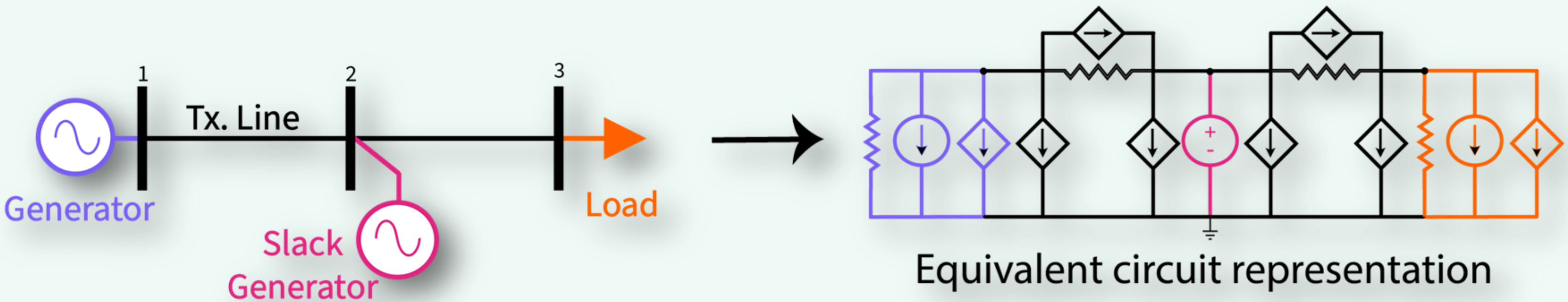
Problem Statement - Infeas T&D Networks

Goal: Model and simulate infeasible large-scale combined positive-sequence T and three-phase D networks within the same solution and modeling framework

Infeasible: A combined T&D network that cannot satisfy power flow network constraints while satisfying device bounds

Model: Equivalent Circuit (Ckt) Formulation

- Insight: Model and analyze combined T&D network as an equivalent circuit[2]
  - KCL: linear network constraints, nonlinear injection models



[2]A. Pandey *et al.* [5]

Combined T&D Modeling – A Ckt Approach

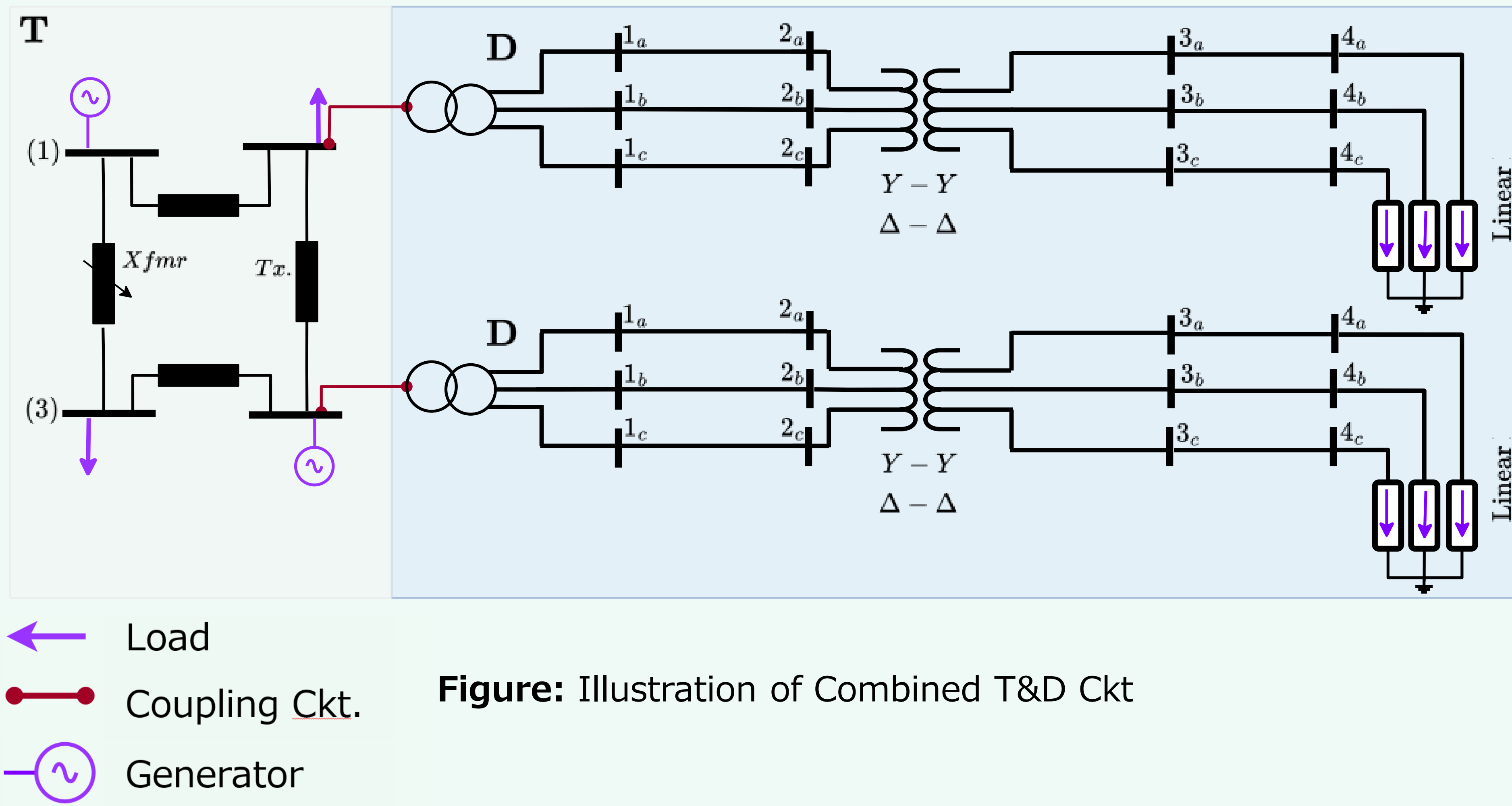


Figure: Illustration of Combined T&D Ckt

Combined T&D Modeling – Coupling Ckt

- Coupling circuits[3] model the interactions between T and D sub-circuits using symmetrical components

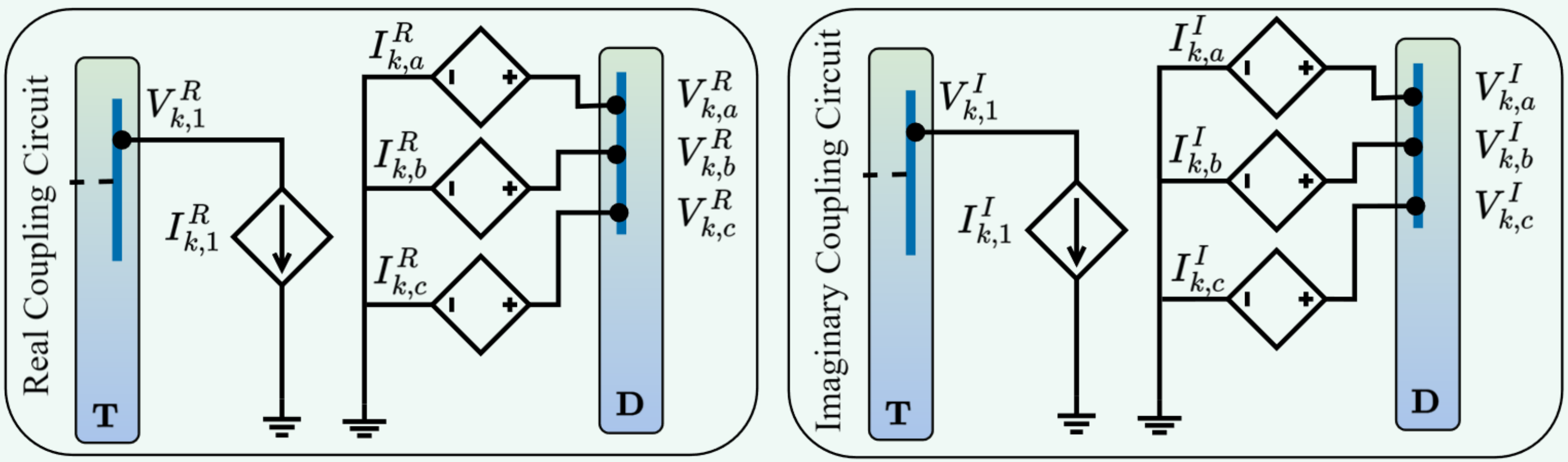
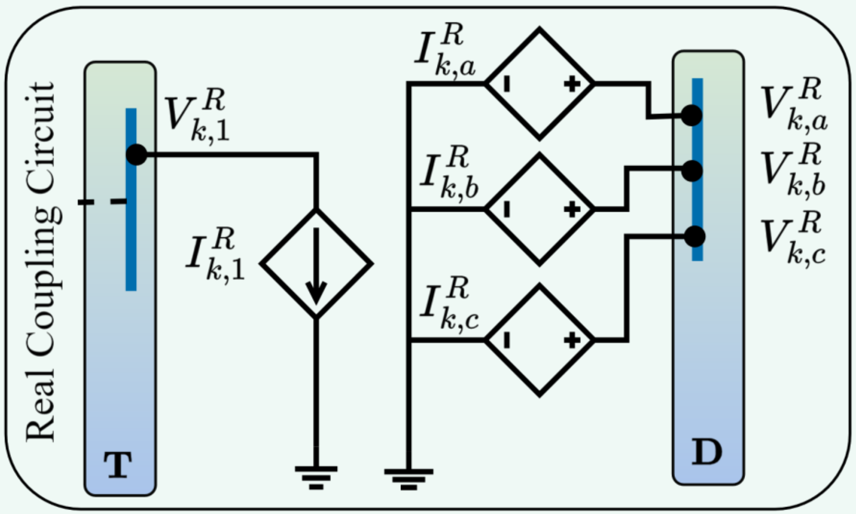


Figure: Coupled T&D layout (coupling circuit).

[3] A. Pandey *et al.* [6]

Combined T&D Modeling – Coupling Ckt

- Real circuit coupling equations (primal setup)



$$V_{k,a}^R = V_{base}\left(V_{k,1}^R\right)$$

$$V_{k,b}^R = V_{base}(\Re(\alpha^2 V_{k,1}^R))$$

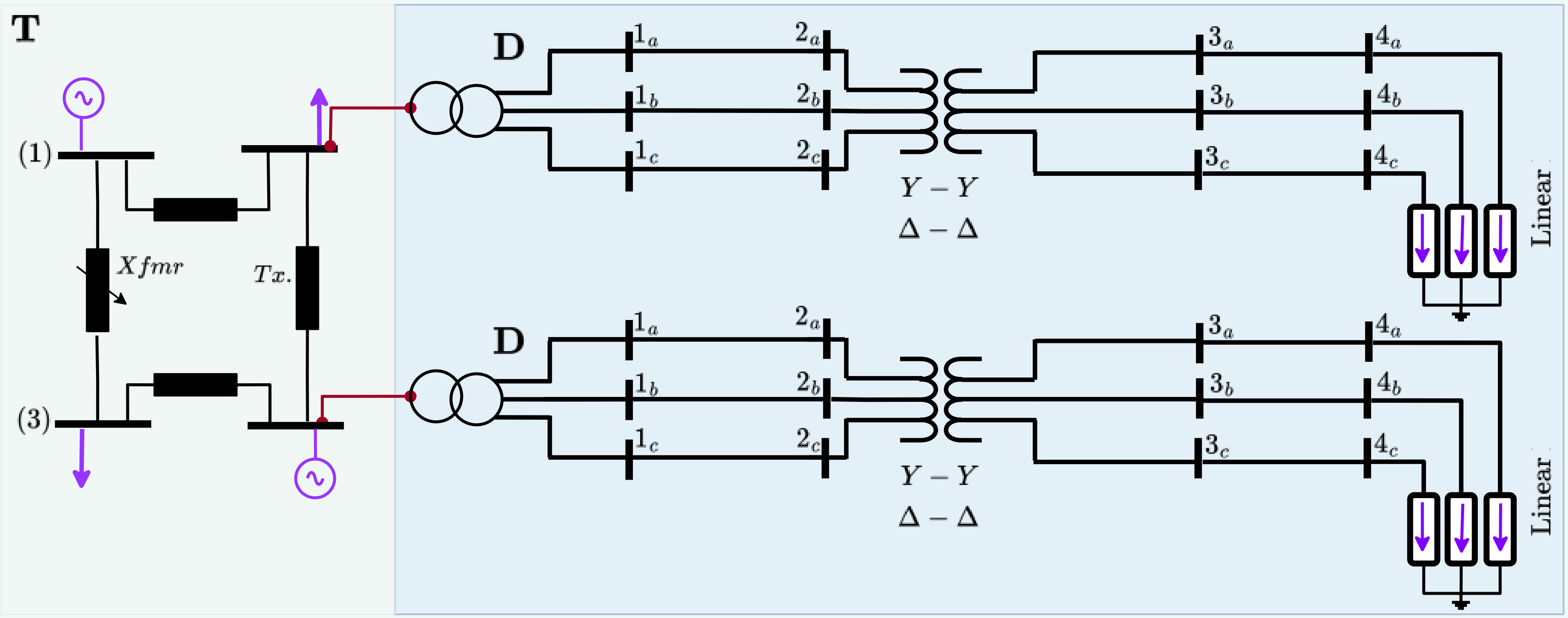
$$V_{k,c}^R = V_{base}(\Re(\alpha V_{k,1}^R))$$

$$I_{k,1}^R = \frac{1}{3t}\Re(I_{k,a}^R + \alpha I_{k,b}^R + \alpha^2 I_{k,c}^R)$$

t: is the normalizing constant
α: is the $2\pi/3$

Combined T&D Solution: A Ckt Approach

- Solving this circuit requires solving the KCL equation at each node and maintaining voltage mag at some



A Circuits Approach: Adding Infeas Sources

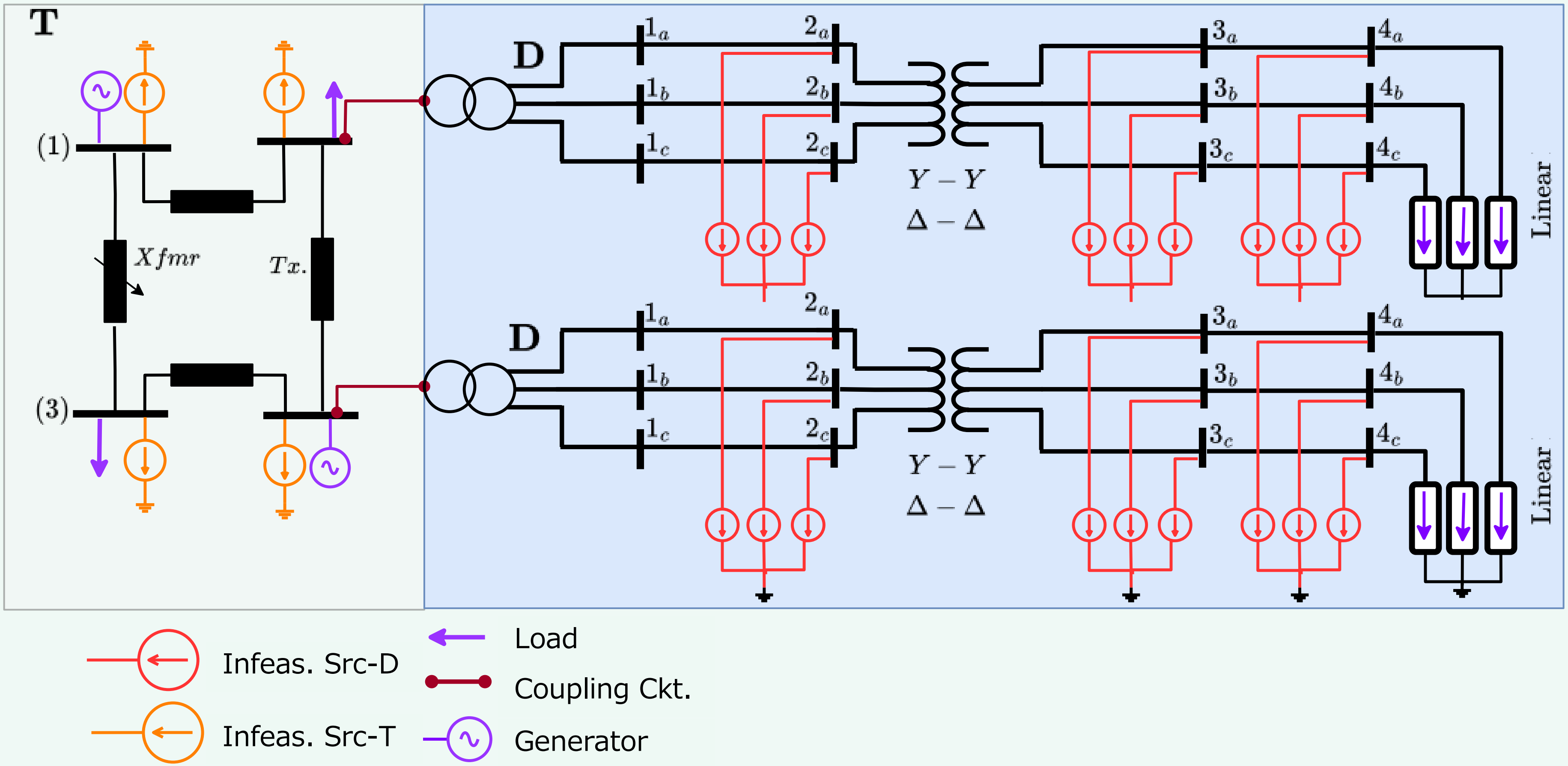


Figure: Illustration of Combined T&D Ckt with Infeasibility Sources

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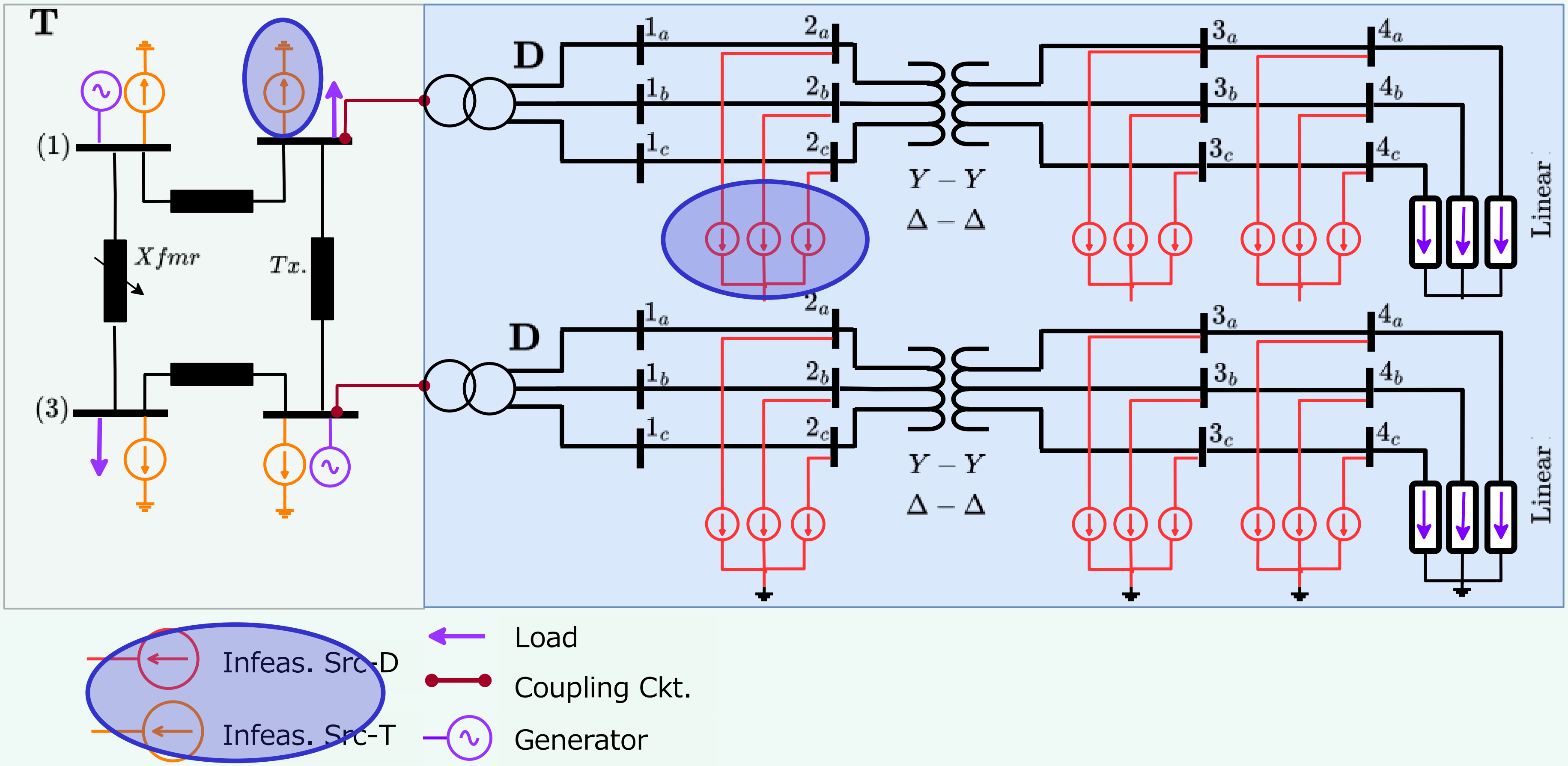


Figure: Illustration of Combined T&D Ckt with Infeasibility Sources

Optimization to Solve Infeasible Networks

- The objective is to minimize the norm of infeasibility currents
  - Feasible network: the objective will be 0; recover the power flow solution from primal variables
  - Infeasible network: the non-zero dual variables localizes the system weaknesses

Choice of norm localizes system weakness differently!

Optimization to Solve Infeasible Networks

$$\min_{X,\mathcal{I}} \sum_{s\in S^T} \|(\mathcal{I}_s^T)\|_p + \sum_{s\in S^D} \sum_{\Omega\in a,b,c} \|(\mathcal{I}_{s,\Omega}^D)\|_p$$ (Minimize infeasibility source)

$s.t.$

$$\mathcal{F}_{s_t}^T(X_{s_t}^T) - \mathcal{I}_{s_t}^T = 0 \quad \forall s_t \in S^T$$ (T&D power flows)

$$\mathcal{F}_{s_d,\Omega}^D(X_{s_d,\Omega}^D) - \mathcal{I}_{s_d,\Omega}^D = 0 \quad \forall \Omega \in \{a,b,c\} \quad \forall s_d \in S^D$$

$$\mathcal{G}_{s_t}^T(X_{s_t}^T) \leq 0 \quad \forall s_t \in S^T$$ (T&D voltage bounds)

$$\mathcal{G}_{s_d,\Omega}^D(X_{s_d,\Omega}^D) \leq 0 \quad \forall \Omega \in \{a,b,c\} \quad \forall s_d \in S^D$$

$$\mathcal{C}_k(X^T, X^D) = 0 \quad \forall k \in K$$ (Coupling constraint)

Methodology – Optimization problem

- The choice of infeasible sources:

$$\mathcal{I} = \begin{cases} I^{R,\text{inf}} + jI^{I,\text{inf}} & \text{if } \mathcal{I} = I^{\text{inf}} \\ (P^{\text{inf}} - jQ^{\text{inf}})/(V^R - jV^I) & \text{if } \mathcal{I} = S^{\text{inf}} \\ (G^{\text{inf}} + jB^{\text{inf}})(V^R + jV^I) & \text{if } \mathcal{I} = Z^{\text{inf}} \end{cases}$$

  - Current infeasibility (I) introduces only linear term
  - Power infeasibility (S) can localize the lack of reactive power implicitly

Methodology – Distributed optimization

- Centralized problem
  - Unable to solve tens of millions of variables under single machine single compute framework
- Distributed problem
  - Preserve privacy between separate T&D utilities
  - Inherent weak coupling between T&D allows natural decomposition

Methodology – Distributed optimization

- Centralized problem
- Distributed problem

In this work, we employ a distributed primal-dual interior point method (PDIP), which follows the framework of the Gauss-Jacobi Newton algorithm

BBD Matrix → Distributed Framework

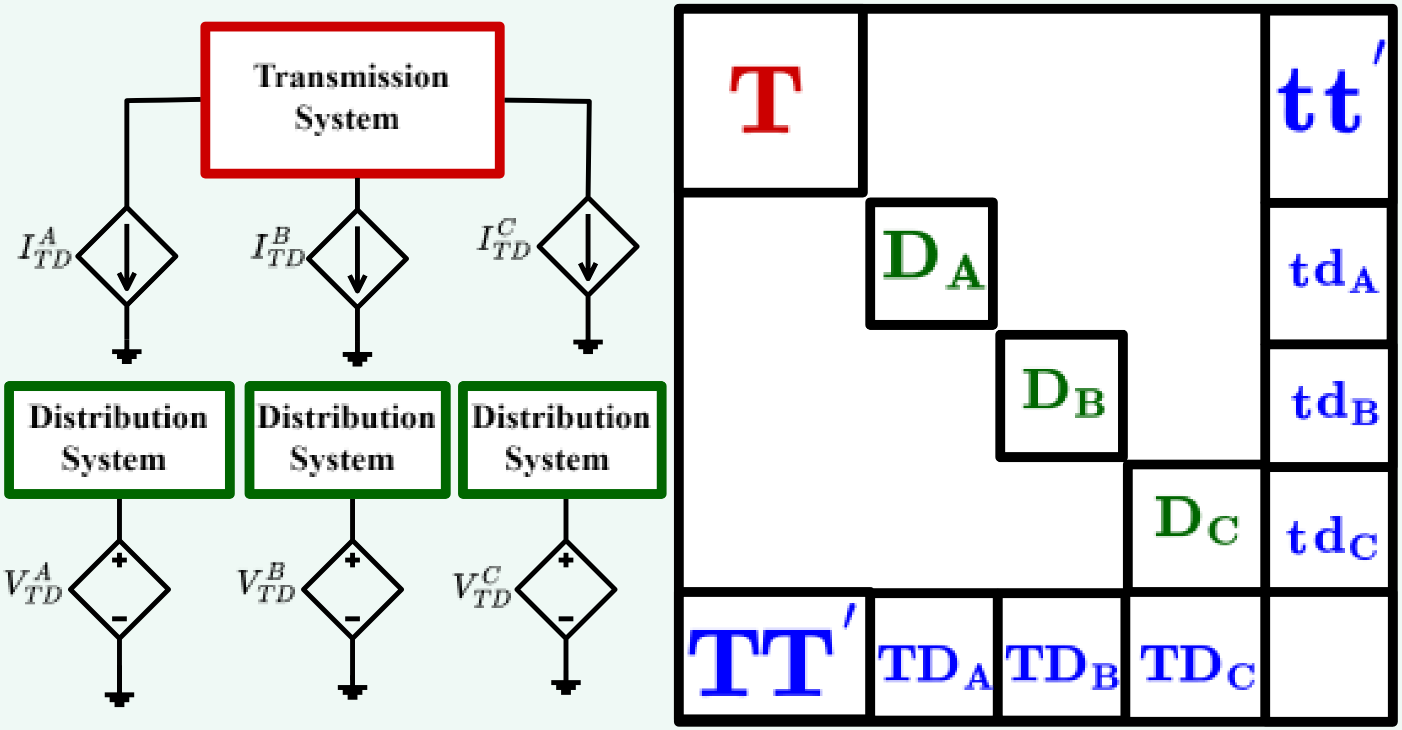
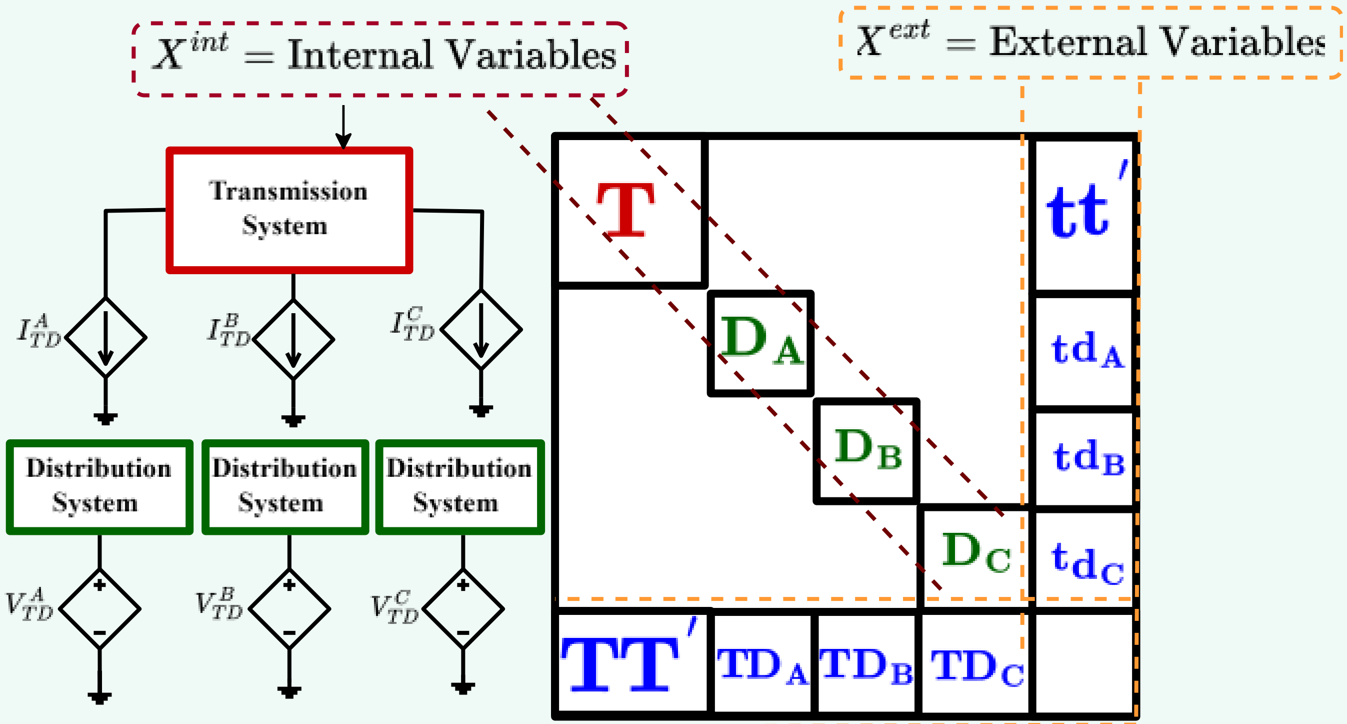


Figure: Coupled T&D layout bordered block diagonal (BBD) structure

BBD Matrix – Int. and Ext. Variables



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Methodology – Distributed PDIP

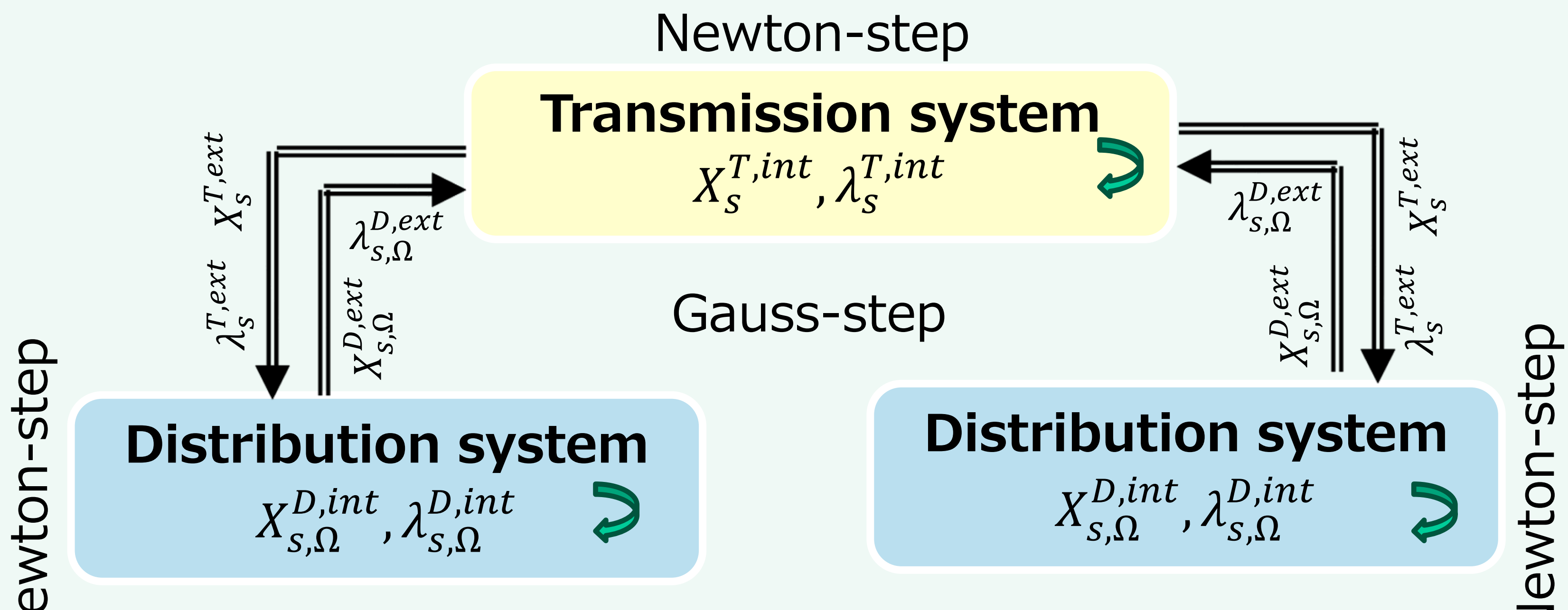
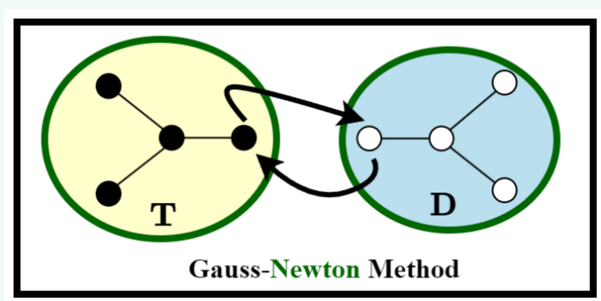


Figure: Distributed primal dual interior point method.



Methodology – Decomposed KKT equations

Newton: Decomposed KKT conditions

$$\forall s \in \{S^T, S^D\}$$

$$\nabla_{\lambda,s}\mathcal{L} = \mathcal{F}_s(X_s^{int}, X_{k,s}^{ext}) - \mathcal{I}_s^{int} = 0$$

$$\nabla_{x,s}\mathcal{L} = \nabla_{x,s}(||\mathcal{I}_s||_p) - \nabla_{x,s}\mathcal{G}_s{}^T \mu_s^{int} + \ \dots$$

$$\nabla_{x,s}(\mathcal{F}_s(X_s^{int}, X_{k,s}^{ext}, \lambda_{k,s}^{ext}) - \mathcal{I}_s^{int})^T \lambda_s^{int} = 0$$

$$-\mu_s^{int}\mathcal{G}_s(X^{int}) + \epsilon = 0$$

$$\mu_s^{int} \geq 0$$

$$\mathcal{G}_s(X^{int}) \leq 0$$

Gauss step: Dual expr. for coupling eqns. (also primal)

$$\begin{bmatrix} \lambda_{k,a}^{R,D} \\ \lambda_{k,a}^{I,D} \\ \lambda_{k,b}^{R,D} \\ \lambda_{k,b}^{I,D} \\ \lambda_{k,c}^{R,D} \\ \lambda_{k,c}^{I,D} \end{bmatrix} = \frac{1}{\kappa} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha^2 & 0 \\ 0 & \alpha^2 \\ \alpha & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} \lambda_k^{R,T} \\ \lambda_k^{I,T} \end{bmatrix}$$

κ is the normalizing constant;
α: is the $2\pi/3$.

Results – Experimental Setup

- Use case: Reactive power compensation
  - Corrective action to improve voltages on transmission net due to increased loading on the distribution nets

Table: Test cases description

Name	T&D test case
Case-118_31	118-bus (T) + GC 12.47.1
Case-2869_31	PEGAS2869 (T) + GC 12.47.1
Case-25k_1420	ACTIVEsg25k (T) + D-net

D-net: Synthetic urban meshed network with 1420 three-phase nodes (4260 single-phase nodes)

Results – ADMM , D-PDIP, C-PDIP

Algorithm	Network	Obj.	Iter.	Time (s)
ADMM	Case-118-31	0.353	71	1.27
D-PDIP	Case-118-31	0.352	28	0.98
ADMM	Case-2869-31	Did not converge		
D-PDIP	Case-2869-31	1.103	32	8.63
ADMM	Case-25k-1420	Did not converge		
D-PDIP	Case-25k-1420	0.109	88	102.04

ADMM runs terminated after 1800 sec.

Solving Real Combined T&D Network

- Combined T&D net with real VT dist. feeder
- Higher electrification → network infeasibility
  - Goal: Localize weak spots

Table: Test cases summary

Name	T&D test case
Case-25k-8.5k	25k (T) + South-HERO (~8.5k 3P)

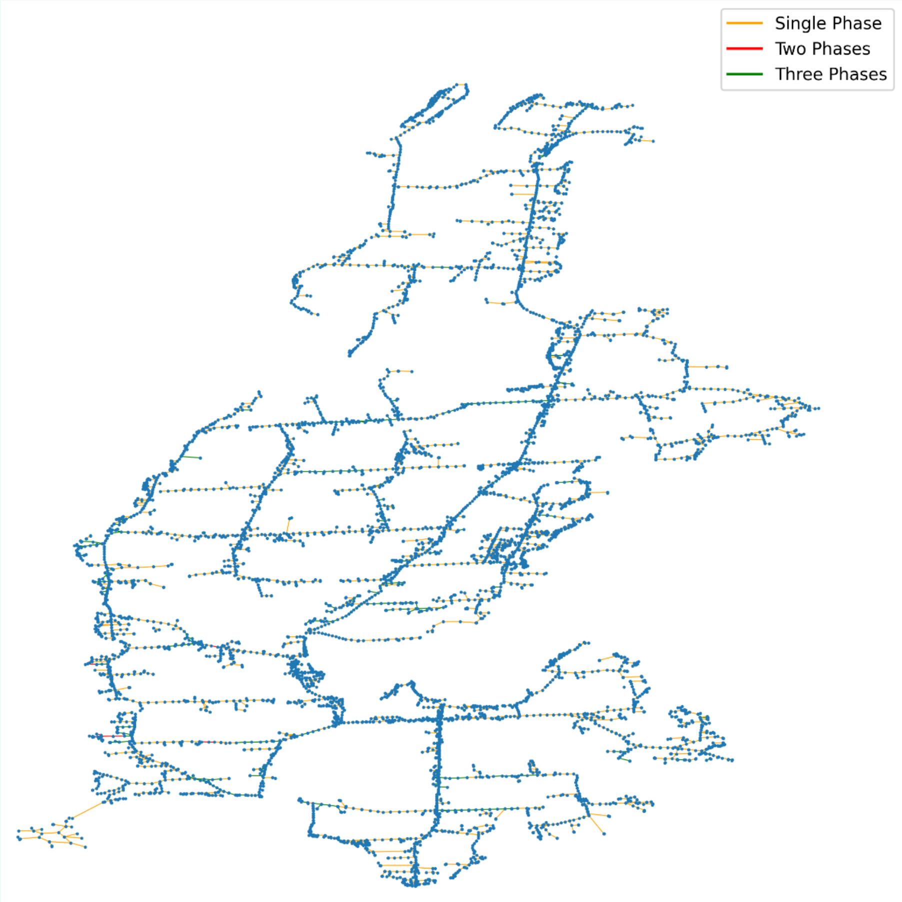
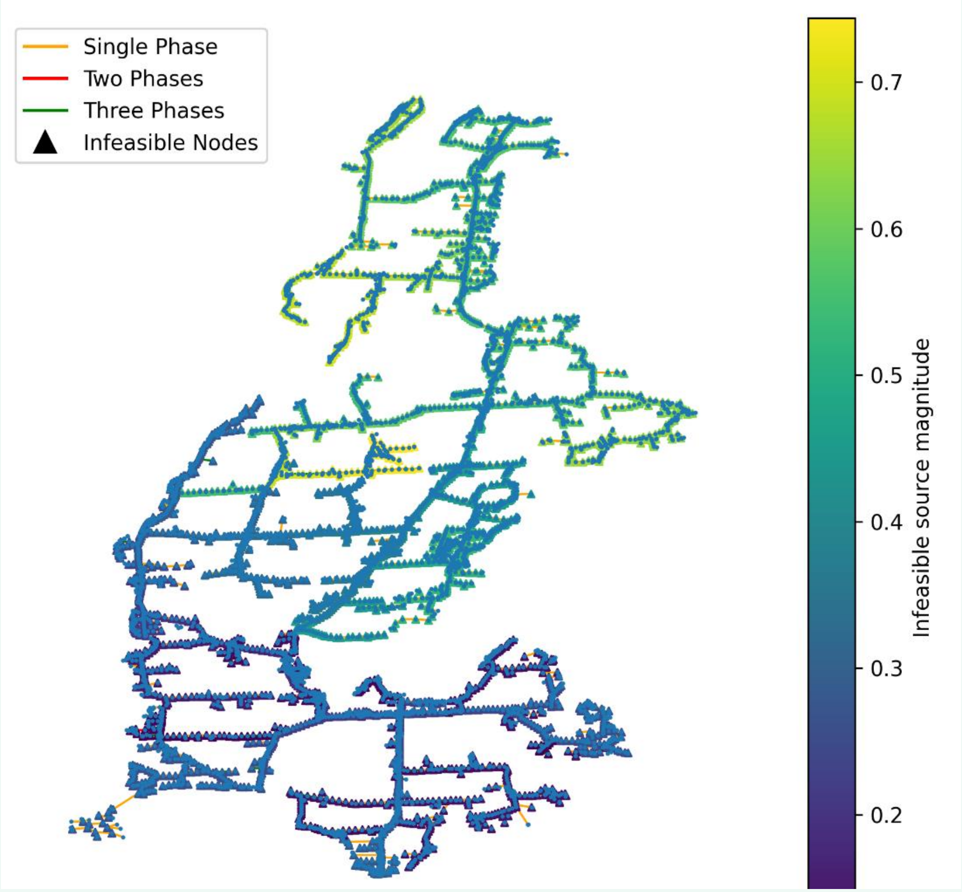


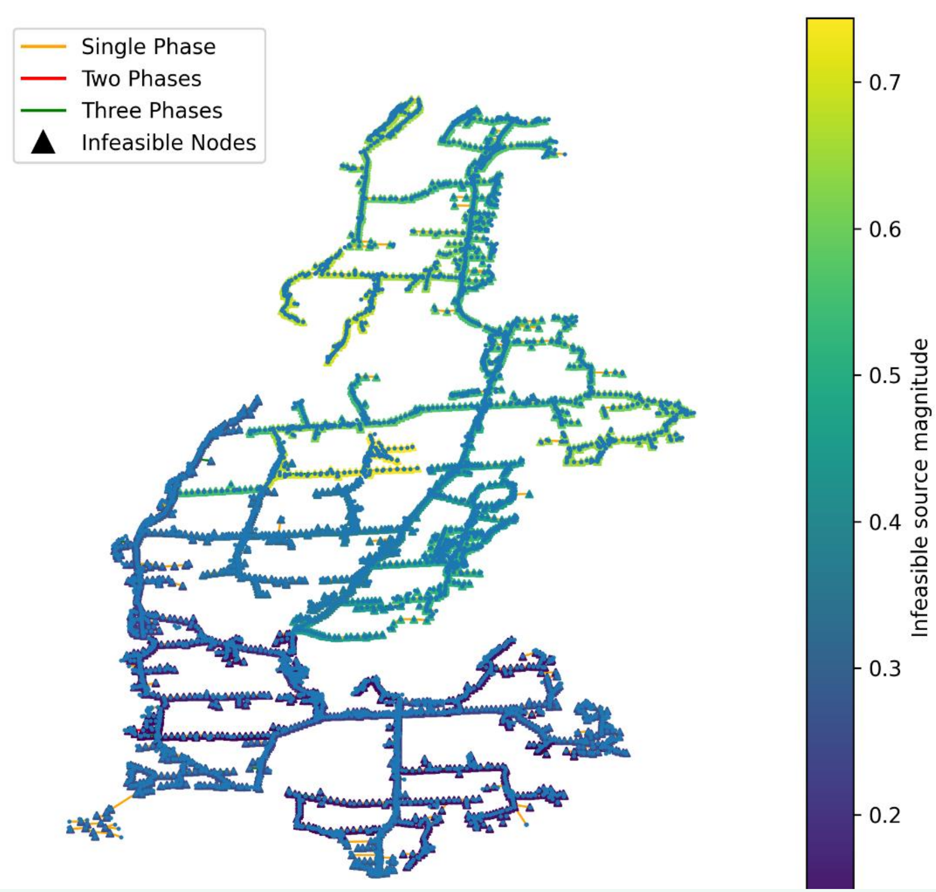
Figure: VT Distribution Feeder

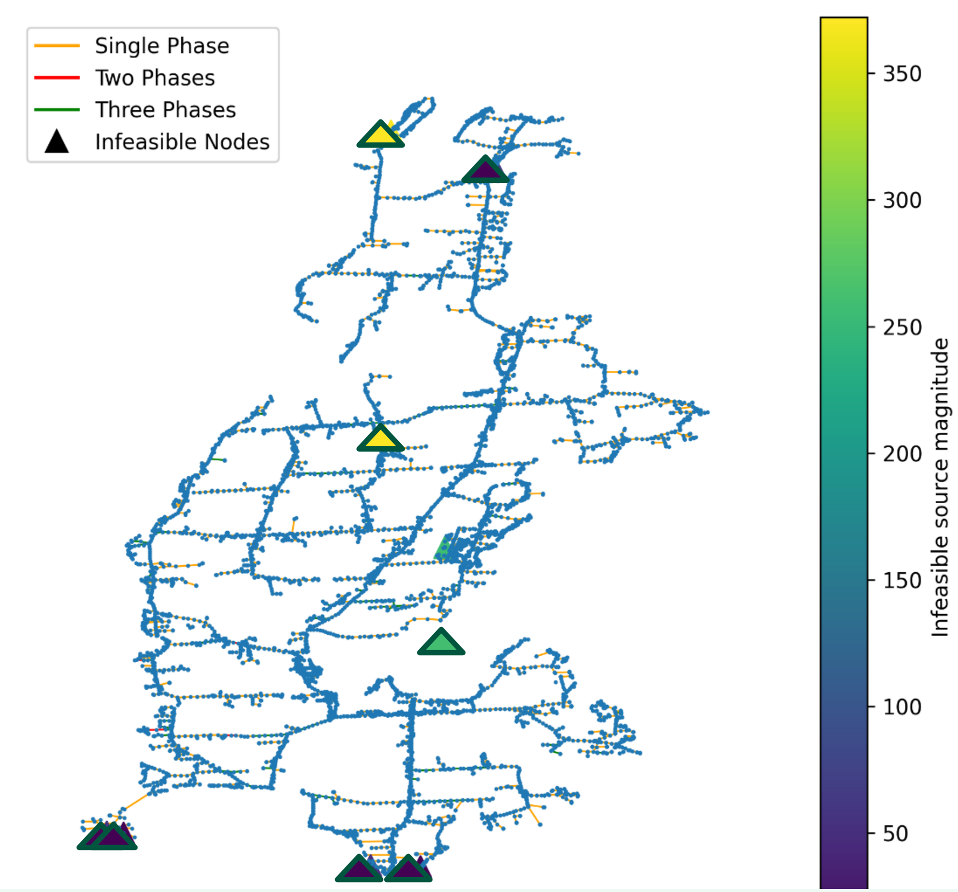
Solving Real Combined T&D Network



With L2 norm, the infeasibility is distributed throughout the system

Solving Real Combined T&D Network





With L2 norm, the infeasibility is distributed throughout the system

With L1 norm, the infeasibility is localized to few locations

Concluding Remarks

- Interactions between T&D nets are growing and grid operators and planners are looking for new tools:
  - Robust: Methods converge with some guarantee
  - Scalable: Can solve real setups with >10e6 variables
  - Generalized: Apply to a variety of problems

Come Visit Us!

Combined T&D – References

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[2] T. J. Overbye, A power flow measure for unsolvable cases," in *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1359-1365, Aug. 1994.

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[7] M. P. Desai and I. N. Hajj, "On the convergence of block relaxation methods for circuit simulation," *IEEE Transactions on Circuits and Systems*, vol. 36, no. 7, pp. 948–958, 1989.